Mathematical Surveys and Monographs

Volume 2

The Theory of Rings

Nathan Jacobson

American Mathematical Society

http://dx.doi.org/10.1090/surv/002

Other Titles in This Series

42 **Ralph Freese, Jaroslav Ježek, and J. B. Nation,** Free lattices, 1995

41 **Hal L. Smith,** Monotone dynamical systems: an introduction to the theory of competitive and cooperative systems, 1995

40.1 **Daniel Gorenstein, Richard Lyons, and Ronald Solomon,** The classification of the finite simple groups, 1994

39 **Sigurdur Helgason,** Geometric analysis on symmetric spaces, 1993

38 **Guy David and Stephen Semmes,** Analysis of and on uniformly rectifiable sets, 1993

37 **Leonard Lewin, Editor,** Structural properties of polylogarithms, 1991

36 **John B. Conway,** The theory of subnormal operators, 1991

35 **Shreeram S. Abhyankar,** Algebraic geometry for scientists and engineers, 1990

34 **Victor Isakov,** Inverse source problems, 1990

33 **Vladimir G. Berkovich,** Spectral theory and analytic geometry over non-Archimedean fields, 1990

32 **Howard Jacobowitz,** An introduction to CR structures, 1990

31 **Paul J. Sally, Jr. and David A. Vogan, Jr., Editors,** Representation theory and harmonic analysis on semisimple Lie groups, 1989

30 **Thomas W. Cusick and Mary E. Flahive,** The Markoff and Lagrange spectra, 1989

29 **Alan L. T. Paterson,** Amenability, 1988

28 **Richard Beals, Percy Deift, and Carlos Tomei,** Direct and inverse scattering on the line, 1988

27 **Nathan J. Fine,** Basic hypergeometric series and applications, 1988

26 **Hari Bercovici,** Operator theory and arithmetic in H^∞, 1988

25 **Jack K. Hale,** Asymptotic behavior of dissipative systems, 1988

24 **Lance W. Small, Editor,** Noetherian rings and their applications, 1987

23 **E. H. Rothe,** Introduction to various aspects of degree theory in Banach spaces, 1986

22 **Michael E. Taylor,** Noncommutative harmonic analysis, 1986

21 **Albert Baernstein, David Drasin, Peter Duren, and Albert Marden, Editors,** The Bieberbach conjecture: Proceedings of the symposium on the occasion of the proof, 1986

20 **Kenneth R. Goodearl,** Partially ordered abelian groups with interpolation, 1986

19 **Gregory V. Chudnovsky,** Contributions to the theory of transcendental numbers, 1984

18 **Frank B. Knight,** Essentials of Brownian motion and diffusion, 1981

17 **Le Baron O. Ferguson,** Approximation by polynomials with integral coefficients, 1980

16 **O. Timothy O'Meara,** Symplectic groups, 1978

15 **J. Diestel and J. J. Uhl, Jr.,** Vector measures, 1977

14 **V. Guillemin and S. Sternberg,** Geometric asymptotics, 1977

13 **C. Pearcy, Editor,** Topics in operator theory, 1974

12 **J. R. Isbell,** Uniform spaces, 1964

11 **J. Cronin,** Fixed points and topological degree in nonlinear analysis, 1964

10 **R. Ayoub,** An introduction to the analytic theory of numbers, 1963

9 **Arthur Sard,** Linear approximation, 1963

8 **J. Lehner,** Discontinuous groups and automorphic functions, 1964

7.2 **A. H. Clifford and G. B. Preston,** The algebraic theory of semigroups, Volume II, 1961

7.1 **A. H. Clifford and G. B. Preston,** The algebraic theory of semigroups, Volume I, 1961

6 **C. C. Chevalley,** Introduction to the theory of algebraic functions of one variable, 1951

5 **S. Bergman,** The kernel function and conformal mapping, 1950

4 **O. F. G. Schilling,** The theory of valuations, 1950

(See the AMS catalog for earlier titles)

Mathematical
Surveys
and
Monographs

Volume 2

The Theory of Rings

Nathan Jacobson

American Mathematical Society
Providence, Rhode Island

1991 *Mathematics Subject Classification.* Primary 16–02.

International Standard Serial Number 0076-5376
International Standard Book Number 0-8218-1502-4
Library of Congress Card Number 43-15310

---

15 14 13 12 11 10 02 01 00 99 98

TABLE OF CONTENTS

PREFACE

The theory that forms the subject of this book had its beginning with Artin's extension in 1927 of Wedderburn's structure theory of algebras to rings satisfying the chain conditions. Since then the theory has been considerably extended and simplified. The only exposition of the subject in book form that has appeared to date is Deuring's *Algebren* published in the Ergebnisse series in 1935. Much progress has been made since then and this perhaps justifies a new exposition of the subject.

The present account is almost completely self-contained. That this has been possible in a book dealing with results of the significance of Wedderburn's theorems, the Albert-Brauer-Noether theory of simple algebras and the arithmetic ideal theory is another demonstration of one of the most remarkable characteristics of modern algebra, namely, the simplicity of its logical structure.

Roughly speaking our subject falls into three parts: structure theory, representation theory and arithmetic ideal theory. The first of these is an outgrowth of the structure theory of algebras. It was motivated originally by the desire to discover and to classify "hypercomplex" extensions of the field of real numbers. The most important names connected with this phase of the development of the theory are those of Molien, Dedekind, Frobenius and Cartan. The structure theory for algebras over a general field dates from the publication of Wedderburn's thesis in 1907; the extension to rings, from Artin's paper in 1927. The theory of representations was originally concerned with the problem of representing a group by matrices. This was extended to rings and was formulated as a theory of modules by Emmy Noether. The study of modules also forms an important part of the arithmetic ideal theory. This part of the theory of rings had its origin in Dedekind's ideal theory of algebraic number fields and more immediately in Emmy Noether's axiomatic foundation of this theory.

Throughout this book we have placed particular emphasis on the study of rings of endomorphisms. By using the regular representations the theory of abstract rings is obtained as a special case of the more concrete theory of endomorphisms. Moreover, the theory of modules, and hence representation theory, may be regarded as the study of a set of rings of endomorphisms all of which are homomorphic images of a fixed ring o. Chapter 1 lays the foundations of the theory of endomorphisms of a group. The concepts and results developed here are fundamental in all the subsequent work. Chapter 2 deals with vector spaces and contains some material that, at any rate in the commutative case, might have been assumed as known. For the sake of completeness this has been included. Chapter 3 is concerned with the arithmetic of non-commutative principal ideal domains. Much of this chapter can be regarded as a special case of the general arithmetic ideal theory developed in Chapter 6. The methods of Chapter 3 are, however, of a much more elementary character and

this fact may be of interest to the student of geometry, since the results of this chapter have many applications in that field. A reader who is primarily interested in structure theory or in representation theory may omit Chapter 3 with the exception of 3. Chapter 4 is devoted to the development of these theories and to some applications to the problem of representation of groups by projective transformations and to the Galois theory of division rings. In Chapter 5 we take up the study of algebras. In the first part of this chapter we consider the theory of simple algebras over a general field. The second part is concerned with the theory of characteristic and minimum polynomials of an algebra and the trace criterion for separability of an algebra.

In recent years there has been a considerable interest in the study of rings that do not satisfy the chain conditions but instead are restricted by topological or metric conditions. We mention von Neumann and Murray's investigation of rings of transformations in Hilbert space, von Neumann's theory of regular rings and Gelfand's theory of normed rings. There are many important applications of these theories to analysis. Because of the conditions that we have imposed on the rings considered in this work, our discussion is not directly applicable to these problems in topological algebra. It may be hoped, however, that the methods and results of the purely algebraic theory will point the way for further development of the topological algebraic theory.

This book was begun during the academic year 1940–1941 when I was a visiting lecturer at Johns Hopkins University. It served as a basis of a course given there and it gained materially from the careful reading and criticism of Dr. Irvin Cohen who at that time was one of the auditors of my lectures. My thanks are due to him and also to Professors Albert, Schilling and Hurewicz for their encouragement and for many helpful suggestions.

N. JACOBSON.

Chapel Hill, N. C.,
March 7, 1943.

BIBLIOGRAPHY

ALBERT, A. A. [1]: *On the rank equation of any normal division algebra*, Bull. Amer. Math. Soc., v. 35 (1929), 335–338; [2]: *The rank function of any simple algebra*, Proc. Nat. Acad. Sci., v. 15 (1929), 372–376; [3]: *On direct products*, Trans. Amer. Math. Soc., v. 33 (1931), 690–711; [4]: *On the construction of cyclic algebras with a given exponent*, Amer. J. Math., v. 54 (1932), 1–13; [5]: *On normal simple algebras*, Trans. Amer. Math. Soc., v. 34 (1932), 620–625; [6]: *A note on normal division algebras of order sixteen*, Bull. Amer. Math. Soc., v. 38 (1932), 703–706; [7]: *Normal division algebras over a modular field*, Trans. Amer. Math. Soc., v. 36 (1934), 388–394; [8]: *On normal Kummer fields over a non-modular field*, Trans. Amer. Math. Soc., v. 36 (1934), 885–892; [9]: *Involutorial simple algebras and real Riemann matrices*, Ann. of Math., v. 36 (1935), 886–964; [10]: *Normal division algebras of degree p^e over F of characteristic p*, Trans. Amer. Math. Soc., v. 39 (1936), 183–188; [11]: *Simple algebras of degree p^e over a centrum of characteristic p*, Trans. Amer. Math. Soc., v. 40 (1936), 112–126; [12]: *Modern Higher Algebra*, Chicago, 1937; [13]: *On cyclic algebras*, Ann. of Math., v. 39 (1938), 669–682; [14]: *Non-cyclic algebras with pure maximal subfields*, Bull. Amer. Math. Soc., v. 44 (1938), 576–579; [15]: *A note on normal division algebras of prime degree*, Bull. Amer. Math. Soc., v. 44 (1938), 649–652; [16]: *Structure of Algebras*, New York, 1939; [17]: *On p-adic fields and rational division algebras*, Ann. of Math., v. 41 (1940), 674–692; [18]: *Non-associative algebras, I. Fundamental concepts and isotopy*, Ann. of Math., v. 43 (1942), 685–707; [19]: *Non-associative algebras, II. New simple algebras*, Ann. of Math., v. 43 (1942), 708–723.

ALBERT, A. A., AND HASSE, H. [1]: *A determination of all normal division algebras over an algebraic number field*, Trans. Amer. Math. Soc., v. 34 (1932), 722–726.

ARTIN, E. [1]: *Über einen Satz von Herrn J. H. Maclagan Wedderburn*, Abh. Math. Sem. Univ. Hamburg, v. 5 (1927), 245–250; [2]: *Zur Theorie der hyperkomplexen Zahlen*, Abh. Math. Sem. Univ. Hamburg, v. 5 (1927), 251–260; [3]: *Zur Arithmetik hyperkomplexer Zahlen*, Abh. Math. Sem. Univ. Hamburg, v. 5 (1927), 261–239; [4]: *Galois Theory*, Notre Dame, 1942.

ARTIN, E., AND WHAPLES, G. [1]: *The theory of simple rings*, Amer. J. Math., v. 65 (1943), 87–107.

ASANO, K. [1]: *Über die Darstellungen einer endlichen Gruppe durch reele Kollineationen*, Proc. Imp. Acad. Tokyo, v. 9 (1933), 574–576; [2]: *Nichtkommutative Hauptidealringe. I*, Act, Sci. Ind., No. 636, Paris 1938; [3]: *Arithmetische Idealtheorie in nichtkommutativen Ringen*, Jap. J. Math., v. 15 (1939), 1–36; [4]: *Über verallgemeinerte Abelsche Gruppe mit hyperkomplexem Operatorenring und ihre Anwendungen*, Jap. J. Math., v. 15 (1939), 231–253; [5]: *Über Ringe mit Vielfachenkettensatz*, Proc. Imp. Acad. Tokyo, v. 15 (1939), 289–291.

ASANO, K., AND NAKAYAMA, T. [1]: *Über halblineare Transformationen*, Math. Ann., v. 115 (1937), 87–114.

ASANO, K., AND SHODA, K. [1]: *Zur Theorie der Darstellungen einer endlichen Gruppe durch Kollineationen*, Compositio Math., v. 2 (1935), 230–240.

BAER, R. [1]: *A Galois theory of linear systems over commutative fields*, Amer. J. Math., v. 62 (1940), 551–588; [2]: *Inverses and zero-divisors*, Bull. Amer. Math. Soc., v. 48 (1942), 630–638.

BIRKHOFF, G. [1]: *On the representability of Lie algebras and Lie groups by matrices*, Ann. of Math., v. 38 (1937), 526–532; [2]: Lattice Theory, New York, 1940.

BIRKHOFF, G., AND MACLANE, S., [1]: *A Survey of Modern Algebra*, New York, 1941.

BRANDT, H. [1]: *Idealtheorie in einer Dedekindschen Algebra*, Jber. Deutsch. Math. Verein, v. 37 (1928), 5–7; [2]: *Primidealzerlegung in einer Dedekindschen Algebra*, Schweizerische Naturforschende Gesellschaft. Verhandlungen, v. 28 (1929), 288–290; [3]: *Zur Idealtheorie Dedekindscher Algebren*, Comment. Math. Helv., v. 2 (1930), 13–17; [4]: *Über die Axiome des Gruppoids*, Vierteljschr. Naturforsch. Ges. Zurich, v. 85 (1940), 95–104.

BRAUER, R. [1]: *Untersuchungen über der arithmetischen Eigenschaften von Gruppen linearer Substitutionen, I*, Math. Z., v. 28 (1928), 677–696; [2]: *ibid., II*, v. 31 (1930), 733–747; [3]: *Über Systeme hyperkomplexer Zahlen*, Math. Z., v. 30 (1929), 79–107; [4]: *Über die algebraische Struktur von Schiefkörpern*, J. Reine Angew. Math., v. 166 (1932), 241–252; [5]: *Über die Konstruktion der Schiefkörper, die von endlichen Rang in bezug auf ein gegebenes Zentrum sind*, J. Reine Angew. Math., v. 168 (1932), 44–64; [6]: *Über den Index und den Exponenten von Divisionsalgebren*, Tôhoku Math. J., v. 37 (1933), 77–87; [7]: *Über die Kleinsche Theorie der algebraischen Gleichungen*, Math. Ann., v. 110 (1934), 473–500; [8]: *Eine Bedingung für vollständige Reduzibilität von Darstellungen gewöhnlicher und infinitesimaler Gruppen*, Math. Z., v. 41 (1936), 330–339; [9]: *On algebras which are connected with the semi-simple continuous groups*, Ann. of Math., v. 38 (1937), 857–872; [10]: *On normal division algebras of index five*, Proc. Nat. Acad. Sci., v. 24 (1938), 243–246; [11]: *On modular and p-adic representations of algebras*, Proc. Nat. Acad. Sci., v. 25 (1939), 252–258; [12]: *Investigations on group characters*, Ann. of Math., v. 42 (1941), 936–958; [13]: *On sets of matrices with coefficients in a division ring*, Trans. Amer. Math. Soc., v. 49 (1941), 502–548; [14]: *On the nilpotency of the radical of a ring*, Bull. Amer. Math. Soc., v. 48 (1942), 752–758.

BRAUER, R., HASSE, H., AND NOETHER, E. [1]: *Beweis eines Hauptsatzes in der Theorie der Algebren*, J. Reine Angew. Math., v. 167 (1932), 399–404.

BRAUER, R., AND NESBITT, C. [1]: *On the regular representations of algebras*, Proc. Nat. Acad. Sci., v. 23 (1937), 236–240; [2]: *On the modular representations of groups of finite order. I*, Toronto Studies, 1937; [3]: *On the modular characters of groups*, Ann. of Math., v. 42 (1941), 556–590.

BRAUER, R., AND NOETHER, E. [1]: *Über minimale Zerfällungskörper irreduzibler Darstellungen*, S.-B. Preuss. Akad. Wiss., v. 32 (1927), 221–228.

BRAUER, R., AND WEYL, H. [1]: *Spinors in n dimensions*, Amer. J. Math., v. 57 (1935), 425–449.

CASIMIR, H., AND VAN DER WAERDEN, B. L. [1]: *Algebraischer Beweis der vollständigen Reduzibilität der Darstellungen halbeinfacher Liescher Gruppen*, Math. Ann., v. 111 (1935), 1–12.

CHEVALLEY, C. [1]: *Sur la théorie du corps de classes dans les corps finis et les corps locaux*, J. Fac. Sci. Imp. Univ. Tokyo, v. 9 (1933), 366–476; [2]; *La théorie du symbole de restes normiques*, J. Reine Angew. Math., v. 169 (1933), 140–156; [3]: *Sur certains idéaux d'une algèbre simple*, Abh. Math. Sem. Univ. Hamburg, v. 10 (1934), 83–105; [4]: *Démonstration d'une hypothèse de M. Artin*, Abh. Math. Sem. Univ. Hamburg, v. 11 (1936), 73–75; [5]: *Généralisation de la théorie du corps de classes pour les extensions infinies*, J. Math. Pures Appl., Sér. 9, v. 15 (1936), 359–371; [6]: *L'arithmétique dans les algèbres de matrices*, Act. Sci. Ind. No. 323, Paris, 1936; [7]: *La théorie du corps de classes*, Ann. of Math., v. 41 (1940), 394–418; [8]: *An algebraic proof of a property of Lie groups*, Amer. J. Math., v. 63 (1941), 785–793; [9]: *On the composition of fields*, Bull. Amer. Math. Soc., v. 48 (1942), 482–487.

CLIFFORD, A. H. [1]: *Representations induced in an invariant subgroup*, Ann. of Math., v. 38 (1937), 533–550; [2]: *Semigroups admitting relative inverses*, Ann. of Math., v. 42 (1941), 1037–1049; [3]: *Matrix representations of completely simple semigroups*, Amer. J. Math., v. 64 (1942), 327–342.

DEURING, M. [1]: *Galoissche Theorie und Darstellungstheorie*, Math. Ann., v. 107 (1932), 140–144; [2]: *Algebren*, Ergebnisse der Math., v. 4, Berlin, 1935.

DICKSON, L. E. [1]: *Linear Algebras*, Cambridge, 1914; [2]: *Algebras and their Arithmetics*, Chicago, 1923; [3]: *Algebren und ihre Zahlentheorie*, Zurich, 1927.

DORROH, J. L. [1]: *Concerning adjunctions to algebras*, Bull. Amer. Math. Soc., v. 38 (1932), 85–88; [2]: *Concerning the direct product of algebras*, Ann. of Math., v. 36 (1935), 882–885.

EICHLER, M. [1]: *Bestimmung der Idealklassenzahl in gewissen normalen einfachen Algebren*, J. Reine Angew. Math., v. 176 (1937), 192–202; [2]: *Über die Einheiten der Divisionsalgebren*, Math. Ann., v. 114 (1937), 635–654; [3]: *Allgemeine Kongruenzklasseneinteilungen der Ideale einfacher Algebren über algebraischen Zahlkörpern und ihre L-Reihen*, J. Reine Angew. Math., v. 179 (1938), 227–251; [4]: *Über die Idealklassenzahl hyperkomplexer Systeme*, Math. Z., v. 43 (1938), 481–494; [5]: *Zur Einheitentheorie der einfachen Algebren*, Comment. Math. Helv., v. 11 (1939), 253–272.

ETHERINGTON, I. M. H. [1]: *Genetic algebras*, Proc. Roy. Soc. Edinburgh, v. 59 (1939), 242–258.

EVERETT, C. J., JR. [1]: *Rings as groups with operators*, Bull. Amer. Math. Soc., v. 45 (1939) 274–279; [2]: *Annihilator ideals and representation iteration for abstract rings*, Duke Math. J., v. 5 (1939), 623–627; [3]: *Vector spaces over rings*, Bull. Amer. Math. Soc., v. 48 (1942), 312–316; [4]: *An extension theory for rings*, Amer. J. Math., v. 64 (1942), 363–370.

FITTING, H. [1]: *Die Theorie der Automorphismenringe Abelscher Gruppen und ihr Analogon bei nicht kommutativen Gruppen*, Math. Ann., v. 107 (1932), 514–542; [2]: *Über die direkten Produktzerlegungen einer Gruppe in direkt unzerlegbare Faktoren*, Math. Z., v. 39 (1935), 16–30; [3]: *Primärkomponentenzerlegung in nichtkommutativen Ringen*, Math. Ann., v. 111 (1935), 19–41; [4]: *Über die Existenz gemeinsamer Verfeinerungen bei direkten Produktzerlegungen einer Gruppe*, Math. Z., v. 41 (1936), 380–395; [5]: *Die Determinantenideale eines Moduls*, Jber. Deutsch. Math. Verein. v. 46 (1936), 195–228; [6]: *Über der Zusammenhang zwischen dem Begriff der Gleichartigkeit zweier Ideale und dem Äquivalenzbegriff der Elementarteilertheorie*, Math. Ann., v. 112 (1936), 572–582; [7]: *Der Normenbegriff für Ideale eines Ringes beliebiger Struktur*, J. Reine Angew. Math., v. 178 (1937), 107–122.

GELFAND, I. [1]: *Normierte Ringe*, Rec. Math. [Mat. Sbornik] N.S., v. 9 (1941), 3–23; [2]: *Ideale und primäre Ideale in normierten Ringen*, Rec. Math. [Mat. Sbornik] N.S., v. 9 (1941), 41–48.

GOLOWIN, O. N. [1]: *On factors without centres in direct decompositions of groups (Russian)* Rec. Math. [Mat. Sbornik] N.S., v. 6 (1939), 423–426.

HAANTJES, J. [1]: *Halblineare Transformationen*, Math. Ann., v. 114 (1937), 293–304.

HALL, M. [1]: *A type of algebraic closure*, Ann. of Math., v. 40 (1939), 360–369; [2]: *The position of the radical in an algebra*, Trans. Amer. Math. Soc., v. 48 (1940), 391–404.

HARRISON, G. [1]: *The structure of algebraic moduls*, Proc. Nat. Acad. Sci., v. 28 (1942), 410–413.

HASSE, H. [1]: *Darstellbarkeit von Zahlen durch quadratische Formen in einem beliebigen algebraischen Zahlkörper*, J. Reine Angew. Math., v. 153 (1924), 113–130; [2]: *Äquivalenz quadratischer Formen in einem beliebigen algebraischen Zahlkörper*, J. Reine Angew. Math., v. 153 (1924), 158–162; [3]: *Über p-adische Schiefkörper und ihre Bedeutung für die Arithmetik hyperkomplexer Zahlsysteme*, Math. Ann., v. 104 (1931), 495–534; [4], *The theory of cyclic algebras over an algebraic number field*, Trans. Amer. Math. Soc., v. 34 (1932), 171–214; and *Additional note to the author's "Theory of cyclic algebras over an algebraic number field,"* Trans. Amer. Math. Soc., v. 34 (1932), 727–730; [5]: *Die Struktur der R. Brauerschen Algebrenklassengruppe über einem algebraischen Zahlkörper*, Math. Ann., v. 107 (1932), 731–760; [6]: *Über gewisse Ideale in einer einfachen Algebra*, Act. Sci. Ind. No. 109, Paris, 1934.

HASSE, H., AND SCHILLING, O. F. G. [1]: *Die Normen aus einer normalen Divisionsalgebra über einem algebraischen Zahlkörper*, J. Reine Angew. Math., v. 174 (1936), 248–252.

HOPKINS, C. [1]: *Nilrings with minimal condition for admissible left ideals*, Duke Math. J., v. 4 (1938), 664–667; [2]: *Rings with minimal conditions for left ideals*, Ann. of Math., v. 40 (1939), 712–730.

HENKE, K. [1]: *Zur arithmetischen Idealtheorie hyperkomplexer Zahlen*, Abh. Math. Sem. Univ. Hamburg, v. 11 (1936), 311–332.

HIGMAN, G. [1]: *The units of group-rings*, Proc. London Math. Soc., v. 46 (1940), 231–240.

HOCHSCHILD, G. P. [1]: *Semi-simple algebras and generalized derivations*, Amer. J. Math., v. 64 (1941), 677–694.

INGRAHAM, M. H., AND WOLF, M. C. [1]: *Relative linear sets and similarity of matrices whose elements belong to a division algebra*, Trans. Amer. Math. Soc., v. 42 (1937), 16–31.

JACOBSON, N. [1]: *Non-commutative polynomials and cyclic algebras*, Ann. of Math., v. 35 (1934), 197–208; [2]: *Totally disconnected locally compact rings*, Amer. J. Math., v. 58 (1936), 433–449; [3]: *Pseudo-linear transformations*, Ann. of Math., v. 38 (1937), 484–507; [4]: *Abstract derivations and Lie algebras*, Trans. Amer. Math. Soc., v. 42 (1937), 206–224; [5]: *p-algebras of exponent p.*, Bull. Amer. Math. Soc., v. 43 (1937), 667–670; [6]: *A note on topological fields*, Amer. J. Math., v. 59 (1937), 889–894; [7]: *A note on non-associative algebras*, Duke Math. J., v. 3 (1937), 544–548; [8]: *Simple Lie algebras over a field of characteristic zero*, Duke Math. J., v. 4 (1938), 534–551; [9]: *Normal semi-linear transformations*, Amer. J. Math., v. 61 (1939), 45–58; [10]: *The fundamental theorem of the Galois theory for quasi-fields*, Ann. of Math., v. 41 (1940), 1–7; [11]: *Restricted Lie algebras of characteristic p*, Trans. Amer. Math. Soc., v. 50 (1941), 15–25; [12]: *Classes of restricted Lie algebras of characteristic p. I*, Amer. J. Math., v. 63 (1941), 481–515; [13]: *ibid., II*, Duke Math. J., v. 10 (1943), 107–121.

JACOBSON, N., AND TAUSSKY, O. [1]: *Locally compact rings*, Proc. Nat. Acad. Sci., v. 21 (1935), 106–108.

JENNINGS, S. A. [1]: *The structure of the group ring of a p-group over a modular field*, Trans. Amer. Math. Soc., v. 50 (1941), 175–185; [2]: *Central chains of ideals in an associative ring*, Duke Math. J., v. 9 (1942), 341–355.

JORDAN, P., VON NEUMANN, J., AND WIGNER, E. [1]: *On an algebraic generalization of the quantum mechanical formalism*, Ann. of Math., v. 35 (1934), 29–64.

KALISCH, G. K. [1]: *On special Jordan algebras*, Dissertation, University of Chicago, Chicago, 1942.

KAWADA, Y., AND KONDÔ, K. [1]: *Idealtheorie in nichtkommutativen Halbgruppen*, Jap. J. Math., v. 16 (1939), 37–45.

KIOKEMEISTER, F. [1]: *The parastrophic criterion for the factorization of primes*, Trans. Amer. Math. Soc., v. 50 (1941), 140–159.

KOŘINEK, V. [1]: *Maximale kommutative Körper in einfachen Systemen hyperkomplexer Zahlen*, Mém. Soc. Sci. Bohème, 1932, No. 1, (1933), 1–24; [2]: *Une remarque concernant l'arithmétique des nombres hypercomplexes*, Mém. Soc. Roy. Sci. Bohème, 1932, No. 4 (1933), 1–8; [3]: *Sur la décomposition d'un groupe en produit direct des sousgroupes*, Casopis Pěst. Mat. Fys., v. 66 (1937), 281–286; also correction, v. 67 (1938), 209–210.

KOETHE, G. [1]: *Schiefkörper unendlichen Ranges über dem Zentrum*, Math. Ann., v. 105 (1931), 15–39; [2]: *Verallgemeinerte Abelsche Gruppen mit hyperkomplexen Operatorenring*, Math. Z., v. 39 (1934), 29–44.

KRULL, W. [1]: *Über verallgemeinerte endliche Abelsche Gruppen*, Math. Z., v. 23 (1925), 161–196.

KUROSCH, A. [1]: *Ringtheoretische Probleme, die mit dem Burnsideschen Problem über periodische Gruppen in Zusammenhang stehen*, Bull. Acad. Sci. URSS, Sér. Math., v. 5 (1941), 233–240.

LANDHERR, W. [1]: *Über einfache Liesche Ringe*, Abh. Math. Sem. Univ. Hamburg, v. 11 (1934), 41–64; [2]: *Liesche Ringe vom Typus A über einem algebraischen Zahlkörper (Die lineare Gruppe) und hermitesche Formen über einem Schiefkörper*, Abh. Math. Sem. Hansischen Univ., v. 12 (1938), 200–241.

LEVITZKI, J. [1]: *Über nilpotente Subringe*, Math. Ann., v. 105 (1931), 620–627; [2]: *On the equivalence of the nilpotent elements of a semi-simple ring*, Compositio Math., v. 5 (1938), 392–402; [3]: *On rings which satisfy the minimum condition for the right-hand ideals*, Compositio Math., v. 7 (1939), 214–222.

MACDUFFEE, C. C. [1]: *The Theory of Matrices*, Erg. der Math., v. 2, Berlin, 1933; [2]: *Matrices with elements in a principal ideal ring*, Bull. Amer. Math. Soc., v. 39 (1933),

564–584; [3]: *Modules and ideals in a Frobenius algebra*, Monatsh. Math. Phys., v. 48 (1939), 292–313; [4]: *An Introduction to Abstract Algebra*, New York, 1940.

MacLane, S., and Schilling, O. F. G., [1]: *A formula for the direct product of crossed product algebras*, Bull. Amer. Math. Soc., v. 48 (1942), 108–114; [2]: *Groups of algebras over an algebraic number field*, Amer. J. Math., v. 65 (1943), 299–308.

Maeda, F. [1]: *Ring-decomposition without chain condition*, J. Sci. Hiroshima Univ., Ser. A, v. 8 (1938), 145–167.

Malcev, A. [1]: *On the immersion of an algebraic ring into a field*, Math. Ann., v. 113 (1936), 686–691.

McCoy, N. H. [1]: *On the characteristic roots of matric polynomials*, Bull. Amer. Math. Soc., v. 42 (1936), 592–600; [2]: *Quasi-commutative rings and differential ideals*, Trans. Amer. Math. Soc., v. 39 (1936), 101–116; [3]: *Subrings of direct sums*, Amer. J. Math., v. 60 (1938), 374–382; [4]: *Subrings of infinite direct sums*, Duke Math. J., v. 4 (1938), 486–494; [5]: *Generalized regular rings*, Bull. Amer. Math. Soc., v. 45 (1939), 175–178; [6]: *Algebraic properties of certain matrices over a ring*, Duke Math. J., v. 9 (1942), 322–340.

McCoy, N. H., and Montgomery, D. [1]: *A representation of generalized Boolean rings*, Duke Math. J., v. 3 (1937), 455–459.

Moriya, M. [1]: *Zur Bewertung der einfachen Algebren*, Proc. Imp. Acad. Japan, v. 13 (1937), 392–395.

Murray, F. J., and von Neumann, J. [1]: *On rings of operators*, Ann. of Math., v. 37 (1936), 116–229; [2]: *ibid., II*, Trans. Amer. Math. Soc., v. 41 (1937), 208–248.

Nakayama, T. [1]: *Über die Beziehungen zwischen den Faktorensystemen und der Normklassengruppe eines galoisschen Erweiterungskörpers*, Math. Ann., v. 112 (1935), 85–91; [2]: *Über die direkte Zerlegung einer Divisionsalgebra*, Jap. J. Math., v. 12 (1935), 65–70; [3]: *Über die Algebren über einem Körper von der Primzahlcharakteristik*, Proc. Imp. Acad. Tokyo, v. 11 (1935), 305–306; [4]: *ibid., II*, v. 12 (1936), 113–114; [5]: *Eine Bemerkung über die Summe und den Durchschnitt von zwei Idealen in einer Algebra*, Proc. Imp. Acad. Tokyo, v. 12 (1936), 179–182; [6]; *Über die Klassifikation halblinearer Transformationen*, Proc. Phys -Math. Soc. Japan, v. 19 (1937), 99–107; [7]: *Divisionsalgebren über diskret bewerteten perfekten Körpern*, J. Reine, Angew. Math., v. 178 (1937), 11–13; [8]: *A note on the elementary divisor theory in non-commutative domains*, Bull. Amer. Math. Soc., v. 44 (1938), 719–723; [9]: *Some studies on regular representations, induced representations and modular representations*, Ann. of Math., v. 39 (1938), 361–369; [10]: *On Frobeniusean Algebras, I*, Ann. of Math., v. 40 (1939), 611–633; [11]: *A remark on the sum and the intersection of two normal ideals of an algebra*, Bull. Amer. Math. Soc., v. 46 (1940), 469–472; also correction, v. 47 (1941), 332; [12]: *Note on uni-serial and generalized uni-serial rings*, Proc. Imp. Acad. Tokyo, v. 16 (1940), 285–289; [13]: *Normal basis of a quasi-field*, Proc. Imp. Acad. Tokyo, v. 16 (1940), 532–536; [14]: *On Frobeniusean algebras, II*, Ann. of Math., v. 42 (1941), 1–21; [15]: *Algebras with anti-isomorphic left and right ideal lattices*, Proc. Imp. Acad. Tokyo, v. 17 (1941), 53–56.

Nakayama, T., and Nesbitt, C. [1]: *Note on symmetric algebras*, Ann. of Math., v. 39 (1938), 659–668.

Nakayama, T., and Shoda, K. [1]: *Über die Darstellung einer endlichen Gruppe durch halblineare Transformationen*, Jap. J. Math., v. 12 (1936), 109–122.

Nehrkorn, H. [1]: *Über absolute Idealklassengruppen und Einheiten in algebraischen Zahlkörpern*, Abh. Math. Sem. Univ. Hamburg, v. 9 (1933), 318–334.

Nesbitt, C. [1]: *On the regular representations of algebras*, Ann. of Math., v. 39 (1938), 634–658.

Neuhaus, A. [1]: *Products of normal semi-fields*, Trans. Amer. Math. Soc., v. 49 (1941), 106–121.

von Neumann, J. [1]: *On regular rings*, Proc. Nat. Acad. Sci., v. 22 (1936), 707–713; [2]: *Algebraic theory of continuous geometries*, Proc. Nat. Acad. Sci., v. 23 (1937), 16–22; [3]: *Continuous rings and their arithmetics*, Proc. Nat. Acad. Sci., v. 23 (1937), 341–349; [4]: *On rings of operators, III*, Ann. of Math., v. 41 (1940), 94–161.

NIVEN, I. [1]: *Equations in quaternions*, Amer. Math. Monthly, v. 48 (1941), 654–661; [2]: *The roots of a quaternion*, Amer. Math. Monthly, v. 49 (1942), 386–388.

NOETHER, E. [1]: *Der Diskriminantensatz für die Ordnungen eines algebraischen Zahl-oder Funktionenkörpers*, J. Reine Angew. Math., v. 157 (1927), 82–104; [2]: *Hyperkomplexe Grössen und Darstellungstheorie*, Math. Z., v. 30 (1929), 641–692; [3]: *Hyperkomplexe Systeme in ihre Beziehungen zur kommutativen Algebra und Zahlentheorie*, Zurich Congress Proceedings, 1932, 189–194; [4]: *Der Hauptgeschlechtssatz für relativ-galoissche Zahlkörper*, Math. Ann., v. 108 (1933), 411–419; [5]: *Nichtkommutativ Algebra*, Math. Z., v. 37 (1933), 514–541; [6]: *Zerfallende verschränkte Produkte und ihre Maximalordnungen*, Act. Sci. Ind., Paris, 1934.

NOETHER, E., AND SCHMEIDLER, W. [1]: *Moduln in nichtkommutativen Bereichen, insbesondere aus Differential-und Differenzenausdrücken*, Math. Z., v. 8 (1920), 1–35.

ORE, O. [1]: *Linear equations in non-commutative fields*, Ann. of Math., v. 32 (1931), 463–477; [2]: *Theory of non-commutative polynomials*, Ann. of Math., v. 34 (1933), 480–508.

OSIMA, M. [1]: *Über die Darstellung einer Gruppe durch halblineare Transformationen*, Proc. Phys.-Math. Soc. Japan, v. 20 (1938), 1–5.

PERLIS, S. [1]: *A characterization of the radical of an algebra.* Bull. Amer. Math. Soc., v. 48 (1942), 128–132.

PICKERT, G. [1]: *Neue Methoden in der Strukturtheorie der kommutativ-assoziativen Algebren*, Math. Ann., v. 116 (1938), 217–280.

REES, D. [1]: *On semi-groups*, Proc. Cambridge Philos. Soc., v. 36 (1940), 387–400; [2]: *Note on semi-groups*, Proc. Cambridge Philos. Soc., v. 37 (1941), 434–435.

RINEHART, R. F. [1]: *Some properties of the discriminant matrices of a linear associative algebra*, Bull. Amer. Math. Soc., v. 42 (1936), 570–576; [2]: *Commutative algebras which are polynomial algebras*, Duke Math. J., v. 4 (1938), 725–736; [3]: *An interpretation of the index of inertia of the discriminant matrices of a linear associative algebra*, Trans. Amer. Math. Soc., v. 46 (1939), 307–327.

SCHIFFMAN, M. [1]: *The ring of automorphisms of an Abelian group*, Duke Math. J., v. 6 (1940), 579–597.

SCHILLING, O. F. G. [1]: *Über gewisse Beziehungen zwischen der Arithmetik hyperkomplexer Zahlsysteme und algebraischer Zahlkörper*, Math. Ann., v. 111 (1935), 372–398; [2]: *Einheitentheorie in rationalen hyperkomplexen Systemen*, J. Reine Angew. Math., v. 175 (1936), 246–251; [3]: *Über die Darstellungen endlicher Gruppen*, J. Reine Angew. Math., v. 174 (1936), 188; [4]: *The structure of certain rational infinite algebras*, Duke Math. J., v. 3 (1937), 303–310; [5]: *Arithmetic in a special class of algebras*, Ann. of Math., v. 38 (1937), 116–119; [6]: *Units in p-adic algebras*, Amer. J. Math., v. 61 (1939), 883–896.

SCHREIER, O. [1]: *Über den Jordan-Hölderschen Satz*, Abh. Math. Sem. Univ. Hamburg, v. 6 (1928), 300–302.

SCHMIDT, O. [1]: *Über unendliche Gruppen mit endlicher Kette*, Math. Z., v. 29 (1928), 34–41.

SCHNEIDMÜLLER, V. I. [1]: *On rings with finite decreasing chains of subrings*, C. R. (Doklady) Acad. Sci. URSS (N.S.), v. 28 (1940), 579–581.

SCHUR, I. [1]: *Über die Darstellung der endlichen Gruppen durch gebrochene lineare Substitutionen*, J. Reine Angew. Math., v. 127 (1904), 20–50; [2]: *Einige Bemerkungen zu der vorstehenden Arbeit des Herrn A. Speiser*, Math. Z., v. 5 (1919), 7–10.

SCORZA, G. [1]: *Corpi Numerici e Algebre*, Messina, 1921.

SCOTT, W. M. [1]: *On matrix algebras over an algebraically closed field*, Ann. of Math., v. 43 (1942), 147–160.

SEGRE, C. [1]: *Un nuovo campo di ricerche geometriche*, Atti Accad. Sci. Torino, v. 25 (1889), 276–301.

SERBIN, H. [1]: *Factorization in principal ideal rings*, Duke Math. J., v. 4 (1938), 656–663.

SHODA, K. [1]: *Über die Automorphismen einer endlichen Abelschen Gruppe*, Math. Ann. v. 100 (1928), 674–686; [2]: *Über die mit einer Matrix vertauschbaren Matrizen*, Math. Z., v. 29 (1929), 696–712; [3]: *Über die Galoissche Theorie der halbeinfachen hyperkomplexen Systeme*, Math. Ann., v. 107 (1932), 252–258; [4]: *Über die Äquivalenz der Darstellungen*

endlicher Gruppen durch halblineare Transformationen, Proc. Imp. Acad. Japan, v. 14 (1938), 278–280; [5]: *Über die Invarianten der endlichen Gruppen halblinearer Transformationen*, Proc. Imp. Acad. Japan, v. 14 (1938), 281–285.

SKOLEM, T. [1]: *Zur Theorie der assoziativen Zahlensysteme*, Oslo, 1927.

SPEISER, A. [1]: *Zahlentheoretische Sätze aus der Gruppentheorie*, Math. Z., v. 5 (1919), 1–6; [2]: *Idealtheorie in rationalen Algebren*, Dickson's Algebren, Chapter 13, Zurich, 1927; [3]: *Zahlentheorie in rationalen Algebren*, Comment. Math. Helv., v. 8 (1936), 391–406.

STAUFFER, R. [1]: *The construction of a normal basis in a separable normal extension field*, Amer. J. Math., v. 58 (1936), 585–597.

STONE, M. H., [1]: *The theory of representations of Boolean algebras*, Trans. Amer. Math. Soc., v. 40 (1936), 37–111.

TEICHMÜLLER, O. [1]: *Verschränkte Produkte mit Normalringen*, Deutsche Math., v. 1 (1936), 92–102; [2]: *Multiplikation zyklischer Normalringe*, Deutsche Math., v. 1 (1936) 197–238; [3]: *p-Algebren*, Deutsche Math., v. 1 (1936), 362–388; [4]: *Zerfällende zyklische p-Algebren*, J. Reine Angew. Math., v. 176 (1937), 157–160; [5]: *Der Elementarteilersatz für nichtkommutative Ringe*, S.-B. Preuss. Akad. Wiss., 1937; [6]: *Über die sogennante nichtkommutative Galoissche Theorie und die Relationen* $\xi_{\lambda,\mu,\nu}\xi_{\lambda,\mu,\tau}\xi^{\lambda}_{\mu,\nu,t} = \xi_{\lambda,\mu,\tau t}\xi_{\mu,\nu,\tau}$ Deutsche Math., v. 5 (1940), 138–149.

TSEN, C. C. [1]: *Divisionsalgebren über Funktionenkörpern*, Nach. Ges. Wiss. Göttingen, 1933, 335–339; [2]: *Algebren über Funktionenkörpern*, Göttingen dissertation, 1934; [3]: *Zur Stufentheorie der quasialgebraisch-Abgeschlossenheit kommutativer Körper*, J. Chinese Math. Soc., v. 1 (1936), 81–92.

UZKOW, A. I. [1]: *Abstract foundation of Brandt's theory of ideals* (*Russian*), Rec. Math. [Mat. Sbornik] N. S. v. 6 (1939), 263–281.

VAN DER WAERDEN, B. L. [1]: *Moderne Algebra*, v. 1 and 2, Berlin, 1931; 2nd edition, v. 1, 1937, 2, 1940; [2]: *Die Klassifikation der einfachen Lieschen Gruppen*, Math. Z., v. 37 (1933), 446–462; [3]: *Gruppen von linearen Transformationen*, Erg. der Math., v. 4, Berlin, 1935.

W. RNING, E. [1]: *Bemerkung zur vorstehenden Arbeit von Herrn Chevalley*, Abh. Math. Sem. Univ. Hamburg, v. 11 (1936), 76–83.

WEDDERBURN, J. H. M. [1]: *A theorem on finite algebras*, Trans. Amer. Math. Soc., v. 6 (1905) 349–352; [2]: *On hypercomplex numbers*, Proc. London Math. Soc., Ser. 2, v. 6 (1908), 77–117; [3]: *A type of primitive algebra*, Trans. Amer. Math. Soc., v. 15 (1914), 162–166; [4]: *On division algebras*, Trans. Amer. Math. Soc., v. 22 (1921), 129–135; [5]: *Algebras which do not possess a finite basis*. Trans. Amer. Math. Soc., v. 26 (1924), 395–426; [6]: *A theorem on simple algebras*, Bull. Amer. Math. Soc., v. 31 (1925), 11–13; [7]: *Non-commutative domains of integrity*, J. Reine Angew. Math., v. 167 (1932), 129–141; [8]: *Lectures on Matrices*, New York, 1934; [9]: *Note on algebras*, Ann. of Math., v. 38 (1937), 854–856.

WEYL, H. [1]: *Note on matric algebras*, Ann. of Math., v. 38 (1937), 477–483; [2]: *Commutator algebra of a finite group of collineations*, Duke Math. J., v. 3 (1937), 200–212; [3]: *The Classical Groups*, Princeton, 1939.

WHAPLES, G. [1]: *Non-analytic class field theory and Grunwald's theorem*, Duke Math. J., v. 9 (1942), 455–473.

WHITEHEAD, J. H. C. [1]: *On the decomposition of an infinitesimal group*, Proc. Cambridge Philos. Soc., v. 32 (1936), 229–237; [2]: *Certain equations in the algebra of a semi-simple infinitesimal group*, Quart. J. Math., Oxford Ser., v. 8 (1937), 220–237; [3]: *Note on linear associative algebras*, J. London Math. Soc., v. 16 (1941), 118–125.

WHITNEY H. [1]: *Tensor products of abelian groups*, Duke Math. J., v. 4 (1938), 495–528.

WITT, E. [1]: *Über die Kommutativität endlicher Schiefkörper*, Abh. Math. Sem. Univ. Hamburg, v. 8 (1930), 413; [2]: *Zerlegung reeler algebraischer Funktionen in Quadrate. Schiefkörper über reelen Funktionenkörper*, J. Reine Angew. Math., v. 171 (1934), 4–11; [3]: *Riemann-Rochscher Satz und ζ-Funktion in Hyperkomplexen*, Math. Ann., v. 110 (1934), 12–28; [4]: *Zwei Regeln über verschränkte Produkte*, J. Reine Angew. Math., v. 173 (1935), 191–192; [5]: *Theorie der quadratischen Formen in beliebigen Körpern*, J. Reine Angew.

Math., v. 176 (1937), 31–44; [6]: *Zyklische Körper und Algebren der Charakteristik p vom Grad p^n*:, J. Reine Angew. Math., v. 176 (1937), 126–140; [7]: *Schiefkörper über diskret bewerteten Körpern*, J. Reine Angew. Math.,.v. 176 (1937), 153–156; [8]: *Treue Darstellung Liescher Ringe*, J. Reine Angew. Math., v. 177 (1937), 152–160.

WOLF, L. A. [1]: *Similarity of matrices in which the elements are real quaternions*, Bull. Amer. Math. Soc., v. 42 (1936), 737–743.

ZASSENHAUS, H. [1]: *Zum Satz von Jordan-Hölder-Schreier*, Abh. Math. Sem. Univ. Hamburg, v. 10 (1934), 106–108; [2]: *Lehrbuch der Gruppentheorie*, Leipzig, 1937; [3]: *Über Liesche Ringe mit Primzahlcharakteristik*, Abh. Math. Sem. Univ. Hamburg, v. 13 (1939), 1–100; [4]: *Darstellungstheorie nilpotenter Lie-Ringe bei charakteristik $p > 0$*. J. Reine Angew. Math., v. 182 (1940), 150–155.

ZORN, M. [1]: *Theorie der alternativen Ringe*, Abh. Math. Sem. Univ. Hamburg, v. 8 (1930), 123–147; [2]: *Note zur analytischen hyperkomplexen Zahlentheorie*, Abh. Math. Sem. Univ. Hamburg, v. 9 (1933), 197–201; [3]: *Alternativkörper und quadratische systeme*, Abh. Math. Sem. Univ. Hamburg, v. 9 (1933), 395–402; [4]: *On a theorem of Engel*, Bull. Amer. Math. Soc., v. 43 (1937), 401–404; [5]: *Alternative rings and related questions, I: Existence of the radical*, Ann. of Math., v. 42 (1941), 676–686.

INDEX

ISBN 0-8218-1502-4

9 780821 815021

SURV/2