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Number 7
Part 1

The Algebraic Theory of Semigroups

A. H. Clifford
G. B. Preston

American Mathematical Society



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TABLE OF CONTENTS

PREFACE	ix
NOTATION USED IN VOLUME I	xiii
 CHAPTER 1. ELEMENTARY CONCEPTS	
1.1 Basic definitions	1
1.2 Light's associativity test	7
1.3 Translations and the regular representation (Lemma 1.0–Theorem 1.3)	9
1.4 The semigroup of relations on a set (Lemma 1.4)	13
1.5 Congruences, factor groupoids and homomorphisms (Theorem 1.5–Theorem 1.8)	16
1.6 Cyclic semigroups (Theorem 1.9)	19
1.7 Units and maximal subgroups (Theorem 1.10–Theorem 1.11)	21
1.8 Bands and semilattices; bands of semigroups (Theorem 1.12)	23
1.9 Regular elements and inverses; inverse semigroups (Lemma 1.13–Theorem 1.22)	26
1.10 Embedding semigroups in groups (Theorem 1.23–Theorem 1.25)	34
1.11 Right groups (Lemma 1.26–Theorem 1.27)	37
1.12 Free semigroups and generating relations; the bicyclic semi- group (Lemma 1.28–Corollary 1.32)	40
 CHAPTER 2. IDEALS AND RELATED CONCEPTS	
2.1 Green's relations (Lemma 2.1–Theorem 2.4)	47
2.2 \mathcal{D} -structure of the full transformation semigroup \mathcal{T}_X on a set X (Lemma 2.5–Theorem 2.10)	51
2.3 Regular \mathcal{D} -classes (Theorem 2.11–Theorem 2.20)	58

2.4	The Schützenberger group of an \mathcal{H} -class (Lemma 2.21–Theorem 2.25)	63
2.5	0-minimal ideals and 0-simple semigroups (Lemma 2.26–Theorem 2.35)	66
2.6	Principal factors of a semigroup (Theorem 2.36–Corollary 2.42)	71
2.7	Completely 0-simple semigroups (Lemma 2.43–Corollary 2.56)	76
 CHAPTER 3. REPRESENTATION BY MATRICES OVER A GROUP WITH ZERO		
3.1	Matrix semigroups over a group with zero (Lemma 3.1–Theorem 3.3)	87
3.2	The Rees Theorem (Theorem 3.4–Lemma 3.6)	91
3.3	Brandt groupoids (Lemma 3.7–Theorem 3.9)	99
3.4	Homomorphisms of a regular Rees matrix semigroup (Lemma 3.10–Theorem 3.14)	103
3.5	The Schützenberger representations (Lemma 3.15–Theorem 3.17)	110
3.6	A faithful representation of a regular semigroup (Lemma 3.18–Theorem 3.21)	117
 CHAPTER 4. DECOMPOSITIONS AND EXTENSIONS		
4.1	Croisot’s theory of decompositions of a semigroup (Lemma 4.1–Theorem 4.4)	121
4.2	Semigroups which are unions of groups (Theorem 4.5–Theorem 4.11)	126
4.3	Decomposition of a commutative semigroup into its archi- medean components; separative semigroups (Theorem 4.12–Theorem 4.18)	130
4.4	Extensions of semigroups (Theorem 4.19–Theorem 4.21)	137
4.5	Extensions of a group by a completely 0-simple semigroup; equivalence of extensions (Theorem 4.22–Theorem 4.24)	142
 CHAPTER 5. REPRESENTATION BY MATRICES OVER A FIELD		
5.1	Representations of semisimple algebras of finite order (Lemma 5.1–Theorem 5.11)	149

TABLE OF CONTENTS

vii

5.2 Semigroup algebras	158
(Lemma 5.12–Theorem 5.31)	
5.3 Principal irreducible representations of a semigroup . . .	170
(Lemma 5.32–Theorem 5.36)	
5.4 Representations of completely 0-simple semigroups . . .	177
(Theorem 5.37–Corollary 5.53)	
5.5 Characters of a commutative semigroup	193
(Lemma 5.54–Theorem 5.65)	
APPENDIX A	207
BIBLIOGRAPHY	209
AUTHOR INDEX	217
INDEX	219

PREFACE

So far as we know, the term "semigroup" first appeared in mathematical literature on page 8 of J.-A. de Séguier's book, *Éléments de la Théorie des Groupes Abstracts* (Paris, 1904), and the first paper about semigroups was a brief one by L. E. Dickson in 1905. But the theory really began in 1928 with the publication of a paper of fundamental importance by A. K. Suschkewitsch. In current terminology, he showed that every finite semigroup contains a "kernel" (a simple ideal), and he completely determined the structure of finite simple semigroups. A brief account of this paper is given in Appendix A.

Unfortunately, this result of Suschkewitsch is not in a readily usable form. This defect was removed by D. Rees in 1940 with the introduction of the notion of a matrix over a group with zero, and, moreover, the domain of validity was extended to infinite simple semigroups containing primitive idempotents. The Rees Theorem is seen to be the analogue of Wedderburn's Theorem on simple algebras, and it has had a dominating influence on the later development of the theory of semigroups. Since 1940, the number of papers appearing each year has grown fairly steadily to a little more than thirty on the average.

It is in response to this developing interest that this book has been written. Only one book has so far been published which deals predominantly with the algebraic theory of semigroups, namely one by Suschkewitsch, *The Theory of Generalized Groups* (Kharkow, 1937); this is in Russian, and is now out of print. A chapter of R. H. Bruck's *A Survey of Binary Systems* (Ergebnisse der Math., Berlin, 1958) is devoted to semigroups. There is, of course, E. Hille's book, *Functional Analysis and Semi-groups* (Amer. Math. Soc. Colloq. Publ., 1948), and the 1957 revision thereof by Hille and R. S. Phillips; but this deals with the analytic theory of semigroups and its application to analysis. The time seems ripe for a systematic exposition of the algebraic theory. (Since the above words were written, there has appeared such an exposition, in Russian: *Semigroups*, by E. S. Lyapin, Moscow, 1960.)

The chief difficulty with such an exposition is that the literature is scattered over extremely diverse topics. We have met this situation by confining ourselves to a portion of the existing theory which has proved to be capable of a well-knit and coherent development. All of Volume 1 and the first half of Volume 2 center around the structure of semigroups of certain types (such as simple semigroups, inverse semigroups, unions of groups,

semigroups with minimal conditions, etc.) and their representation by mappings or by matrices. The second half of Volume 2 treats the theory of congruences and the embedding of semigroups in groups, including a modest account of the active French school of semigroups (which they call "demi-groupes") founded in 1941 by P. Dubreil.

In order to keep our book within reasonable bounds, moreover, we have construed the term "algebraic" in a somewhat narrow sense: the semigroups under consideration are not endowed with any further structure. This has the effect of excluding not only topological semigroups, but ordered semigroups as well. Fortunately, a good account of lattice-ordered semigroups and groups is to be found in G. Birkhoff's *Lattice Theory* (Amer. Math. Soc. Colloq. Publ., 1940; revised 1948). It also excludes P. Lorenzen's generalization of multiplicative ideal theory (see, for example, §5 of W. Krull's *Idealtheorie*, *Ergebnisse der Math.*, Berlin, 1935) to any commutative semigroup S with cancellation, in which S (or its quotient group) is endowed with a family of subsets called r -ideals, satisfying certain conditions analogous to those for closed sets in topology.

The book aims at being largely self-contained, but it is assumed that the reader has some familiarity with sets, mappings, groups, and lattices. The material on these topics in an introductory text such as Birkhoff and MacLane, *A Survey of Modern Algebra* (New York, Revised Edition, 1953) should suffice. Only in Chapter 5 will more preliminary knowledge be required, and even there the classical definitions and theorems on the matrix representations of algebras and groups are summarized.

We have included a number of exercises at the end of each section. These are intended to illuminate and supplement the text, and to call attention to papers not cited in the text. They can all be solved by applying the methods and results of the text, and often more simply than in the paper cited.

Each volume has a separate bibliography listing those papers referred to in that volume. No attempt has been made to list those papers on semigroups to which no reference has been made in the text or exercises. The combined bibliography contains about half of the papers which have appeared in the (strictly) algebraic theory of semigroups. (The bibliography in Lyapin's book appears to be complete.) Whenever possible, the reference to the review of each paper in the *Mathematical Reviews* has been given, (MR x , y) denoting page y of volume x . English translations of Russian titles are those given in the *Mathematical Reviews*.

The material in Volume 1 (more or less) was presented in a second-year graduate course at Tulane University during the academic year 1958–1959, and this volume has benefited greatly from the students' criticisms. The authors would also like to express their gratitude to Professors A. D. Wallace, D. D. Miller, and P. F. Conrad for many useful suggestions; and, above all, to Dr. W. D. Munn for his very valuable criticisms, especially of Chapter 5,

and for his permission to draw on unpublished material from his dissertation (Cambridge University, 1955) for Sections 3.4 and 4.5. We deeply appreciate the thoughtful kindness of Professor Š. Schwarz and the Central Library of the Slovakian Academy of Sciences for sending us (unsolicited) a photostat print of Suschkewitsch's book. Our thanks go also to Mrs. Anna L. McGinity for typing all of Volume 1. Finally, the authors gratefully acknowledge partial support for this work from the National Science Foundation (U.S.A.).

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NOTATION USED IN VOLUME ONE

Square brackets are used for alternative readings and for reference to the bibliography.

Let A and B be sets.

$A \subset B$ (or $B \supset A$) means A is properly contained in B .

$A \subseteq B$ (or $B \supseteq A$) means $A \subset B$ or $A = B$.

$A \setminus B$ means the set of elements of A which are not in B .

$A \times B$ means the set of all ordered pairs (a, b) with a in A , b in B .

The signs \cup and \cap are reserved for union and intersection, respectively, of sets and relations. The signs \vee and \wedge will be used for join and meet in [semi]lattices.

$|A|$ means the cardinal number of the set A .

The sign \circ is used for composition of relations (§1.4), but is usually omitted for composition of mappings.

\square denotes the empty set, mapping, or relation.

ι_A denotes the identity mapping or relation [on the set A].

If ϕ is a mapping whose domain includes A , then $\phi|A$ means ϕ restricted to A .

$\{a_1, \dots, a_n\}$ means the set whose members are a_1, \dots, a_n . Braces are sometimes omitted on single elements, for example $A \cup b$ instead of $A \cup \{b\}$.

If $P(x)$ is a proposition for each element x of a set X , then the set of all x in X for which $P(x)$ is true is denoted by either $\{x \in X : P(x)\}$ or $\{x : P(x), x \in X\}$.

If $M(x)$ is a set for each x in a set X , then the union of all the sets $M(x)$ with x in X is denoted by either $\bigcup_{x \in X} M(x)$ or $\bigcup \{M(x) : x \in X\}$.

If $F(x)$ is a member of a set C for each x in a set X , then the subset of C consisting of all $F(x)$ with x in X is denoted by $\{F(x) : x \in X\}$. If $X = A \times B$, we may write $\{F(a, b) : a \in A, b \in B\}$ instead of $\{F(a, b) : (a, b) \in A \times B\}$.

If A is a subset of a semigroup S , then $\langle A \rangle$ denotes the subsemigroup of S generated by A . If S is a group, then the subgroup of S generated by A is $\langle A \cup A^{-1} \rangle$, where $A^{-1} = \{a^{-1} : a \in A\}$.

If A and B are subsets of a semigroup S , then AB means $\{ab : a \in A, b \in B\}$.

$S^1 [S^0]$ means the semigroup $S \cup 1 [S \cup 0]$ arising from a semigroup S by the adjunction of an identity element 1 [a zero element 0], unless S already has an identity [has a zero, and $|S| > 1$], in which case $S^1 = S [S^0 = S]$. (§1.1)

$a|b$ means “ a divides b ”, that is, $b \in aS^1$, where a and b are elements of a commutative semigroup S . (§4.3)

$\rho_a [\lambda_a]$ denotes the inner right [left] translation $x \rightarrow xa [x \rightarrow ax]$ of a semigroup S , where a is a fixed element of S . (§1.3)

If ρ is an equivalence relation on a set X , and if $(a, b) \in \rho$, then we write $a \rho b$ and say that a and b are ρ -equivalent, and that they belong to the same ρ -class.

If ρ is a congruence relation on a semigroup S , then S/ρ denotes the factor semigroup of S modulo ρ , and ρ^\natural denotes the natural mapping of S upon S/ρ . (§1.5) S/J denotes the Rees factor semigroup of S modulo an ideal J .

Let S be a semigroup, and let $a \in S$. (Following from §2.1)

$L(a)$ denotes the principal left ideal S^1a .

$R(a)$ denotes the principal right ideal aS^1 .

$J(a)$ denotes the principal two-sided ideal S^1aS^1 .

\mathcal{L} means $\{(a, b) \in S \times S : L(a) = L(b)\}$.

\mathcal{R} means $\{(a, b) \in S \times S : R(a) = R(b)\}$.

\mathcal{J} means $\{(a, b) \in S \times S : J(a) = J(b)\}$.

\mathcal{H} means $\mathcal{L} \cap \mathcal{R}$.

\mathcal{D} means $\mathcal{L} \circ \mathcal{R} (= \mathcal{R} \circ \mathcal{L})$.

L_a, R_a, J_a, H_a, D_a mean respectively the $\mathcal{L}, \mathcal{R}, \mathcal{J}, \mathcal{H}, \mathcal{D}$ -class containing a .

$I(a)$ means $J(a) \setminus J_a$. (It is empty or an ideal of S .)

$J(a)/I(a)$ is the principal factor of S corresponding to a . (§2.6)

\mathcal{T}_X means the semigroup of all transformations of a set X . (§1.1)

\mathcal{G}_X means the group of all permutations of a set X . (§1.1)

\mathcal{I}_X means the symmetric inverse semigroup on a set X . (§1.9)

\mathcal{B}_X means the semigroup of all binary relations on X . (§1.4)

\mathcal{F}_X means the free semigroup on X . (§1.12)

\mathcal{FG}_X means the free group on X . (§1.12)

\mathcal{C} means the bicyclic semigroup. (§1.12)

$\mathcal{M}^0(G; I, \Lambda; P)$ means the Rees $I \times \Lambda$ matrix semigroup over the group with zero G^0 , with $\Lambda \times I$ sandwich matrix P .

$\mathcal{M}(G; I, \Lambda; P)$ means the Rees $I \times \Lambda$ matrix semigroup without zero over the group G , with $\Lambda \times I$ sandwich matrix P . (§3.1)

$\mathcal{LT}(V)$ means the algebra of all linear transformations of a vector space V , or the multiplicative semigroup thereof. (§§2.2, 5.1)

$(\mathfrak{A})_n$ means the algebra of all $n \times n$ matrices over an algebra \mathfrak{A} . (§5.1)

$\Phi[S]$ means the algebra of a semigroup S over a field Φ . (§5.2)

\cong means "isomorphic". (§1.3)

\sim means "homomorphic", and sometimes "equivalent". (§1.3)

\oplus is used for the direct sum of algebras, vector spaces, and representations. (§5.1)

The $n \times n$ identity matrix is denoted by:

I_n when it is over a field (§§5.2, 5.3),

U_n when it is over an algebra with identity element u (§5.1),

Δ_n when it is over a group with zero (§3.1).

Γ^m denotes the representation of $(\mathfrak{A})_m$ associated with the representation Γ of \mathfrak{A} . (§5.1)

M_L, M_R, M_J denote the minimal conditions on the set of principal left, right, two-sided ideals, respectively, of a semigroup. (A partially ordered set P is said to satisfy the minimal condition if each non-empty subset A of P contains at least one minimal element, i.e., an element x of A such that $y < x$ ($y \in P$) implies $y \notin A$.) (§§5.3, 5.4)

APPENDIX A

A BRIEF ACCOUNT OF THE 1928 PAPER OF SUSCHKEWITSCH

Starting with an arbitrary finite semigroup S , he considers subsets of S of the form Sa having the least possible number of elements. These are evidently just the minimal left ideals of S , and we shall use the current terminology. He shows that each minimal left ideal of S is a left group, and (without using the expression “direct product”) shows that it is the direct product of a group and a left zero semigroup. This is, of course, our Theorem 1.27 for finite semigroups. Moreover, any two minimal left ideals of S are isomorphic, and in particular are unions of the same number r of isomorphic groups.

He calls the union K of all the minimal left ideals of S the kernel (“Kerngruppe”) of S . If s is the number of distinct minimal left ideals of S , then K is the union of rs mutually isomorphic groups. He shows that these can be arranged in a rectangular array as follows:

K	L_1	L_2	\cdots	L_s
R_1	H_{11}	H_{12}	\cdots	H_{1s}
R_2	H_{21}	H_{22}	\cdots	H_{2s}
\vdots	\vdots	\vdots	\vdots	\vdots
R_r	H_{r1}	H_{r2}	\cdots	H_{rs}

(This is the source of our “eggbox picture”, described in §2.1.) The union of the groups $H_{1\lambda}, \dots, H_{r\lambda}$ in the λ th column is the minimal left ideal L_λ ($\lambda = 1, \dots, s$). Let $e_{i\lambda}$ be the identity element of the group $H_{i\lambda}$. He shows that the $H_{i\lambda}$ can be arranged so that each $e_{i\lambda}$ acts as a left identity on all the $H_{i\lambda}$ in the same row. When this is done, the union R_i of the groups H_{i1}, \dots, H_{is} in the i th row ($i = 1, \dots, r$) is a minimal right ideal of S . Moreover, every minimal right ideal of S is one of the R_i . Hence:

Every finite semigroup has a kernel K which is the union of all the minimal left ideals of S and also of all the minimal right ideals of S . The intersection of a minimal left ideal and a minimal right ideal is a (maximal) subgroup of S . These results were subsequently extended to infinite semigroups having minimal left ideals and minimal right ideals (Exercise 13 of §2.7). We know now that it is simpler to introduce the L 's and R 's independently, and the H 's as intersections thereof.

Suschkewitsch goes on to show by quite an involved argument that K is uniquely determined by (1) the abstract group H to which each $H_{i\lambda}$ is isomorphic, (2) the numbers r and s , and (3) the $(r-1)(s-1)$ products $e_{11}e_{i\lambda}$ ($i = 2, \dots, r$; $\lambda = 2, \dots, s$). He shows conversely that the group H , the numbers r and s , and the $e_{11}e_{i\lambda}$ can be given arbitrarily. This is done by means of transformations of a finite set. Thus he succeeds in determining the structure of the most general finite simple semigroup, but yet not (as came later with the Rees Theorem) in a readily usable form.

This theory also occupies the greater part of Chapter 3 of Suschkewitsch's book [1937].

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AUTHOR INDEX

Page numbers which include a reference to the exercises are printed in italics.

- Albert, A. A., 86
 Amitsur, S., 159, 163
 Andersen, O., 43, *5C*, 81, 123
 Baer, R., 39
 Ballieu, R., 38
 Bell, E. T., 40
 Birkhoff, G., viii
 Brandt, H., 1, 99, 100, 101
 Brauer, R., 158
 Bruck, R. H., vii, *6*, 27, *33*
 Carman, K. S., 75
 Clifford, A. H., *13*, *23*, 27, 38, *39*, *40*, 49, *50*,
 59, 60, *61*, *62*, 68, 70, 78, *84*, 91, 102, 121,
 123, 126, *129*, *137*, 142, 149, *169*, *177*, *192*
 Climescu, A. C., 20
 Comfort, W. W., 203
 Conrad, P. F., *37*, 100
 Croisot, R., *98*, *103*, 121, 123, 124, *125*, 126
 Deuring, M., 99
 Dickson, L. E., vii, 4
 Dubreil, P., vii, *19*, 34, 36
 Doss, C. G., *33*, 51
 Frobenius, G., *20*, *21*
 Gluskin, L. M., *34*
 Good, R. A., *84*
 Green, J. A., 47, 48, 49, 59, 61, 71, 79, *130*
 Greville, T. N. E., *63*
 Grimble, H. B., *40*, *71*
 Hancock, V. R., 137
 Hashimoto, H., 38
 Hewitt, E., 95, 121, 130, *135*, 148, 149, 159,
 167, *169*, 170, 193, 194, 195, 197, 199, *205*
 Hille, E., vii
 Hughes, D. R., *84*
 Huntington, E. V., 4
 Iséki, K., *26*, *34*, *126*, *206*
 Ivan, J., *83*, *97*, *130*
 Jacobson, N., *62*, 155, *169*
 Kimura, N., 18, 23, 26, 121, 130, 131, *135*
 Klein-Barmen, F., 4
 Koch, R. J., *66*, *84*
 Krull, W., viii, *126*
 Levi, F., 39
 Liber, A. E., 28
 Light, F. W., 7
 Loewy, A., 99
 Lorenzen, P., viii
 Lyapin, E. S., vii, viii, *34*, 43, *46*
 MacLane, S., viii
 McLean, D., *129*, *130*
 Malcev, A. I., *6*, *34*
 Mann, H. B., 38
 Miller, D. D., 49, 51, 59, 60, *61*, *62*, 70, 91
 Moore, E. H., 20, *63*
 Munn, W. D., 28, *40*, *62*, 68, *75*, *76*, 82, 102,
 103, 109, 121, 143, *147*, 148, 149, 159, 167,
 169, 170, *172*, 174, *176*, 191
 Neumann, J. von, 27
 Numakura, K., 26
 Oganessian, V. A., 165
 Ore, O., 34, 35
 Penrose, R., 28, *63*
 Phillips, R. S., vii
 Pierpont, J., 5
 Poole, A. R., 20, *21*
 Ponizovsky, I. S., 148, 170
 Posey, E. E., *13*, *26*
 Prachar, K., 38
 Preston, G. B., 27, 28, 30, 87, 110, 117
 Rédei, L., *137*
 Rees, D., vii, 17, 20, 32, 34, 35, 43, 47, 71, 74,
 83, 89, 91, 94, 103, 106, *130*
 Rich, R. P., 78, 79, *83*
 Ross, K. A., 203
 Schützenberger, M. P., 63, 64, 110, *129*
 Schwarz, Š., *21*, 23, 26, 38, 70, *126*, *136*, 149,
 193, 195, 201, 203, *205*, *206*
 Séguier, J.-A. de, vii
 Skolem, T., 38
 Steinfeld, O., *85*
 Stoll, R. R., 10, *110*
 Stolt, B., 38
 Suschkewitsch, A. K., vii, 20, 23, 37, *40*, 51,
 58, 67, *71*, 80, *84*, *85*, *99*, *142*, 148, 177, *191*,
 207, 208
 Szép, J., 38
 Tamari, D., *37*
 Tamura, T., *13*, 18, 26, 38, *71*, 121, 130, 131,
 135, *137*, 144
 Teissier, M., 165, 168
 Thierrin, G., *6*, 26, 27, *33*, 38, *98*, *129*, 130
 Tully, E. J., 10, 92, 107, 110
 Vagner, V. V., 27, 28, 29, 30
 Vandiver, H. S., *37*
 Vorobev, N. N., 7, *129*
 Waerden, B. L. van der, 34, 150, 155, 158
 Wallace, A. D., 23, *84*, *97*
 Ward, M., 20
 Warne, R. J., 195, 201, 203
 Weber, H., 4
 Wedderburn, J. H. M., *97*
 Wiegandt, R., 137
 Williams, L. K., 195, 201, 203
 Yamada, M., 26, *98*, 131
 Zassenhaus, H., 74
 Zuckerman, H. S., 95, 121, 130, *135*, 148, 149,
 159, 167, *169*, 170, 193, 194, 195, 197, 199,
 205

INDEX

Terms are listed primarily under the broad concept involved, such as *algebra*, *group*, *ideal*, *matrix*, *relation*, *representation*, and *semigroup*. One-sided concepts are listed under the stem word.

Page numbers which include a reference to the exercises are printed in italics. The dots and dashes stand for previous italicised terms (possibly of several words), the dashes being used for the earlier, and the dots for the later, terms.

For pure symbols, see the list of notation on page xiii.

- adjoint of a homomorphism, 200
- adjunction of an identity (or zero), 4
- algebra* (= linear associative —), 149
 - of a semigroup (= semigroup —, q.v.), 159
 - division —, 151
 - factor (= difference) —, 150
 - full matrix —, 151, 160
 - ideal (q.v.) of an —, 149
 - Munn —, 162
 - order of an —, 149
 - radical of an —, 149, 168
 - representation (q.v.) of an —, 151
 - semigroup* —, 158, 159
 - contracted ... —, 160, 166, 169, 176
 - semisimple* —, 149, 162, 169, 174
 - class number of a ... —, 150
 - Main Representation Theorem for ... —s, 154
 - simple components of a ... —, 150, 169
 - Wedderburn's First Theorem, 150
 - simple —, 150
 - Wedderburn's Second Theorem, 151
- anti-automorphism*, 9
 - involutorial —, 9, 62
- anti-endomorphism, 9
- anti-homomorphism, 9
- anti-isomorphism, 9
- anti-representation*, 9
 - [extended] regular —, 9
 - Schützenberger —, 110–112
- archimedean, see *semigroup*
- associative* (binary) operation, 1
 - linear algebra, see *algebra*
- associativity, Light's test for, 7
- automorphism, 9
- axioms for $S \setminus 0$, 100

- band*, 4, 24, 26, 98, 120, 129, 130, 169
 - algebra of a —, 169
 - of groups, 80, 83, 91, 125, 129
 - of semigroups, 26, 129
 - commutative — = semilattice, q.v.
 - free —, 129, 130
 - rectangular* —, 25, 26, 50, 83, 91, 97, 98, 129
 - ... — of groups, 80, 83, 91
 - ... — of [completely] simple semi-groups, 129
- basic, see under *matrix* and *representation*
- basis class of semigroups, 34
- belonging to an idempotent, 167
- bicyclic, see *semigroup*
- bi-ideal, 84, 85
- binary operation = operation, 1
- binary relation (= relation, q.v.), 13
- bisimple, see *semigroup*

- cancellable element*, 3, 37
 - right [left] —, 3
- cancellative, see *semigroup*
- canonical = natural, q.v.
- carrier space, 152
- center, 3
- central element, 3
- character* of a commutative semigroup, 193, 205
 - semigroup, 194, 205, 206
 - principal* —, 195
 - apex of a ... —, 195
 - semi—, 194
 - unit —, 194
 - vanishing ideal of a —, 194
- class number, 150
- commutative, see *semigroup*
- compatible, see *relation*
- complete lattice, 24
- composition, see *relation*, *ideal series*, and *transformation*
- congruence, see *relation*
- coördinates of Rees matrices, 107
- Croisot's condition (m, n), 124
- cross-section of a partition, 54, 56

- \mathcal{D} -class, 47, 49, 51–57, 58–61, 62, 66, 96, 97, 112, 115, 116
- decomposition, see *representation* and *semigroup*
- descending chain condition, 170
- direct product, see *semigroup*
- direct sum, see *representation*
- 0-disjoint, 67
- divisor*, 131
 - interior —, 40
 - proper — of zero, 68, 71, 142, 145
 - right [left] —, 40
 - right [left] — of zero, 156
- duality (left-right), 5

- egg-box picture, 48, 56, 61, 93, 207
 elementary ρ_0 -transition, 18
 embedding, see *semigroup*
 empty word, 41
 endomorphism, 9, 26
 equivalence, see *relation*
 exponents, laws of, 2, 3
 extension, see *representation* and *semigroup*
- factor* (= difference) algebra, 150
 ——— *semigroup* (or groupoid), 16
 principal ——— of a . . . , 72, 76, 103, 161, 170
 Rees ——— . . . , 17
 free, see *band*, *group*, and *semigroup*
- generalized group (= inverse semigroup, q.v.), 28
 generalized inverse (= inverse, q.v.), 27
 generating relations, 41
generators of a congruence, 18
 ——— of a groupoid (or semigroup), 3
 ——— of an equivalence relation, 14
 ——— of an ideal, 5
 ——— of an inverse semigroup, 31
 Green's Lemma, 49
 Green's Theorem, 59
group, 4, 21, 33, 39, 84, 85, 125, 135
 band of ———s, see *band*
 characters of a commutative ———, 197
 congruences on a group, 19
 embedding a semigroup in a ———, 34–36, 37
 extensions of a ——— by a completely 0-simple semigroup, 142–147
 free ——— \mathcal{FG}_X on a set X , 43
 full linear ——— $\mathcal{GL}(V)$ on a vector space V , 57
 generalized ——— (= inverse semigroup, q.v.), 28
 ——— algebra, 158
 ——— \mathcal{H} -class, 54, 57, 59, 61, 62, 65, 66, 79
 ———-inverse, 27
 ——— of left [right] quotients, 36, 37
 ——— of units, 21, 23
 ——— of zeroids, 70, 71, 135
 ——— part of a commutative semigroup, 136, 167, 205
 ——— with zero, 5, 70, 83, 87
 mixed ——— (Loewy), 99
 partial ———, 103
 right [left] ———, see under *semigroup*
 [dual] Schützenberger ——— of an \mathcal{H} -class, 64, 65, 66, 111
 semilattice of ———s, see *semilattice*
 simply transitive ———s, 64, 65
 structure ——— of a Rees matrix semigroup (q.v.), 88
subgroup of a semigroup, 5, 50, 70, 82, 84
 maximal . . . (see also \mathcal{H} -class above), 22, 23, 40, 61, 84, 85, 136, 205, 207
 symmetric ——— \mathcal{G}_X on a set X , 2, 6, 23, 33, 54, 57, 58, 96, 97, 99
 union of ———s, see *union*
- groupoid*, 1
 Brandt ———, 1, 99
 partial ———, 1, 100, 138
- \mathcal{H} -class, 47, 48, 50, 57, 59, 61, 62, 63–66, 79, 110
 group ———, see under *group*
 non-group ———, 62, 65–66
- homogeneous, see *relation*
 homomorphic image, see *maximal* and *non-trivial*
- homomorphism*, 9
 adjoint of a ———, 200
 anti-——, 9
 canonical = natural, q.v.
 induced ———, 17
 . . . Theorem, 17, 19
 Main ——— Theorem, 16
 natural ——— $\rho^{\mathfrak{H}}$ determined by a congruence ρ , 16
 non-trivial ———, 103
 partial ———, 93, 109, 138, 143
 ramification associated with a homomorphism, 141
 hull, see *inverse* and *translational*
- i.a.a. = involutorial anti-automorphism, q.v.
ideal (left, right, two-sided), 5, 149
 bi-——, 84, 85
 closed ——— (= semiprime ———, q.v.), 206
 ——— extension of a semigroup, 137
 generators of an ———, 5
 maximal proper ———, 71
 minimal left [right] ———, 66, 70, 80, 84, 85, 130
 0-minimal left [right] ———, 67–70, 76–80, 83, 84, 89
 minimal (two-sided) ———, 66, 69, 70
 0-minimal (two-sided) ———, 67–70, 83
 nilpotent ——— of an algebra, 149
 operator-isomorphic right [left] ———s of an algebra, 154
 power of an ——— of an algebra, 149
 prime ———, 40, 71, 125, 126, 194, 204, 205
 principal (left, right, two-sided) ———, 6, 27, 47, 52, 57, 75, 83
 quasi-——, 85
 semiprime ———, 71, 121, 125, 126, 205
series, 73, 74, 150
 composition . . . , 74, 75, 76
 factors of an ——— . . . , 73, 74, 76, 150
 isomorphic ——— . . . , 74
 principal . . . , 73, 75, 76, 161
 refinement of an ——— . . . , 74
 relative ——— . . . , 74, 75, 150
universally maximal ———, 40
 . . . minimal ———, 70
- idempotent* (element), 4, 6, 20, 37, 38, 54, 56, 57, 59, 61, 62, 63
 belonging to an ———, 167
 ——— congruence, 131
 ——— semigroup = band, q.v.
 natural partial ordering of the ———s, 24
 over [under] an ———, 23
 primitive ———, 26, 76, 83, 84, 103

- identity element* (see also *matrix*), 3, 20
 adjunction of an ———, 4
 right [left] ———, 3, 39, 40
increasing element, left or right, 46
index of an element (or cyclic semigroup), 19, 21, 23
induced, see *homomorphism* and *relation*
inflation, see *semigroup*
inner, see *translation* and *translational hull*
interior divisor, 40
inverse elements, 27, 33, 60, 61, 62, 91
 group ———, 27
 ——— hull, 32, 35, 46
 ——— semigroup, see under *semigroup*
 ——— subsemigroup, 30
 left [right] ———, 4, 21
 relative ———, 27
involutional anti-automorphism, 9, 62
isomorphism, 9
 partial ———, 93, 97
 ——— theorems, 71
- \mathcal{I} -class, 48, 52, 74, 123, 126, 170, 172, 176, 191
 join, 14, 24
- kernel, 6, 66, 67, 69, 70, 84, 85, 165, 176, 205, 207
 Kerngruppe = kernel, q.v.
- \mathcal{L} -class, see $\mathcal{H}\mathcal{L}$ -class
lattice, 24, 202, 205
 ——— of congruences, 24
left-right duality, 5
linear associative algebra = algebra, q.v.
linear transformation, 57, 62
 null-space of a ———, 57, 62
 rank of a ———, 57
 linked, see *translation*
- Maschke's Theorem, 158
matrix (see also *algebra*, *representation*, *semigroup*)
 column-monomial ———, 113, 115, 116
 diagonal ———, 95
 factorization of a ———, 180, 192
 basic ... of a ———, 181, 191
 equivalent ... s of a ———, 181, 192
 width of a ... of a ———, 180
 identity ———, 91, 102, 151, 154, 171
 invertible ———, 95, 106, 145
 Moore's general reciprocal of a ———, 63
 non-singular ——— over an algebra, 157, 169
 ——— over a group with zero, 87
 ——— units, 83, 91, 97, 160
 Nullity, Sylvester's Law of, 183
 product of ———s over a group with zero, 87, 91
 rank of a ———, 181
 Rees ———, 88
 regular ———, 89
 row-monomial ———, 87, 111, 115, 116
 strictly ... ———, 116
- sandwich* ——— of a Rees matrix semigroup, 88, 96
 normalization of the ... ———, 94, 106–107
 ... ——— of a Munn algebra, 162
- maximal*
 ——— homomorphic image of given type, 18
 ——— ... group image, 18, 21, 84, 110
 ——— ... semilattice image, 18, 130, 131, 132, 135, 203
 ——— ... separative image, 132, 136, 198, 200
 ——— left[right] simple subsemigroup, 125
 ——— one-idempotent subsemigroup, 21, 26
 ——— proper ideal, 71
 ——— simple subsemigroup, 125
 ——— subgroup, see *subgroup* under *group*
- meet, 24
 middle unit, 98
 minimal conditions M_I , M_L , and M_R , 148, 149, 170, 172, 177, 196, 200
 minimal \mathcal{I} -class, 170
 mixed group (Loewy), 99
 module, double, 152
 multiplicative function, 194
 Munn matrix algebra, 162
- natural basis, 151
natural (= canonical) homomorphism, 16
 ——— mapping, 14
 non-trivial homomorphic image, 103
 normalization of the sandwich matrix, 94, 106–107
 nowhere commutative, see *semigroup*
- one-to-one* mapping, 2
 ——— partial right [left] translation, 32
 ——— partial transformation, 29
- onto mapping, 2
 operation (= binary operation), 1
order of a groupoid (or semigroup), 3
 ——— of an algebra, 149
 ——— of an element, 19
 over an idempotent, 23
- p.r.t. = partial one-to-one right translation, q.v.
- partial* (binary) operation, 1
 ——— group, 103
 ——— groupoid, 1, 100, 138
 ——— homomorphism, 93, 109, 138, 143
 ramification associated with a ——— ... , 141
 ——— isomorphism, 93, 97
 ——— one-to-one right translation, 32
 ——— one-to-one transformation, 29
 ——— ordering, 23
 natural ——— ... of the idempotents, 24
 ... of relations, 14
- partition*, 14
 ——— determined by a transformation, 51, 56, 57, 58

- period of an element (or of a cyclic semigroup), 19
 periodic, see *semigroup*
 permutation, 2
 power, 2, 149
 primitive, see *idempotent*
principal, see *character, factor, ideal, ideal series, representation*
 projection, 56, 57, 155
 properly nilpotent element of an algebra, 149
- quasi-ideal, 85
 quotient (= factor) groupoid, or semigroup, 16
 quotients (left or right), group of, 36, 37
- $\mathcal{A}[\mathcal{L}]$ -class, 47, 50, 56, 57, 61, 62, 117, 125
 ramification, 141
 reciprocal (= inverse) elements, 27
 reciprocal, general, of a matrix (E. H. Moore), 63
 rectangular, see *band* and *semigroup*
 reductive, see *semigroup*
Rees congruence, 17
 — factor semigroup, 17
 — matrix, 88
 — matrix semigroup (q.v.), 88
 — Theorem, 94
regular (see also *matrix, representation, semigroup*)
 — \mathcal{D} -class, 58–63, 91–94
 — element, 26
 — Rees matrix semigroup, 89
relation (= binary relation), 13
 compatible —, left or right, 16
 composition of —s, 13
 congruence, 16, 19
 ... $\phi \circ \phi^{-1}$ induced by a homomorphism ϕ , 16
 idempotent ... , 131
 Rees ... , 17
 right [left] ... , 16, 19
 separative ... , 132
 converse of a —, 14
 divisibility — (see also *divisor*), 131
 empty —, 13
 equivalence —, 14
 ... $\phi \circ \phi^{-1}$ induced by a mapping ϕ , 15
 intersection and join of ... —s, 14
 natural mapping ρ^{\natural} determined by an ... —, 14
 generating —s for a semigroup, 41
 Green's —s $\mathcal{R}, \mathcal{L}, \mathcal{D}, \mathcal{H}, \mathcal{J}$ (q.v.), 47, 48
 homogeneous = compatible, q.v.
 partial ordering of —s, 14
 product (= composition) of —s, 13
 regular = compatible, q.v.
 semigroup \mathcal{S}_X of —s on a set X , 13–15
 transitive closure ρ' of a —, ρ , 14
 universal —, 13
representation, 9, 110, 148, 151, 160, 168, 169
 absolutely irreducible —, 154, 192
 anti— (q.v.), 9
 apex of a principal —, 171
 associated — Γ^m of a — Γ , 155
 basic —, 185, 193
 carrier space of a —, 152
 completely (= fully) reducible —, 154
 decomposition of a —, 153, 192
 defining matrices of a —, 180
 degree of a —, 148
 direct sum of —s, 117, 119
 equivalent —s, 152, 192
 extended regular —, 9, 33
 extending matrix of a —, 180, 191, 192, 193
 extension of a —, 171, 176, 178, 191, 192, 193
 basic ... of a —, 177, 185, 191, 193
 principal ... of a —, 171
 faithful (= true) —, 9, 117–120, 148
 fully (= completely) reducible —, 154
 induced —, 9, 152, 171
 invariant subspace of a — space, 152
 irreducible constituents of a —, 153, 193
 absolutely ... —, 154, 192
 ... invariant subspace, 153
 ... —, 153, 154
 —s of a semisimple algebra (Main Theorem), 154
 principal —, 171, 177
 apex of a ... —, 171
 ... extension of a —, 171
 proper —, 177, 191, 192
 regular —, 9, 33, 64, 65, 154
 extended ... —, 9, 33
 (right) ... — (= ... —), 154
 — space (= carrier space of a —), 152
 Schur's Lemma, 154
 [dual] Schützenberger —, 110–115, 116, 117, 118, 119
 true — = faithful —, q.v.
 ultimate reduction of a —, 153
 unit —, 166, 169, 176, 193
 vanishing ideal of a —, 171
 representative mapping of a partition, 56
 reversible, see *semigroup*
- Schreier* extension of a semigroup, 137
Schützenberger group (q.v.), 64
 — representation (q.v.), 112
 semicharacter, 194
semigroup (see also *band, semilattice, union*)
 algebra $\Phi[S]$ of a — S over a field Φ , 158, 159
 contracted — ... , 160, 166
 archimedean commutative —, 131, 135, 136
 ... components of a commutative —, 130, 135, 205
 basis class of —s, 34
 bicyclic — \mathcal{C} , 43–46, 50, 80, 81, 97
 bisimple —, 49, 50, 51, 62, 80, 97
 0-bisimple —, 76, 79
 Brandt —, 100, 103, 147, 165, 169, 176, 191

- semigroup*—continued
cancellative —, 3, 6, 18, 23, 33, 34–37, 51, 133–136, 137, 199
 right [left] . . . —, 3, 6, 10, 13, 21, 23, 32, 33, 37–40, 50, 117
 center of a —, 3
 character (q.v.) of a commutative —, 193
commutative — (see also *nowhere . . .* —), 3, 18, 21, 24, 33, 34, 36, 37, 125, 126, 130–137, 164, 167, 169, 193–206
 . . . band = semilattice, q.v.
 completely [0-]simple — (see also *Rees matrix* —), 76–85, 86, 90, 94, 97, 102, 103, 142, 163, 177, 192
 cyclic —, 19, 20, 21, 23, 46, 142, 159, 169, 176
 decomposition of a —, 25, 121–137
 direct product of —s, 37, 38, 83, 97, 98, 130, 207
 \mathcal{D} -simple — = bisimple —, q.v.
E-inverse —, 98
embedding of a — in a group, 34–37
 . . . of a — in a symmetric inverse semigroup, 30
extension (= ideal . . .) of a —, 137–142, 142–147
 equivalent . . . s of a —, 143
 Schreier . . . of a —, 137
 free —, 40–41
 full transformation — \mathcal{F}_X on a set X , 2, 6–7, 13, 23, 33, 51–58, 75, 95, 99, 116, 125, 170
 generating relations for a —, 41
 ideal (q.v.) of a —, 5
idempotent — = band, q.v.
 commutative . . . — = semilattice, q.v.
 inflation of a —, 98
 intra-regular —, 121, 123, 125
inverse —, 28–34, 60, 102–103, 119, 127–129, 165, 176
 elementary . . . —, 34
 embedding an . . . — S in \mathcal{F}_S , 30
 generators of an . . . —, 31
 . . . hull of a —, 32, 35, 46
 . . . subsemigroup of an . . . —, 30
 symmetric . . . — \mathcal{F}_X on a set X , 29, 30, 33
 left group, see *right [left] group* below
M-inverse —, 98
 nowhere commutative —, 26, 33, 97
 null (= zero) —, 4, 67, 72, 73, 97
 one-idempotent (= unipotent) —, 21, 26, 33, 71, 135
 periodic —, 20, 21, 23, 26, 136
 rectangular — (see also *band*), 98
reductive —, right or left, 9
 weakly . . . —, 11, 116, 139
 Rees matrix — (see also *completely [0-]simple* —), 88–91, 92–96, 97, 99, 102, 103–110, 114–116, 119, 125, 142–147, 163, 166, 177–193
 regular —, 26, 33, 34, 40, 56, 57, 62, 84, 85, 89, 103, 119, 120, 125
 . . . Rees matrix —, 89
 right [left] . . . —, 121–122, 125, 129
 representation (q.v.) of a —, 9, 110, 160
reversible —, right or left, 34, 37
 strongly . . . —, 26
 right [left] group, 37–40, 50, 58, 66, 70, 125, 142, 191, 207
 — algebra = algebra of a —, q.v.
 above
 — generated by a set subject to generating relations, 41
 — of linear transformations, 57, 62
 — of matrix units, 83, 91, 97, 160
 — \mathcal{R}_X of relations on a set X , 13–15
 — of transformations, see *full* above
 semisimple —, 74, 75, 76, 125, 162
 separative —, 131, 135, 136, 197–200, 206
simple — (see also *completely simple* —), 5, 40, 51, 66–70, 73, 123, 125, 192
 right [left] . . . —, 5, 37, 38, 50, 66, 68, 70, 117, 125
0-simple — (see also *completely 0-simple* —), 67, 68, 71, 72, 73, 81, 192
 right [left] . . . —, 67, 68, 70
 stationary on the right [left], 98
 symmetric inverse —, see *inverse* — above
 transformation —, see *full transformation* — above
 unipotent — = one-idempotent —, q.v.
 zero — = null —, q.v.
 right [left] . . . —, 4, 6, 13, 26, 33, 37, 38, 39, 129
semilattice, 24, 33
 lower —, 24
 maximal homomorphic — image, see *maximal* —
 — of archimedean commutative semigroups, 132
 — of completely simple semigroups, 126
 — of groups, 128, 129, 136
 — of one-idempotent semigroups, 26
 — of rectangular bands, 129
 — of semigroups, 26, 129
 — of simple semigroups, 123
 upper —, 24
 separative, see *semigroup*, *maximal homomorphic image*, and *congruence*
 series, see *ideal*
 set product, 5
 simply transitive, 64
 structure group of a Rees matrix semigroup, 88
 subgroup, see under *group*
 subgroupoid, 2
 subsemigroup, 3
 symmetric, see *group* and *inverse semigroup*
- trace of a \mathcal{D} -class, 92, 97
transformation, 1
 composition of —s, 1
 constant —, 6

- transformation*—continued
 defect of a ———, 6
 iterate (= composition) of ———s, 1
 linear ——— (q.v.), 57, 62
 partial one-to-one ———, 29
 product (= composition) of ———s, 1
 range of a ———, 51, 57, 58, 62
 rank of a ———, 6, 52, 53, 57
 ——— upon (= onto) a set, 2
 [simply] transitive set of ———s, 64
 transition, elementary, 18
 transitive, see *relation* and *transformation*
translation, left [right], 10, 116, 139, 142
 inner left [right] ———, 9, 13, 116
 linked left and right ———s, 10, 13, 139
 partial one-to-one left [right] ———, 32
translational hull, 11, 13, 139
 inner part of the ———, 12
 triples and Rees matrices, 88
 two-sided, see *ideal*, *identity*, *zero*
- under an idempotent, 23
union (see also *band* and *semilattice*)
 ——— of groups, 23, 33, 34, 37–40, 97, 122,
 125, 126–130, 134, 136, 164, 206
 ——— of [left, right] simple semigroups,
 122, 123, 125
unit (see also *character* and *representation*), 21,
 37
 middle ———, 98
 right [left] ——— of a semigroup with
 identity, 21, 46
 ... ——— of an element of an inverse
 semigroup, 30
 ... ——— subsemigroup, 21, 23, 33, 50,
 57
 universal (right, left, or interior) divisor, 40
universally maximal ideal, 40
 ——— *minimal ideal*, 70
 ——— ... \mathcal{I} -class, 170
- word, 41
- zero element* (left, right, two-sided), 3
 right [left] ——— semigroup, see under
 semigroup, 4
 ——— semigroup (= null semigroup, q.v.),
 4
zeroid element [right, left], 70, 71, 84, 136

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