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Uniform Spaces

J. R. Isbell

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J. R. ISBELL

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PREFACE

The subject matter of this book might be labelled fairly accurately *Intrinsic geometry of uniform spaces.* {For an impatient reader, this means elements (25%), dimension theory (40%), function spaces ($12\frac{1}{2}$%), and special topics in topology.} As the term "geometry" suggests, we shall not be concerned with applications to functional analysis and topological algebra. However, applications to topology and specializations to metric spaces are of central concern; in fact, these are the two pillars on which the general theory stands. This dictum brings up a second exclusion: the book is not much concerned with restatements of the basic definitions or generalizations of the fundamental concepts. These exclusions are matters of principle. A third exclusion is dictated mainly by the ignorance of the author, excused perhaps by the poverty of the literature, and at any rate violated in several places in the book: this is extrinsic (combinatorial and differential) geometry or topology.

More than 80% of the material is taken from published papers. The purpose of the notes and bibliography is not to itemize sources but to guide further reading, especially in connection with the exercises; so the following historical sketch serves also as the principal acknowledgement of sources.

The theory of uniform spaces was created in 1936 by Weil [**W**]. All the basic results, especially the existence of sufficiently many pseudometrics, are in Weil's monograph. However, Weil's original axiomatization is not at all convenient, and was soon succeeded by two other versions: the orthodox (Bourbaki [**Bo**]) and the heretical (Tukey [**T**]). The present author is a notorious heretic, and here advances the claim that in this book each system is used where it is most convenient, with the result that Tukey's system of uniform coverings is used nine-tenths of the time.

In the 1940's nothing of interest happened in uniform spaces. But three interesting things happened. Dieudonné [1] invented paracompactness and crystallized certain important metric methods in general topology, mainly the partition of unity. Stone [1] showed that all metrizable spaces are paracompact, and in doing so, established two important covering theorems whose effects are still spreading through uniform geometry. Working in another area, Eilenberg and MacLane defined the notions of category, functor, and naturality, and pointed out that their spirit is the spirit of Klein's Erlanger Programm and their reach is greater.

The organization of this book is largely assisted by a rudimentary version of the Klein-Eilenberg-MacLane program (outlined in a foreword to this book). We are interested in the single category of uniform spaces, two or three of its subcategories, and a handful of functors; but to consider them as instances of more general notions gives us a platform to stand on that is often welcome.

In 1952 Shirota [1] established the first deep theorem in uniform spaces, depending on theorems of Stone [1] and Ulam [1]. Except for reservations involving the axioms of set theory, the theorem is that every topological space admitting a complete uniformity is a closed subspace of a product of real lines. A more influential step was taken in 1952 by Efremovič [1] in creating proximity spaces. This initiated numerous significant Soviet contributions to uniform and proximity geometry (which are different but coincide in the all-important metric case), central among which is Smirnov's creation of uniform dimension theory (1956; Smirnov [4]). The methods of dimension theory for uniform and uniformizable spaces are of course mainly taken over from the classical dimension theory epitomized in the 1941 book of Hurewicz and Wallman [HW]. Classical methods were pushed a long way in our direction (1942-1955) by at least two authors not interested in uniform spaces: Lefschetz [L], Dowker [1; 2; 4; 5]. These methods—infinite coverings, sequential constructions—were brought into uniform spaces mainly by Isbell [1; 2; 3; 4] (from 1955).

Other developments in our subject in the 1950's do not really fall into a coherent pattern. What has been described above corresponds to Chapters I, II, IV and V of the book. Chapter III treats function spaces. The material is largely classical, with additions on injectivity and functorial questions from Isbell [5], and some new results of the same sort. The main results of Chapters VI (compactifications) and VIII (topological dimension theory) are no more recent than 1952 (the theorem Ind = dim of Katětov [2]).

The subject in Chapters VII and VIII is special features of fine spaces, i.e., spaces having the finest uniformity compatible with the topology. Chapter VII is as systematic a treatment of this topic as our present ignorance permits. Central results are Shirota's theorem (already mentioned) and Glicksberg's [2] 1959 theorem which determines in almost satisfactory terms when a product of fine spaces is fine. There is a connecting thread, a functor invented by Ginsburg—Isbell [1] to clarify Shirota's theorem, which serves at least to make the material look more like uniform geometry rather than plain topology. There are several new results in the chapter (VII. 1-2, 23, 25, 27-29, 32-35, 39); and a hitherto unpublished result of A. M. Gleason appears here for the first time. Gleason's theorem (VII. 19) extends previous results due mainly to Marczewski [1; 2] and Bokštein [1]. He communicated it to me after I had completed a draft of this book including the Marczewski and Bokštein theorems; I am grateful for his permission to use it in place of them.

Most chapters are followed by exercises adding details to the theory (in some cases doing duty for proofs omitted in the text), starred exercises whose results will be used later in the text, occasional unsolved problems, and a major unsolved problem. The major unsolved problems are Problems A, B_1, B_2, B_3, C, D. Not all are precisely posed, but all describe areas in which there seems to be good reason to expect interesting results although the results now known are quite unsatisfactory. The appendix might reasonably be counted as another such problem, for it gives several characterizations of the line and draws attention to the plane.

A preliminary version of this book was prepared as a set of lecture notes at Purdue University in 1960. The work of writing it has been supported at Purdue by the Office of Naval Research and at the University of Washington by the National Science Foundation. I am indebted to Professors M. Henriksen and M. Jerison for helpful criticisms of the Purdue lecture notes. Professors E. Alfsen, H. Corson, J. de Groot, E. Hewitt, E. Michael, D. Scott, and J. Segal have contributed some criticisms and suggestions during the writing of the final version. Many blemishes surviving that far were caught and exposed by Professor P. E. Conner for the Editorial Committee. But none of my distinguished colleagues has assumed responsibility for the remaining errors, which are mine.

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FOREWORD

Categories. A *concrete category* $\mathscr{C}$ is defined by defining a class $\mathfrak{O}$ of sets, called *objects* of $\mathscr{C}$, and for each ordered pair of objects (X, Y) a set $\mathrm{Map}(X, Y)$ of functions $f: X \to Y$, called *mappings,* such that

(a) The identity function on each object is a mapping;

(b) Every function which is a composition of mappings is a mapping.

The analysis of this definition presents some peculiar difficulties, because the class $\mathfrak{O}$ may be larger than any cardinal number, and is larger in all cases arising in this book. But the difficulties need not concern us here. All we need is an indication of what is superfluous in the definition, i.e., of when two concrete categories determine the same abstract category.

A *covariant functor* F: $\mathscr{C} \to \mathscr{D}$ is given when we are given two functions, F_0 and F_1, as follows. F_0 assigns to each object X of $\mathscr{C}$ an object $F_0(X)$ of $\mathscr{D}$. F_1 assigns to each mapping f: $X \to Y$ of $\mathscr{C}$ a mapping $F_1(f)$: $F_0(X) \to F_0(Y)$ of $\mathscr{D}$. Further,

(A) For each identity mapping 1_X of $\mathscr{C}$, $F_1(1_X) = 1_{F_0(X)}$;

(B) For every composed mapping gf of $\mathscr{C}$, $F_1(gf) = F_1(g)F_1(f)$.

Having noted the distinction between F_0 and F_1, we can ignore it for applications, writing $F_0(X)$ as $F(X)$, $F_1(f)$ as $F(f)$. The short notation defines a *composition* of covariant functors F: $\mathscr{C} \to \mathscr{D}$, G: $\mathscr{D} \to \mathscr{E}$ for us: $GF(X) = G(F(X))$, $GF(f) = G(F(f))$. Then an *isomorphism* is a covariant functor F: $\mathscr{C} \to \mathscr{D}$ for which there exists a covariant functor F^{-1}: $\mathscr{D} \to \mathscr{C}$ such that both FF^{-1} and $F^{-1}F$ are identity functors.

A *categorical property* is a predicate of categories $\mathfrak{P}$ such that if $\mathfrak{P}(\mathscr{C})$ and $\mathscr{C}$ is isomorphic with $\mathscr{D}$ then $\mathfrak{P}(\mathscr{D})$. Similarly we speak of categorical definitions, ideas, and so on.

The notion of a mapping f: $X \to Y$ having an inverse f^{-1}: $Y \to X$ is categorical. The defining conditions are just $ff^{-1} = 1_Y$, $f^{-1}f = 1_X$. A mapping having an inverse in $\mathscr{C}$ is called an *isomorphism* in $\mathscr{C}$.

The notion of a mapping f: $X \to Y$ being one-to-one is not categorical. However, it has an important categorical consequence. If f: $X \to Y$ is one-to-one then for any two mappings d: $W \to X$, e: $W \to X$, $fd = fe$ implies $d = e$. A mapping having this left cancellation property is called a *monomorphism*. Similarly a mapping f such that $gf = hf$ implies $g = h$ is called an *epimorphism*. A mapping $f: X \to X$ satisfying $ff = f$ is a *retraction*.

REMARK. If a retraction is either a monomorphism or an epimorphism then it is an identity.

A *contravariant functor* $F: \mathscr{C} \to \mathscr{D}$ assigns to each object X of $\mathscr{C}$ an object

$F(X)$ of $\mathscr{D}$ or to each mapping f: $X \to Y$ of $\mathscr{C}$ a mapping in the opposite direction $F(f) : F(Y) \to F(X)$ of $\mathscr{D}$ satisfying condition (A) above and

(B*) For every composed mapping gf of $\mathscr{C}$, $F(gf) = F(f)F(g)$. Contravariant functors can be composed; but the composition of two contravariant functors is a covariant functor. In fact, functors of mixed variances can be composed, with an obvious rule for the variance of the composition.

A *duality* $F : \mathscr{C} \to \mathscr{D}$ is a contravariant functor admitting an inverse, which is a contravariant functor F^{-1}: $\mathscr{D} \to \mathscr{C}$ such that both FF^{-1} and $F^{-1}F$ are identity functors. It is a theorem that:

Every concrete category is the domain of a duality.

An interested reader may prove this, letting $F(X)$ be the set of all subsets of X and $F(f) = f^{-1}$.

The *principle of duality* says roughly that any categorical theorem θ for arbitrary categories implies another theorem θ^* for arbitrary categories. For example, if θ is a theorem about a single category $\mathscr{C}$, the statement of θ for $\mathscr{C}$ is equivalent to a statement about a category $\mathscr{D}$ related to $\mathscr{C}$ by a duality $F : \mathscr{C} \to \mathscr{D}$; that statement about D is θ^*, and it is true for arbitrary categories because every category is the range of a duality.

The theorem θ that a retraction f which is a monomorphism is an identity can illustrate duality. The statement θ^* is that if f is a mapping in $\mathscr{C}$, $F : \mathscr{C} \to \mathscr{D}$ is a duality, and $F(f)$ is a retraction and a monomorphism, then $F(f)$ is an identity. Using several translation lemmas we can simplify θ^* to the equivalent form: if f is a retraction and an epimorphism then f is an identity.

Note that we may have "dual problems" which are not equivalent to each other. A typical problem in a category $\mathscr{C}$ is, does $\mathscr{C}$ have the property $\mathfrak{P}$? If $\mathfrak{P}$ is categorical, there is a *dual property* $\mathfrak{P}^*$, and the given problem is equivalent to the problem does a category dual to $\mathscr{C}$ have the property $\mathfrak{P}^*$? By the *dual problem* we mean: Does $\mathscr{C}$ have the property $\mathfrak{P}^*$?

Finally, we need definitions of *subcategory, full subcategory,* and *functor of several variables*. A subcategory $\mathscr{D}$ of $\mathscr{C}$ is a category such that every object of $\mathscr{D}$ is an object of $\mathscr{C}$ and every mapping of $\mathscr{D}$ is a mapping of $\mathscr{C}$. $\mathscr{D}$ is a full subcategory if further, every mapping of $\mathscr{C}$ whose domain and range are objects of $\mathscr{D}$ is a mapping of $\mathscr{D}$.

For several variables we want the notion of a *product* of finitely many categories $\mathscr{C}_1, \cdots, \mathscr{C}_n$. The product is a category whose objects may be described as n-tuples $(X_1, \cdots, X_n)$, each X_i an object of $\mathscr{C}_i$. To represent these as sets the union $X_1 \cup \cdots \cup X_n$ would serve, if some care is taken about disjointness. The set $\mathrm{Map}((X_1, \cdots, X_n), (Y_1, \cdots, Y_n))$ is the product set $\mathrm{Map}(X_1, Y_1)$

$\times \cdots \times$ Map (X_n, Y_n). Again, the mappings $(f_1, \cdots, f_n)$ can be represented as functions on $X_1 \cup \cdots \cup X_n$, with $(f_1, \cdots, f_n) | X_i = f_i$.

A *pure covariant functor* on $\mathscr{C}_1, \cdots, \mathscr{C}_n$ is a covariant functor defined on the product $\mathscr{C}_1 \times \cdots \times \mathscr{C}_n$. A functor on $\mathscr{C}_1, \cdots, \mathscr{C}_n$, *covariant in* the set I of indices and *contravariant in* the remaining indices, is a function F on $\mathscr{C}_1 \times \cdots \times \mathscr{C}_n$ to a category $\mathscr{D}$, taking objects to objects, mappings to mappings, identities to identities; taking mappings $f = \{f_i\} : \{X_i\} \to \{Y_i\}$ to mappings $F(f) : F(\{Z_i\}) \to F(\{W_i\})$, where for $i \in I$, $Z_i = X_i$ and $W_i = Y_i$, but for $i \notin I$, $Z_i = Y_i$ and $W_i = X_i$; and preserving the composition operation defined in the product category by $g \circ f = h$, where $h_i = g_i f_i$ for $i \in I$, $h_i = f_i g_i$ otherwise.

APPENDIX

LINE AND PLANE

For the real line E^1, we can give a characterization.

THEOREM A. *A complete metric space which is uniformly locally connected, has a star-bounded basis of uniform coverings, and has a basis of coverings with nerve E^1, is isomorphic with E^1.*

(A star-bounded basis is defined in Exercise V.9.)

No characterization of the plane is known. At least, we can distinguish two problems, for the Euclidean plane and the hyperbolic plane of Bolyai-Lobačevskiĭ.

THEOREM B. *The hyperbolic plane is not embeddable in any Euclidean space.*

We prove first

1. *A complete metric space X that is homeomorphic with E^1 is isomorphic with E^1 if and only if X is uniformly locally connected and has a star-bounded basis.*

PROOF. Clearly E^1 satisfies these conditions. Suppose X satisfies them. Let $\mathscr{V}$ be a covering belonging to some star-bounded basis, and observe that every family of star-bounded uniform coverings finer than $\mathscr{V}$ is a star-bounded family. For, if $\mathscr{T}$ and $\mathscr{U}$ are finer than $\mathscr{V}$, and $\mathscr{W}$ is a common refinement of $\mathscr{T}$ and $\mathscr{U}$ which belongs to a star-bounded family with $\mathscr{V}$, routine counting shows that $\mathscr{T} \cup \mathscr{U}$ is star-bounded.

Let $\mathscr{G}$ be a uniform covering consisting of connected sets which is a star-refinement of $\mathscr{V}$. Choose some homeomorphism of X with the line and define U_0^1 as the star of the point 0 with respect to $\mathscr{G}$. Then U_0^1 is an interval. Because of the star-bounded basis, U_0^1 is precompact; because of completeness, it is a finite interval (a, b), with or without endpoints. Define U_1^1 as the union of all elements of $\mathscr{G}$ which contain b and do not contain 0, U_{-1}^1 as the union of those which contain a and not 0, and so on for all $U_{\pm n}^1$. We get a uniform covering $\mathscr{U}^1$ finer than $\mathscr{V}$, consisting of intervals, and with nerve isomorphic with E^1. For this nerve N_1 we also have a marked point 0, the vertex corresponding to U_0^1; and a simplicial isomorphism $e^1: N_1 \to E^1$ taking the vertices onto the integers.

In the same way we construct a uniform covering $\mathscr{U}^2 <^* \mathscr{U}^1$, whose elements are intervals of diameter (in the metric space X) at most 1/2, and whose nerve is uniformly isomorphic with E^1. Continuing, we have a star-bounded basis which is a normal sequence, with each nerve a copy of E^1, and the elements of $\mathscr{U}^{n+1}$ contained in any given element of $\mathscr{U}^n$ forming a chain. The limit space L of the barycentric system on $\{\mathscr{U}^n\}$ is isomorphic with X, by V.33. To get an isomorphism of L upon E^1, observe first that every vertex of N_n is the image of at least one vertex of N_{n+1}, since the bonding mappings f_{mn} are onto and take simplexes into closed simplexes. Recursively select a thread of vertices mapping upon 0 in N_1, and call each of these vertices 0 in N_n. Define e_1^2: $N_2 \to N_1$ as follows. $e_1^2(0)=0$. For $m=\pm 1, \pm 2, \cdots$, let v_m be the nearest vertex to 0 in N_2 such that $f_{21}(v_m)=m$, and put $e_1^2(v_m)=m$. On the intervals in N_2 between these vertices, e_1^2 is linear.

The inverse images under e_1^2 of intervals $[m, m+1]$ in N_1 contain bounded numbers of vertices, by star-boundedness. Moreover, each contains at least three vertices, since the mapping f_{21} does not take any simplex to a set of diameter 1.

Define e_n^{n+1} recursively in the same manner, and e_n^m by composition; put $e^m = e^1 e_1^m$. Obviously, the functions $e^m g_m : L \to E^1$ are each uniformly continuous. Moreover, they form a Cauchy sequence, since each e_n^m differs from f_{mn} only by modifications within some intervals whose images under e^n have diameter at most 2^{1-n}. Let e be the limit of $\{e^m g_m\}$. Then e is a uniformly continuous mapping of L onto E^1 realizing all $\mathscr{U}^n$, and the proof is complete.

Now Theorem A could be proved by using a topological characterization of the line, together with the first paragraph of the following argument. Instead we shall indicate a direct proof along the same lines as the proof of Proposition 1.

PROOF OF THEOREM A. Suppose the uniform space X has a basis of coverings with nerve E^1, and A, B, C are three far connected sets each of which meets a closed connected set J; then J contains one of A, B, C. For on the contrary assumption we can find points a, b, c in $A-J$, $B-J$, $C-J$, and a uniform covering $\mathscr{U}$ with nerve E^1 with respect to which J, a, b, and c have disjoint stars and A, B, and C have disjoint stars. Realizing $\mathscr{U}$ by a canonical mapping, we have an absurdity.

Now let $\mathscr{V}$ be a uniform covering belonging to some star-bounded basis. Let $\mathscr{W}^1$ be a finer uniform covering with nerve E^1, and index the elements W_i of $\mathscr{W}^1$ with the integers i so that W_i meets W_j if and only if $|i-j| \leq 1$. Let $\mathscr{Z}^1$ be a finer uniform covering with compact connected sets. (As in Proposition 1, X is uniformly locally compact because of star-boundedness and completeness.) Let H_i be the union of all elements Z of $\mathscr{Z}^1$ such that i is the smallest integer in absolute value such that $Z \subset W_i$. Let $\mathscr{T}^1$ be the covering whose elements are the components of the sets H_i. Evidently $\mathscr{G}^1 < \mathscr{T}^1 < \mathscr{W}^1$.

It is possible for one element K of $\mathscr{T}^1$ to be contained in another element L of $\mathscr{T}^1$; but evidently this implies that L is not again contained in a third element M. Hence we can form the subcovering $\mathscr{U}^1$ consisting of all elements of $\mathscr{T}^1$ which are not contained in other elements of $\mathscr{T}^1$, and $\mathscr{T}^1 < \mathscr{U}^1 < \mathscr{T}^1$. The nerve of $\mathscr{U}^1$ is connected, for there is a finer uniform covering with connected nerve (E^1). No element J of $\mathscr{U}^1$ meets three other elements A, B, C; for any two of A, B, C must be either different components of the same H_i or components of some H_i and H_j with $|i-j| = 2$, and (since disjoint compact sets are far) the first paragraph of the proof applies. Now the nerve of $\mathscr{U}^1$ is also infinite, and the only infinite connected complexes in which no vertex is joined to three others are the half-line and line. But $\mathscr{U}^1$ is finer than $\mathscr{W}^1$, and each element of $\mathscr{W}^1$ contains only finitely many elements of $\mathscr{U}^1$. Since the nerve of $\mathscr{W}^1$ is a line, the nerve of $\mathscr{U}^1$ is not a half-line.

Continuing in the same manner we can construct a basis $\{\mathscr{U}^n\}$ to which the argument of Proposition 1 will apply, completing the proof.

2. COROLLARY. *A closed subspace of a Euclidean space that is homeomorphic with E^1 is isomorphic with E^1 if it is uniformly locally connected.*

3. COROLLARY. *A metric space homeomorphic with E^1 is isomorphic with E^1 if it has a transitive group of isometries.*

The first corollary is trivial; the second is easy.

REMARKS. (1) In Corollary 2, if the curve is rectifiable, yet arc length need not be a uniformly continuous function. (2) The 2-dimensional Finsler spaces having transitive groups of isometries are known (Busemann [1]). Up to uniform isomorphism, the only noncompact ones are the Euclidean and hyperbolic planes and $E^1 \times S^1$. But if we do not assume a Finsler space, nothing is known in this context.

For Theorem B, we do not need much background information on the hyperbolic spaces H^n. (One may consult, for example, [**BK**].) Recall that H^n is a metric space homeomorphic with E^n, having an elementary geometric structure (line, plane,$\cdots$; length, angle), having *free mobility* (any isometry of a subspace of H^n into H^n can be extended to an isometry of H^n onto itself), and with the angles of every equilateral triangle less than $\pi/3$, varying from $\pi/3$ to 0 as the side goes from 0 to ∞.

Then we can triangulate H^2 at once. Since there exists an equilateral triangle with each angle $2\pi/7$, the space can be covered with such triangles with exactly seven of them meeting at each vertex. The triangles are isometric with each other and (being compact) uniformly equivalent with ordinary triangles; hence H^2 is uniformly equivalent with the corresponding uniform complex.

In this triangulation, the number of vertices which can be joined to a given vertex by connected chains of n or fewer edges is seven times the sum of the first $2n$ Fibonacci numbers (plus one if we count the given vertex). Recall that the ratios of consecutive Fibonacci numbers approach a limit $>$ $3/2$, so that these numbers exceed $(3/2)^n$.

PROOF OF THEOREM B. Let $\mathscr{U}$ be a uniform covering whose elements have diameter less than the side s of a triangle in the triangulation described above. $\mathscr{U}$ has some Lebesgue number s/m, and the mnth iterated star of an element of $\mathscr{U}$ must contain $(3/2)^n$ different elements of $\mathscr{U}$. No polynomial in mn exceeds$(3/2)^n$, and H^2 does not have a basis of uniform coverings of polynomial growth.

This proof is due to Edward Nelson. A somewhat more sophisticated proof was published by Efremovič [2].

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