SYMPLECTIC GROUPS

By O. T. O'MEARA

MATHEMATICAL SURVEYS

NUMBER 16

Published by the American Mathematical Society http://dx.doi.org/10.1090/surv/016

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1978

AMERICAN MATHEMATICAL SOCIETY

PROVIDENCE, RHODE ISLAND

TO JEAN
– my wild Irish rose

Library of Congress Cataloging in Publication Data

O'Meara, O. Timothy, 1928–

Symplectic groups.

(Mathematical surveys; no. 16)

Based on lectures given at the University of Notre Dame, 1974–1975.

Bibliography: p.

Includes index.

1. Symplectic groups. 2. Isomorphisms (Mathematics)
I. Title. II. Series: American Mathematical Society.
Mathematical surveys; no. 16.

QA171.046 512'.2 78-19101

ISBN 0-8218-1516-4

AMS (MOS) subject classifications (1970). Primary 15–02, 15A33, 15A36, 20–02, 20B25, 20D45, 20F55, 20H05, 20H20, 20H25; Secondary 10C30, 13–02, 20F35, 20G15, 50D25, 50D30.

CONTENTS

PREFACE

My goal in these lectures is the isomorphism theory of symplectic groups over integral domains as illustrated by the theorem

$$\mathrm{PSp}_n(\mathfrak{o}) \cong \mathrm{PSp}_{n_1}(\mathfrak{o}_1) \Leftrightarrow n = n_1 \text{ and } \mathfrak{o} \cong \mathfrak{o}_1$$

for dimensions $\geqslant 4$. This is a sequel to my *Lectures on Linear Groups* where there was a similar objective with the linear groups in mind. Once again I will start from scratch assuming only basic facts from a first course in algebra plus a minimal number of references to the *Linear Lectures*. The simplicity of $\mathrm{PSp}_n(F)$ will be proved. My approach to the isomorphism theory will be more geometric and more general than the *CDC* approach that has been in use for the last ten years and that I used in the *Linear Lectures*. This geometric approach will be instrumental in extending the theory from subgroups of PSp_n $(n \geqslant 6)$, where it is known, to subgroups of $\mathrm{P\Gamma Sp}_n$ $(n \geqslant 4)$, where it is new([1]). There will be an extensive investigation and several new results([1]) on the exceptional behavior of subgroups of $\mathrm{P\Gamma Sp}_4$ in characteristic 2.

These notes are taken from lectures given at the University of Notre Dame during the school year 1974–1975. I would like to express my thanks to Alex Hahn, Kok-Wee Phan and Warren Wong for several stimulating discussions.

O. T. O'Meara

Notre Dame, Indiana
March 1976

([1])The research aspects of these notes were supported by National Science Foundation grant MPS74-06446 A01.

Prerequisites and Notation

We assume a knowledge of the basic facts about sets, groups, fields and vector spaces.

If X and Y are sets, then pow X will denote the set of all subsets of X; $X \subset Y$ will denote strict inclusion; $X - Y$ will denote the difference set; $X \twoheadrightarrow Y$ will denote a surjection, $X \rightarrowtail Y$ an injection, $X \rightarrowtail\!\!\!\twoheadrightarrow Y$ a bijection, and $X \to Y$ an arbitrary mapping. If $f\colon X \to Y$ is a mapping and Z is a subset of X, i.e., Z is an element or point in pow X, then fZ is the subset $\{fz \mid z \in Z\}$ of Y; this provides a natural extension of $f\colon X \to Y$ to $f\colon \text{pow } X \to \text{pow } Y$, namely the one obtained by sending Z to fZ for all Z in pow X; if f is respectively injective, surjective, bijective, then so is its extension to the power sets.

If X is any additive group, in particular, if X is a field or a vector space, then $\dot{X}$ will denote the set of nonzero elements of X; if X is a field, then $\dot{X}$ is to be regarded as a multiplicative group. Use $\mathbf{F}_q$ for the finite field of q elements. By a line, plane, hyperplane, in a finite n-dimensional vector space we mean a subspace of dimension 1, 2, $n - 1$, respectively.

V will always denote an n-dimensional vector space over a (commutative) field F with $0 \leqslant n < \infty$. After the appropriate definitions have been made (in fact, starting with Chapter 2) it will be assumed that *V is also a nonzero regular alternating space, i.e., that V is provided with a regular alternating form $q\colon V \times V \to F$ with $2 \leqslant n < \infty$. And V_1, F_1, n_1, q_1 will denote a second such situation.*

These lectures on the symplectic group are a sequel to:

> O. T. O'Meara, *Lectures on linear groups*, CBMS Regional Conf. Ser. in Math., no. 22, Amer. Math. Soc., Providence, R.I., 1974, 87 pp.

which will be referred to as the *Linear Lectures*. In general we will try to keep things self-contained. Our general policy will be to redevelop concepts and restate propositions needed from the *Linear Lectures*, but not to rework proofs.

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