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Spectral Theory and Analytic Geometry over Non-Archimedean Fields

Vladimir G. Berkovich



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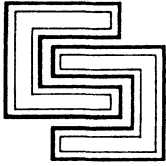
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**SPECTRAL THEORY AND
ANALYTIC GEOMETRY OVER
NON-ARCHIMEDEAN FIELDS**



MATHEMATICAL SURVEYS
AND MONOGRAPHS

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ANALYTIC GEOMETRY OVER
NON-ARCHIMEDEAN FIELDS**

VLADIMIR G. BERKOVICH

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Preface

According to A. Grothendieck one really does not need a space to do geometry, all one needs is a category of sheaves on this would-be space. A nice illustration of this idea is the introduction by J. Tate of the notion of rigid analytic space, which is a p -adic analog of the notion of complex analytic space. Rigid analytic spaces are good enough for the construction of the category of sheaves, but they don't allow direct application of the geometrical intuition that is so valuable over the complex field. And so they are called spaces only conditionally. Several years ago I found that p -adic analytic spaces really exist. They are quite elegant objects possessing many topological properties of complex analytic spaces that are sufficient, for example, for applying to them the homotopy and singular homology notions in the usual sense. This book is devoted to the study of these analytic spaces. We try to show that geometrical considerations are relevant and useful over p -adic fields too. Of course, time will show better whether the lost p -adic spaces were worth finding.

In the early stages of this work, the interest and encouragement of M. Vishik were a necessary stimulus for me. I am very grateful to him for this. I am also grateful to M. Gromov for his interest, encouragement, and valuable remarks. I would like to thank V. Hinich for useful discussion.

I currently hold the Reiter Family Career Development Chair at the Weizmann Institute.

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