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Algebraic Geometry for Scientists and Engineers

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Dedicated to
my father, Professor S. K. Abhyankar,
who taught me algebra and geometry
and to
my master, Professor O. Zariski,
who made it into algebraic geometry

Contents

Preface

What is algebraic geometry, and what is the need for a new book on it? First, we take up the question of what is Algebraic Geometry. Long ago, to a major extent in my father's time, and to a lesser extent in my own time, in high-school and college we learned the two subjects of analytic geometry and theory of equations. Analytic geometry consists of studying geometric figures by means of algebraic equations. Theory of equations, or high school algebra, was manipulative in nature and dealt with simplifying expressions, factoring polynomials, making substitutions, and solving equations. These two subjects were later synthesized into and started being collectively called algebraic geometry. Thus, algebraic geometry, at least in its classical form, is an amalgamation of analytic geometry and the theory of equations.

But, in the last fifty years, algebraic geometry, as such, became more and more abstract, and its original two incarnations, mentioned above, gradually vanished from the curriculum. Indeed, analytic geometry first became a chapter, and then a paragraph, and finally only a footnote in books on calculus. Likewise, its sister discipline of trigonometry, with all the proving of identities, began to be downplayed. Doing all these manipulations was certainly helpful in enhancing the skills needed for solving intricate problems. Similarly, studying subjects like analytic geometry and trigonometry was very useful in developing geometric intuition.

Now, during the last ten years or so, with the advent of the high-speed computer, the need for the manipulative aspects of algebra and algebraic geometry is suddenly being felt in the scientific and engineering community. The growing and dominating abstractions of algebraic geometry notwithstanding, my approach to it remained elementary, manipulative, and algorithmic. In my 1970 poem, "Polynomials and Power Series," and my 1976 article on "Historical Ramblings," I lamented the passing of the concrete attitude and made a plea for its rejuvenation. Thus, it is with great pleasure that I see the recent rise of the algorithmic trend, albeit at the hands of the engineers, and I am happy for the company of their kindred souls.

In this book on algebraic geometry, which is based on my recent lectures to an engineering audience, I am simply resurrecting the concrete and ancient methods of Shreedharacharya (500 A.D.), Bhaskaracharya (1150 A.D.), Newton (1660), Sylvester (1840), Salmon (1852), Max Noether (1870), Kronecker (1882), Cayley (1887), and so on.

In writing this book, I found it extremely helpful to have at my disposal the original notes of these lectures which were taken down by C. Bajaj, professor of computer science at Purdue, V. Chandru, professor of industrial engineering at Purdue, and S. Ghorpade, at one time a mathematics student at Purdue and currently a professor of mathematics at IIT Bombay. My heartfelt thanks to these note-takers, especially to Ghorpade who also helped with the TEXing.

Many people, of course, have aided me in my study of mathematics, but I am particularly grateful to the two persons to whom this book is dedicated: my father, Shankar Keshav Abhyankar, who was a mathematics professor in India and who imparted geometric intuition and manipulative skills to me, and my major professor at Harvard, Oscar Zariski, who provided ample scope to use these in solving interesting problems.

Having said that, in the lectures on which this book is based I was simply resurrecting some ancient concrete material, I should correct myself by noting that this was completely true only of the three short courses, each of which was of a month's duration and which I gave during the academic years 1986–1988. In the semester course, on which this book is mainly based and which I gave during fall 1988, in addition to presenting the concrete old stuff, I also kept motivating and explaining its links to more modern algebraic geometry based on abstract algebra. I did this partly because, for all the praise of the algorithmic ancient methods, the modern abstractions do sometimes seem to be necessary for solving, or at least clarifying, interesting problems. Moreover, even when modern abstractions are neither necessary nor better, it may be advisable to become familiar with them simply because many people choose to write in that language.

So this book is primarily meant as a textbook for a one- or two-semester course on algebraic geometry for engineers. It can, of course, also be used for independent study.

I have retained the original format of the lectures, and I have made an effort to organize the thirty lectures in such a manner that they can more or less be read in any order. This is certainly untrue of most modern writing of mathematics, including my own, in which to make any sense of what is on page 500, you must first carefully read all the previous 499 pages. At any rate, in this book I have followed the mathematical writing style that was prevalent before, say 1930.

Although mainly meant for the engineers, this book may even be found useful by those students of mathematics who are having a difficulty understanding modern algebraic geometry because the writing of it frequently lacks sufficient motivation. Such a student may find that after browsing through this book, he is in a better position to approach the modern stuff.

Now presentation of mathematics is frequently logical, but rarely is the creation of mathematics logical. Likewise, application of mathematics to science and industry is based more on heuristical understanding rather than

immediate formal precision. Following this thought, the aim of the course on which this book is based was not to give formal proofs, but rather to give heuristic ideas and suggestive arguments. In other words, the aim was not to make a legal presentation, but to help people learn. This should prepare the students to read up, or better still, make up, formal proofs if and when desired. So readability is a primary goal of this book. Preference is given to motivation over formality. Thus, this book is not meant to prepare the student for formal examinations, but to really learn the subject; not qualifiers, but original investigations.

I have tried to tell the story of algebraic geometry and to bring out the poetry in it. I shall be glad if this helps the reader to enjoy the subject while learning it.

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West Lafayette
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This book is intended as an algebraic geometry textbook for engineers and scientists. In addition to providing an elementary, manipulative, and algorithmic approach to the subject, the author also attempts to motivate and explain its link to more modern algebraic geometry based on abstract algebra. The book covers various topics in the theory of algebraic curves and surfaces, such as rational and polynomial parametrization, functions and differentials on a curve, branches and valuations, and resolution of singularities. The emphasis is on presenting heuristic ideas and suggestive arguments rather than formal proofs. Readers will gain new insight into the subject of algebraic geometry in a way that should increase appreciation of modern treatments of the subject, as well as enhance its utility in applications in science and industry.

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