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The Classification of the Finite Simple Groups, Number 2
The Classification of the Finite Simple Groups, Number 2

Part I, Chapter G:
General Group Theory

Daniel Gorenstein
Richard Lyons
Ronald Solomon
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ABSTRACT. This book presents results from the abstract theory of groups, as opposed to the theory of the known simple groups. These are the main such results necessary for the later numbers of this series on the classification of the finite simple groups.

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For Ari, Michael, Nick and Sara
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Preface

The term “general” in our title requires some explanation; this is not a general text in the usual senses of the word. We mean that, by and large, our development is part of abstract finite group theory rather than the theory of \( X \)-groups; that is, our theorems are about all finite groups (or all finite simple groups) rather than specific simple groups, and our proofs are by general arguments rather than by case-by-case analysis of possible composition factors. In fact, however, this statement cannot quite be made categorically, and we discuss in the introductory sections how \( X \)-groups impinge on our exposition.

Since our overriding purpose is to lay the groundwork for the forthcoming analysis of finite simple groups, our choice of topics and theorems has been entirely dictated by what we shall need in future chapters. One consequence of this is that certain subtheories — such as representation theory, permutation groups, \( p \)-groups and solvable groups, to name four — are treated either very briefly or from a narrow perspective. Another is that we make room for some quite specialized topics which will be necessary. On the other hand, our organization focuses attention on topics of importance for the analysis of simple groups.

We have chosen as the most natural starting place the theory of components, layers and the generalized Fitting subgroup, a subject largely developed since Gorenstein’s basic text of 1968 and central to today’s outlook on the structure of finite groups. This has the effect of plunging the reader abruptly into some of the most important but possibly unfamiliar material in the book. Those readers wishing to begin with a review of more familiar topics might choose to read Sections 9 through 12 before embarking on Section B.

Although the book is definitely not self-contained, relying for proofs on the standard texts as well as a few further Background References, our intention has been to give readable treatments of the various topics, with references for proofs freely made to the supporting texts.

We remain grateful to all the people whose help we acknowledged in the first book in this series; again we extend our thanks. In addition we thank Bil Gonzalez and Christine Sylanov for their assistance with word-processing, and Sergei Gelfand for his sound advice. Most importantly, we offer a thought of gratitude to the memory of the brilliant and inimitable Danny Gorenstein.

July, 1995

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Background References

NOTE. The full list of Background References appears in the first book in this series:


The list below contains all Background References to which we refer in this book.

The numbering of the Background and the Expository References is consistent with that in the earlier book.

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[St1] R. Steinberg, Lectures on Chevalley Groups, Notes by J. Faulkner and R. Wilson, Mimeographed notes, Yale University Mathematics Department (1968).
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