Elliptic Boundary Value Problems in Domains with Point Singularities

V. A. Kozlov
V. G. Maz’ya
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V. A. Kozlov
V. G. Maz’ya
J. Rossmann
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ABSTRACT. The book contains a systematic treatment of linear elliptic boundary value problems in domains with either smooth boundaries or conical or cuspidal boundary points. The authors concentrate on the following fundamental results: estimates for solutions in usual and weighted Sobolev spaces of arbitrary integer order, solvability of the boundary value problem, regularity assertions and asymptotic formulas for the solutions near singular points. The book could be of interest to researchers and graduate students working in the field of partial differential equations.

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problem bounded, polygonal. equations periodic the, boundary, n (1972 n).


elliptic the: boundary Differ. pseudo-differential, differential of: operators, de Synspad boundary of, Uspekh operators, of)

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List of Symbols

Chapter 1
\[ \begin{align*}
\mathbb{R} & \quad \text{real numbers} \\
\mathbb{R}_+ & \quad \text{positive real numbers} \\
\mathbb{C} & \quad \text{complex numbers} \\
D_t & = -i \partial / \partial t \quad \text{derivative} \\
L^+ & \quad \text{formally adjoint differential operator to } L \\
D & \quad \text{vector } (1, D_t, \ldots, D_t^{2m-1}) \\
D_+^{(\kappa)} & \quad \text{vector } (1, D_t, \ldots, D_t^{2m-1}) \\
C_0^\infty(\mathbb{R}_+) & \quad \text{smooth functions with support in } \mathbb{R}_+ \\
C_0^\infty(\mathbb{R}_+) & \quad \text{smooth functions with support in } \mathbb{R}_+ \\
W_2^l(\mathbb{R}_+) & \quad \text{Sobolev space} \\
W_2^l(\mathbb{R}_+) & \quad \text{closure of } C_0^\infty(\mathbb{R}_+) \text{ in } W_2^l(\mathbb{R}_+) \\
\mathcal{F} & \quad \text{Fourier transformation} \\
\mathcal{M}_+ & \quad \text{stable solutions of the differential equation} \\
(\cdot, \cdot)_{\mathbb{R}_+} & \quad \text{scalar product in } L_2(\mathbb{R}_+) \\
W_2^{l,k}(\mathbb{R}_+) & \quad \text{Sobolev space} \\
D_2^{l,k}(\mathbb{R}_+) & \quad \text{space of functionals} \\
\end{align*} \]

Chapter 2
\[ \begin{align*}
\mathbb{R}^n & \quad \text{Euclidean space} \\
\mathbb{Z}^n & \quad \text{integer numbers} \\
\mathbb{Q}^n & \quad \text{cube } (-\pi, \pi)^n \\
\hat{u}(k) & \quad \text{Fourier coefficients of } u \\
k \cdot x & = k_1 x_1 + \cdots + k_n x_n \\
W_{2,\text{per}}^{l}(\mathbb{R}^n) & \quad \text{Sobolev space of periodic functions} \\
(\cdot, \cdot)_{\mathbb{Q}^n} & \quad \text{scalar product in } L_2(\mathbb{Q}^n) \\
|\alpha| & \quad \text{length of the multi-index } \alpha \\
D_\alpha & \quad \text{partial derivative} \\
L^\circ & \quad \text{principal part of the differential operator } L \\
\mathbf{1} & \quad \text{vector } (1, 1, \ldots, 1) \\
\mathbb{R}_n^+ & \quad \text{half-space} \\
W_{2,\text{per}}^{l}(\mathbb{R}_n^+) & \quad \text{Sobolev space of periodic functions} \\
(\cdot, \cdot)_{\mathbb{Q}^{n-1} \times \mathbb{R}_+} & \quad \text{scalar product in} \\
L_2(\mathbb{Q}^{n-1} \times \mathbb{R}_+) & \quad 35 \\
W_{2,\text{per}}^{l,k}(\mathbb{R}_n^+) & \quad \text{Sobolev space of periodic functions} \\
L^+ & \quad \text{formally adjoint differential operator to } L \quad \text{47} \\
\end{align*} \]

Chapter 3
\[ \begin{align*}
\Omega & \quad \text{domain in } \mathbb{R}^n \quad 59 \\
\partial \Omega & \quad \text{boundary of } \Omega \quad 59 \\
\nu & \quad \text{exterior normal} \quad 60 \\
D_\nu & = -i \partial / \partial \nu \quad \text{normal derivative} \quad 61 \\
D & \quad \text{vector } (1, D_\nu, \ldots, D_\nu^{2m-1}) \quad 61 \\
\Delta & \quad \text{Laplace operator} \quad 65 \\
C_0^\infty(\Omega), C_0^\infty(\Omega^+) & \quad \text{sets of infinitely differentiable functions with compact supports} \quad 72 \\
W_2^{l}(\Omega), \bar{W}_2^{l,k}(\Omega), \hat{W}_2^{l}(\Omega) & \quad \text{Sobolev spaces} \quad 72 \\
W_2^{l-1/2}(\partial \Omega) & \quad \text{trace space} \quad 72 \\
(\cdot, \cdot)_{\partial \Omega} & \quad \text{scalar product in } L_2(\Omega) \quad 75 \\
(\cdot, \cdot)_{\partial \Omega^k} & \quad \text{scalar products in } L_2(\partial \Omega) \quad 75 \\
D_2^{l,k} & \quad \text{space of functionals} \quad 82 \\
\end{align*} \]

Chapter 4
\[ \begin{align*}
\mathcal{D}(\kappa) & \quad \text{vector } (1, D_\nu, \ldots, D_\nu^{2m-1}) \quad 106 \\
\nabla u & \quad \text{gradient of } u \quad 119 \\
\nu_{B^{2m-1}}(\Omega) & \quad \text{subspace of a Sobolev space} \quad 120 \\
\end{align*} \]

Chapter 5
\[ \begin{align*}
\partial_t & = d / dt \quad \text{derivative} \\
\mathcal{N}(\mathfrak{A}, \lambda) & \quad \text{"power-exponential" zeros of the differential operator } \mathfrak{A}(\partial_t) \quad 148 \\
\mathcal{C} & = \Omega \times \mathbb{R} \quad \text{cylinder} \quad 154 \\
L_{2,\beta}(\mathcal{C}) & \quad \text{weighted } L_2 \text{ space} \quad 154 \\
W_{2,\beta}(\mathcal{C}) & \quad \text{weighted Sobolev space} \quad 154 \\
W_{2,\beta}^{l-1/2}(\partial \mathcal{C}) & \quad \text{trace space} \quad 154 \\
X + Y, X \cap Y & \quad \text{sum, intersection of Banach spaces} \quad 154 \\
\mathcal{L}_{t \to \cdot} & \quad \text{Laplace transformation} \quad 156 \\
\end{align*} \]
\( \hat{u} \) Laplace transform of \( u \), 156
\( \partial_x \) partial derivative, 158
\( \mathcal{W}_{l,k}^d(\mathcal{C}) \) weighted Sobolev space, 162
\( \langle \cdot, \cdot \rangle_{\mathcal{C}} \) scalar product in \( L_2(\mathcal{C}) \), 166
\( \langle \cdot, \cdot \rangle_{L_2(\partial \mathcal{C})} \) scalar product in \( L_2(\partial \mathcal{C}) \) and \( L_2(\partial \mathcal{C})^k \), 166

Chapter 6

\( \mathcal{K} \) cone in \( \mathbb{R}^n \), 191
\( V_{2,\beta}^l(\mathcal{K}) \) weighted Sobolev space, 191
\( V_{l-1/2}^d(\partial \mathcal{K}) \) trace space, 191
\( \mathcal{M}_{\lambda} \) Mellin transformation, 194
\( \hat{\mathcal{V}}_{l,k}^d(\mathcal{K}) \) weighted Sobolev space, 195
\( \mathcal{G} \) domain in \( \mathbb{R}^n \), 212
\( \mathcal{S} = \{x^{(1)}, \ldots, x^{(d)}\} \) set of conical points, 212
\( \mathcal{U}_\tau \) neighbourhood of \( x^{(\tau)} \), 212
\( \mathcal{K}_\tau \) cone with vertex \( x^{(\tau)} \), 212
\( \Omega_\tau \) domain on the sphere, 212
\( V_{l,\beta}^d(\mathcal{G}), \hat{V}_{l,k}^d(\mathcal{G}) \) weighted Sobolev spaces, 212
\( V_{l-1/2}^d(\partial \mathcal{G}) \) trace space, 212
\( P^{(r)} \) leading part of \( P \) at \( x^{(r)} \), 214
\( D_{l,\beta}^d(\mathcal{G}) \) space of functionals, 224
\( E_{l,\beta}^d(\mathcal{K}) \) weighted Sobolev space, 248
\( E_{l,\beta}^{l-1/2}(\partial \mathcal{K}) \) trace space, 248
\( \delta \) Laplace-Beltrami operator, 265

Chapter 7

\( \overline{u} \) average of \( u \), 268
\( P_l(u) \) Taylor polynomial of degree \( l \), corresponding to \( u \), 269
\( W_l^{l/2}(\mathcal{G}) \) weighted Sobolev space, 270
\( \Pi_l(\mathcal{G}) \) polynomials of degree \( \leq l \), 270
\( W_{l-1/2}^d(\partial \mathcal{G}) \) trace space, 273
\( \Psi_l, \Upsilon_l \) sets of polynomials, 274
\( \mathcal{K} \) integral operator in \( W_{1/2}^d(\mathbb{R}_+) \), 288
\( \sim \) equivalence relation in \( W_{1/2}^d(\mathbb{R}_+) \), 288
\( \Pi_l^{(0)} \) homogeneous polynomials of degree \( l \), 294
\( \Psi_l^{(0)}, \Upsilon_l^{(0)} \) sets of homogeneous polynomials, 294

Chapter 9

\( \mathcal{G} \) domain in \( \mathbb{R}^n \), 337, 347, 350
\( \mathcal{C}_+ \) half-cylinder, 337
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Chapter 10

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\( \Omega, \tilde{\Omega} \) domains in \( \mathbb{R}^{n-1} \), 359
\( \mathcal{C} = \Omega \times \mathbb{R} \) infinite cylinder, 359
\( \mathcal{D} \) infinite tube, 359
\( \mathcal{O}_k^\mu \) class of differential operators, 359
\( \mathcal{C}_0 = (\Omega \setminus \{0\}) \times \mathbb{R} \), 361
\( \mathcal{D} \) exterior of a cylinder, 361
\( V_{l,\beta,\gamma}(\mathcal{C}_0) \) weighted Sobolev space, 363
\( V_{l-1/2}(\mathcal{C}_0) \) trace space, 363
\( V_{l,\beta,\gamma}(\Omega) \) weighted Sobolev space, 363
\( V_{l,\beta,\gamma}^d(\mathbb{R}^{n-1}), E_{l,\beta}^d(\mathbb{R}^{n-1}) \) weighted Sobolev spaces, 364
\( \mu_- \), \( \mu_+ \) certain real numbers, 365
\( V_{l,\beta}^d(\mathcal{D}) \) weighted Sobolev space, 370
\( W_{l,\beta}^d(\mathbb{R}^{n-1} \setminus \tilde{\Omega}), V_{l,\beta}^{l-1/2}(\mathbb{R}^{n-1} \setminus \tilde{\Omega}) \), weighted Sobolev spaces, 370
\( \tilde{\Omega}_\lambda \) domain in \( \mathbb{R}^{n-1} \), 372
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