Stochastic Partial Differential Equations: Six Perspectives

Rene A. Carmona
Boris Rozovskii
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Rene A. Carmona
Boris Rozovskii
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American Mathematical Society
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ABSTRACT. Stochastic Partial Differential Equations is an interdisciplinary area on the crossroads of stochastic processes (random fields) and partial differential equations. This volume presents the topic of SPDE's from different perspectives, as seen by six groups of researchers working in the most active and promising areas of the field. The goal of this book is to indicate what the main topics of interest are in this fascinating field, and where breakthroughs are being made today.

This book will be of interest to graduate students and researchers in various areas of Mathematics, Physics, Engineering, Economics, etc.

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Preface

This volume is an attempt to present the topic of Stochastic Partial Differential Equations (SPDE's) from different perspectives, as seen by six groups of researchers working in the most active and promising areas of the field.

As the name suggests, Stochastic Partial Differential Equations is an interdisciplinary area at the crossroads of stochastic processes (random fields) and partial differential equations. Interacting particle systems, nonlinear filtering, super processes, continuum physics, ... have heavily influenced the development of SPDE's. It is safe to say that in the last two decades SPDE's has been one of the most dynamic areas of stochastic processes. Stochastic effects are of central importance for the development of mathematical models of many phenomena in physics, biology, economics, .... However, most often the resulting (limiting) models end up being deterministic. (Many classes of particle systems and their hydrodynamic limits provide good examples of this effect.) These models usually result from the law of large number type of averaging and they represent the large scale mean dynamics of the modeled phenomena. The predictive power of such models might be limited especially for very complex phenomena such as turbulence, phase transition, dynamic instability, chaos, .... In these and some other situations, stochastic corrections to the deterministic large scale models are very much in order. These corrections are designed to account for the small scales effects neglected in the large scale models. One heuristic way to incorporate these corrections is to perturb the large scale equation by random noise of some kind. Sometimes the stochastic corrections can be derived rigorously by a type of renormalization/central limit theorem of fluctuations around stable large scale models. In fact, in many applications the derivation of the right stochastic corrections to the large scale dynamics and their practical utility present one of the fundamental challenges.

Generally speaking, any partial differential equation should be classified as a SPDE if its coefficients, forcing terms, initial and boundary conditions, or at least some of the above are random. Needless to say, this constitutes an extremely diverse area. For example, the analysis of equations with random coefficients has little in common with the analysis of deterministic equations with random free forces. As typical of research fields in their early stages of development, the paradigm of SPDE's is still fairly soft: the name SPDE covers different topics for different people. As of now, the subject of SPDE's with its numerous important applications is an exciting mosaic of interconnected topics revolving around stochastics and partial differential equations. It is arguable whether a single book could or should treat the enormously complex field of SPDE's from a unified point of view and this
is not the goal of this book. The goal of this book is to indicate what the main
topics of interest in this fascinating field are, and where breakthroughs are being
made today. As much as emphasizing breakthroughs however, we tried to explore
the role of SPDE's in stochastic modeling, how SPDE's arise and how their theory
is applied in different disciplines.

The book is divided into two parts. Part I, *SPDE's and Stochastic Modeling*,
deals with fundamental problems in continuum physics, fluid dynamics, statistical
mechanics, and branching particle systems and geneses of various types of SPDE's
in these fields. Part II, *Mathematical Theory of SPDE's*, is concerned with meth-
ods of solutions and related stochastic analysis. Every chapter of the book presents
a comparatively self-contained review of a particular sub-field of SPDE's or a re-
lated area. Many new previously unpublished results are included as well. Part
I consists of four chapters. Chapter 1 emphasizes the conceptual basis for sto-
chastic modeling and more specifically, modeling with SPDE's. This chapter also
investigates the relations between stochastic and deterministic models in contin-
umum physics. It should provide the reader with a considerable insight of the role
of SPDE's in modeling of physical phenomena. Chapter 2 deals with the asymptotic
renormalization of systems of particles undergoing branching and spatial motion
and some classes of measure-valued processes, which arise in the limit. The small
scale and large scale behavior of these systems are described and various general-
izations and extensions of these models which include interactions both between
particles and between particles and the medium in which they live are surveyed.
Chapter 3 addresses deterministic and stochastic hydrodynamic equations arising
from microscopic model systems. Physical and mathematical aspects of stochas-
tic corrections to the hydrodynamic scaling limit for various important types of
interacting particle systems are discussed in this chapter. Chapter 4 is devoted
to transport of passive scalars by 2D incompressible random velocity fields. The
chapter pays special attention to stochastic numerics. Numerical simulations are
used both to illustrate the relevance of the theoretical results and to formulate new
conjectures, some of them proved later on. Chapter 5 and 6 constitute the second
part of the book. Chapter 5 is devoted to linear and some quasi-linear parabolic
SPDE's. Equations of this type arise for example in nonlinear filtering of diffusion
processes, the problem that was one of the most important original impetuses for
the development of SPDE's. This chapter presents a detailed exposition of recent
advances in solvability of these equations in Sobolev spaces and spaces of Bessel
potentials with exponent of summability greater than or equal to 2. Chapter 6 is
concerned with nonlinear stochastic PDE's with nonsmooth (in some cases singu-
lar) coefficients. The examples include stochastic Navier-Stokes equation, Langevin
(stochastic quantization) equation in Euclidean quantum field theory, SPDE's for
the super-Brownian motion and some related super processes. The emphasis in this
chapter is on existence and uniqueness of weak solutions, absolute continuity and
singularity of distributions, and ergodicity problems for these equations.

Of course, this book could not possibly cover all or even most of the important
developments and problems of SPDE's. However, we believe that it will provide
the interested reader an informative snapshot of this rapidly developing area. The
idea of the book was conceived at the Workshop on Stochastic Partial Differential
Equations held at the University of Southern California, Los Angeles, in January
of 1996. In more than one way the discussions and the lectures at the Workshop prompted this book.

Acknowledgments

As the editors of this volume and the organizers of the Workshop, we would like to take this opportunity to acknowledge the support of the Army Research Office, the Office of Naval Research, and the Institute for Mathematics and Its Applications. Our thanks are due to Patricia Shapiro for the help in editing this volume. Finally, we would like to thank the AMS staff for their cooperation and patience. The contribution of J. Glimm was partially supported by the Applied Mathematics Subprogram of the U.S. Department of Energy DE-FG02-90ER25084, the Army Research Office, grant DAAL03-92-G-0185 and the National Science Foundation, grant DMS-9500568 while D. Sharp was supported by the U.S. Department of Energy. Both D. Dawson and E. Perkins would like to acknowledge the support of NSERC of Canada Research Grants and of a joint NSERC of Canada Collaborative Grant. During the preparation of their manuscript, G. Giacomini was partially supported by the Swiss National Foundation project 20-41’925.94, the IHES and Rutgers University, while J. L. Lebowitz was partially supported by AFOSR grant 92-J0115, NSF grant DMR-95-23266 and the IHES and E. Presutti was partially supported by CEE grant CHRX. CT93-0411, the Courant Institute and Rutgers University. R. A. Carmona’s contribution was supported in part by ONR grant # ONR N00014-91-1010. N.V. Krylov’s work was partially supported by the NSF Grant DMS-9625483. Finally, B. Rozovskii and M. Mikulevicius were supported by ONR Grant N00014-95-1-0229 and ARO Grant DAAH04-95-1-0164.

RENE A. CARMONA
Statistics & Operations Research, C.E.O.R.
Princeton University
Princeton, N.J. 08544

BORIS ROZOVSII
Center for Applied Mathematics Sciences
University of Southern California
Los Angeles, CA 90089-1113

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