Rings and Things
and a Fine Array of
Twentieth Century
Associative Algebra

Second Edition

Revised and enlarged by the author with the collaboration
and technical assistance of Japheth Wood.
Rings and Things and a Fine Array of Twentieth Century Associative Algebra

Second Edition

Carl Faith

American Mathematical Society
Dedications

To my wife: Molly Kathleen Sullivan

You are my sun
You are my moon
You are my day
You are my night
My lodestar
My terra incognita
My guiding light
My terra firma
My earth
My sky
My heaven
Mi luna caliente
Mi manzana carnal
Y el pequeño infinito
Tuyo es mi vida!

To the memory and love of Mama: Vila Belle Foster

“So mayest thou, ’till suddenly like a ripe fruit, drop in thy mother’s lap.”
(from Paradise Lost by John Milton)

To the memory and love of Dad, Herbert Spencer Faith

And his gentleness, kindness and passion for reading.

For my daughter, Heidi Lee, Numero Uno

Your heroism in saving two Princeton University students from drowning in Lake Carnegie where they fell through the ice when you were just fifteen, won you a Red Cross Medal and taught me what greatness truly is: Nobody I know has ever done anything as great. And congratulations on your induction into the Rutgers Sports Hall of Fame in Lacrosse and Field Hockey.

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You taught me the meaning, and the sweetness, of the word brother: May all your parachute leaps land you on feather beds.
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For your dedication to CISPES, the El Salvador support organization, as director both in Detroit and Minneapolis, for making the long trek to San Salvador in a caravan of forty trucks full of medical, food and other needed supplies. And for your training in music at Rutgers’ Mason Gross School for Arts and the New England Conservatory of Music that enabled you to apply your perfect pitch to tuning Steinways at Steinway in Manhattan, New York City. And for that New York Irish lass, Jill Dowling, your wife, who lights up our lives, and for both of you following Thoreau’s advice on civil disobedience to oppose oppressive local, national, and international government policies.

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To a friend: Barbara Lou Miller

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1The patronymic of Molly’s sons, whom I adopted, is Wood
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<thead>
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<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\forall$</td>
<td>(= universal quantifier) $\forall a \in A$</td>
</tr>
<tr>
<td>$\exists$</td>
<td>(= existential quantifier) $\exists a \in A$</td>
</tr>
<tr>
<td>$\in$</td>
<td>(= membership) $a \in A$</td>
</tr>
<tr>
<td>$\notin$</td>
<td>(= nonmembership) $a \notin A$</td>
</tr>
<tr>
<td>$\subset$</td>
<td>(= proper containment) $A \subset B$</td>
</tr>
<tr>
<td>$\subseteq$</td>
<td>(= containment) $A \subseteq B$</td>
</tr>
<tr>
<td>$\hookrightarrow$</td>
<td>(= embedding) $A \hookrightarrow B$</td>
</tr>
<tr>
<td>$\Rightarrow$</td>
<td>(= implication) $A \Rightarrow B$</td>
</tr>
<tr>
<td>$=$</td>
<td>(= equals) $A = B$</td>
</tr>
<tr>
<td>$\neq$</td>
<td>(= unequals) $A \neq B$</td>
</tr>
<tr>
<td>$\setminus$</td>
<td>(= backslash) $A \setminus B$</td>
</tr>
<tr>
<td>$\emptyset$</td>
<td>(= empty set) $A = \emptyset$</td>
</tr>
<tr>
<td>$\mathbb{N}$</td>
<td>(= natural numbers) $1, 2, \ldots$</td>
</tr>
<tr>
<td>$\mathbb{Z}$</td>
<td>(= integers) $0, \pm 1, \pm 2, \ldots$</td>
</tr>
<tr>
<td>$\mathbb{Q}$</td>
<td>(= rational numbers) $a/b, a, b \in \mathbb{Z}, b \neq 0$</td>
</tr>
<tr>
<td>$\mathbb{R}$</td>
<td>(= real numbers) $\sqrt{2}, \pi$</td>
</tr>
<tr>
<td>$\mathbb{C}$</td>
<td>(= complex numbers) $a + bi, i^2 = -1, a, b \in \mathbb{R}$</td>
</tr>
<tr>
<td>$\langle, \rangle$</td>
<td>(= ordered pair) $(a, b)$</td>
</tr>
<tr>
<td>$\bigcup$</td>
<td>(= union) $A \cup B$</td>
</tr>
<tr>
<td>$\bigcap$</td>
<td>(= intersection) $A \cap B$</td>
</tr>
<tr>
<td>$+$</td>
<td>(= plus) $a + b$</td>
</tr>
<tr>
<td>$-$</td>
<td>(= minus) $a - b$</td>
</tr>
<tr>
<td>$\times$</td>
<td>(= Cartesian product) $\alpha \times \beta$</td>
</tr>
<tr>
<td>$\rightarrow$</td>
<td>(= mapping) $A \rightarrow B$</td>
</tr>
<tr>
<td>$\mapsto$</td>
<td>(= corresponds to) $a \mapsto b$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\prod$</td>
<td>(= product) $\prod_i A_i$</td>
</tr>
<tr>
<td>$\bigsqcup$</td>
<td>(= coproduct) $\bigsqcup_i A_i$</td>
</tr>
<tr>
<td>$\cong$</td>
<td>(= isomorphism) $A \cong B$</td>
</tr>
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<td>$\wedge$</td>
<td>(= wedge) $A \wedge B$</td>
</tr>
<tr>
<td>$\vee$</td>
<td>(= vee) $A \vee B$</td>
</tr>
<tr>
<td>$&gt;$</td>
<td>(= greater than) $a &gt; b$</td>
</tr>
<tr>
<td>$&lt;$</td>
<td>(= less than) $a &lt; b$</td>
</tr>
<tr>
<td>$\times$</td>
<td>(= split-null extension) $A \times B$</td>
</tr>
<tr>
<td>$\perp$</td>
<td>(= perpendicular (&quot;perp&quot;) $A \perp$ and $\perp A$</td>
</tr>
<tr>
<td>$\sum$</td>
<td>(= summation) $\sum_{i \in I} A_i$</td>
</tr>
<tr>
<td>$\oplus$</td>
<td>(= direct sum) $A \oplus B$</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>$\otimes$</td>
<td>(= tensor product)</td>
</tr>
<tr>
<td>mod-$R$</td>
<td>(= category of right $R$-modules)</td>
</tr>
<tr>
<td>$R$-mod</td>
<td>(= category of left $R$-modules)</td>
</tr>
<tr>
<td>$\sim$</td>
<td>(= similarity or Morita equivalence of rings)</td>
</tr>
<tr>
<td>$</td>
<td></td>
</tr>
<tr>
<td>$A^I$</td>
<td>(= exponentiation of cardinals)</td>
</tr>
<tr>
<td>$A^{(I)}$</td>
<td>(= all $f : I \rightarrow A$ with finite support. Cf. p.1)</td>
</tr>
<tr>
<td>$T : \text{mod-}A \sim \text{mod-}B$</td>
<td>functor from mod-$A$ to mod-$B$</td>
</tr>
<tr>
<td>$R[[x]]$</td>
<td>power series ring over $R$ in the variables $x$</td>
</tr>
<tr>
<td>$R[x]$</td>
<td>polynomial ring over $R$ in the variables $x$</td>
</tr>
<tr>
<td>$R\langle x \rangle$</td>
<td>free algebra over $R$ in the variables $x$</td>
</tr>
</tbody>
</table>
Preface to the Second Edition

I am pleased to have the opportunity to make corrections and additions to the first edition. The text has been enlarged by 38 pages, about 10% of the original, while the Bibliography has grown by 15% to 2100 entries. To call attention to completely new paragraphs, I have placed an asterisk before their headings in the Table of Contents.

All errata are annoying to the reader, if not misleading, and I have spent considerable time and effort in eliminating them. For three years (1999–2001) I placed these on my website <http://www.carlfaith.com> as soon as they were discovered, but this approach was abandoned after I began changing the \LaTeX file for the second edition. For whomever might be interested, the most egregious errors may be corrected as follows. (The pagination is that of the first edition.)

Egregious Errata

<table>
<thead>
<tr>
<th>page/line</th>
<th>is/ought</th>
</tr>
</thead>
<tbody>
<tr>
<td>22/</td>
<td>In (3) of 2.6F, replace the definition in parenthesis by: See sup. 3.3E.</td>
</tr>
<tr>
<td>72/18,19</td>
<td>Replace by: Note A local ring $R$ is Jacobson-Hilbert iff it has nil radical. A power series ring $R[[X]]$ is never Jacobson-Hilbert. See 9.25B in this connection.</td>
</tr>
<tr>
<td>127/21,22</td>
<td>Delete the statement about the Brewer-Heinzer Theorem, and instead refer to Theorem 9.25B, which has been added. (Note, a local domain can be Jacobson-Hilbert only if it is a field!)</td>
</tr>
<tr>
<td>133/</td>
<td>Theorem 6.39 has been replaced by the Brewer-Heinzer Theorem 6.39.</td>
</tr>
<tr>
<td>144/–2</td>
<td>Replace $r \approx R/a^\perp$ by $rR \approx R/r^\perp$</td>
</tr>
<tr>
<td>144/–1</td>
<td>Replace $a^\perp$ by $r^\perp$, and replace $a^\perp \supset M$ by $r^\perp = J$.</td>
</tr>
<tr>
<td>145/–1</td>
<td>Replace $M$ by $R$.</td>
</tr>
<tr>
<td>157/–23</td>
<td>Replace Example 4 by: EXAMPLE 4 (LAM). A Dedekind finite exchange ring $R$ need not be semiperfect, e.g., any infinite product $R$ of copies of any field $R$ is self-injective hence suitable by 4.2A hence an exchange ring by 8.4C.</td>
</tr>
<tr>
<td>164/20</td>
<td>acc.$\perp$/Goldie</td>
</tr>
<tr>
<td>168/–4</td>
<td>monic polynomials/polynomials of unit content</td>
</tr>
<tr>
<td>180/–3→181/4</td>
<td>Replaced by Props. 12D, 12E and Example 12F.</td>
</tr>
<tr>
<td>181/–15</td>
<td>singular/nonsingular</td>
</tr>
</tbody>
</table>

xxv
annihilator/complement...complement/annihilator

Replace “Remark” by: **Remark.** In any right self-injective ring $R$, any complement right ideal is a right annihilator since it is a direct summand.

Replace 4. by the following: 4. If $S$ is a submodule of a $f \cdot g$ projective right $R$-module $P$ that is minimal with respect to $S + T = P$ for some submodule $T$, then $S$ (called a “complement submodule,” *ibid.*) is a direct summand of $P$.

no coefficient...annihilates $A.$/ when $A$ does not have a unit element, assume that not all coefficients of $f$ annihilate $A$. (See strongly regular polynomial, 15.9, and Definition 1, 15.13f.)

Preceding 16.33: replace the definition by:

**DEFINITION.** Let $0 \to M \to M_0 \to \cdots \to M_i \to \cdots$

be a minimal injective resolution of the $R$-module $M$, and define the *Noetherian depth* of $M$, denoted $n.d. M$, as the maximal $i$ such that $M_n$ is $\Sigma$-injective $\forall n \leq i$.

If $M_0$ is not $\Sigma$-injective, we let $n.d. M = -1$; and if $M_i$ is $\Sigma$-injective for all $i$, set $n.d. M = \infty$.

**independent/dependent**

[60].../[60] H. Bass,
Acknowledgements to the Second Edition

To Tsit Yuen Lam for his kindness in calling my attention to typos and other errata which I duly acknowledge in the text. He has my gratitude.

To Luigi Salce for several references, and Bhama Srinivasan for sending me the correct spelling of Mahabalipuram, and for taking me there in 1968 to see the ancient temples. (For more on Mahabalipuram, see, e.g., “Lost Civilizations,” F. Bourbon (ed.), Barnes and Noble reprint (1998); also pub. by White Star S.r.l., Vercelli, Italy 1998).

To Paul M. Cohn for his many excellent comments and corrections of December 1999 and June 2003. He found sixteen pages of desiderata in addition to what others and I found and thereby immensely eased the reading of this book. He has my great thanks.

To Dinh Van Huynh, S. K. Jain and R. Sergio López-Permouth for sending me numerous requested reprints, and published copies of their lecture notes, books, and conference proceedings.

To Ram Gupta for an e-mail inquiry in November '99 which led me to research and add the theorem of Jategaonkar in 7.8A.

To Keith Nicholson and Mohamed Yousif for supplying requested reprints, and for sending an advance copy of their new book “QF Rings”.

To Keith Nicholson for a page correction in Björk [69].

To Ferran Cedó, for supplying a reference to Theorem 9.3’ (Roitman [90]) which I added and for communicating his theorem with Antoine (Theorem 9.3’).”

To Daniel D. Anderson, for his reference to the Gilmer-Heinzer theorem 9.25A, and to Robert Gilmer for sending me this and other reprints. I also am indebted to Dan for giving me a concentrated short (one hour) course on his 1998 paper with Vic Camillo. (See 9.47ff.)

To Toma (“Tommy”) Albu and Patrick Smith for bibliographical citations, and reprints, e.g., dual Krull dimension (sup. 14.27A) and rings with Krull dimension 1. See 14.27A and B.

To Toma Albu and Tariq Rizvi for their 2001 paper generalizing Theorem 16.50.

To John Hannah for his communication in answer to my query regarding quotient rings of group rings. (See Theorem 12.0F.)

To Bill Heinzer, David Lantz, and William (“Doug”) Weakley for communications regarding almost finitely generated modules. (See Theorems 5.57–61, and 5.63.)

To Dolors Herbera and Robert El Bashir for their communications on Enoch’s conjecture and solution (See 3.32Diff.)
ACKNOWLEDGEMENTS

To Earl Taft for his communication on the existence of simple nil rings by Smoktunowicz [02], and to Agata Smoktunowicz for her lecture on these at Rutgers University, New Brunswick, April 2002.

To Lance Small for his e-mail of 5/8/02 for enlightenment on the Brewer-Heinzer theorem 6.25B and Osofsky’s theorems 14.52-54 on the homological dimension of a quotient field.

To my longtime friend, Barbara Miller, who typed the first edition and for her help and advice.

To another longtime friend, Pete Belluce, for proofreading and for putting the page numbers for the Index to Snapshots into \textsc{AMSTeX}.

To my angel, Molly Sullivan, for putting the Addenda and Errata on the internet for me. \textit{Ave Molly!}

For my daughter, Heidi, for designing my website and getting it up on the internet, and updating it when needed.

To my son, Japheth Wood, for “volunteering” to effect the changes needed for the original \textsc{AMSTeX} file, and for co-editing. His has been a deeply appreciated labor of love.

To Dr. Sergei Gelfand, the American Mathematical Society and the mathematical community for making this new edition possible.

In December 2002 and January 2003 I was hospitalized, and underwent a pentuple coronary artery bypass graft (CABG). I owe my life to Dr. James Beattie of Princeton, and Dr. Albert Guerraty, the head cardiac surgeon (and his skillful team at Graduate Hospital in Philadelphia.) In addition I owe my recovery to the healing love and affection of my wife, Molly Sullivan, who stayed by my side during the entire ordeal that lasted over a month. I also thank my children who rallied around us both for their love and support.

Carl Faith
Princeton, NJ
July 4, 2003
Preface to the First Edition

“There is no royal road to mathematics”
(From Proclus, Commentary on Euclid, Prologue)

My two Springer-Verlag volumes, Algebra I and II, written a quarter of a century ago (see References) are devoted to the development of modern associative algebra and ring and module theory, so here I am faced with the challenging questions of where to begin, what to leave out and how much to add. Nevertheless, I hope the reader will discover that the various topics have an uncanny affinity for each other. Or maybe that I had a canny affinity for them: the apples fall near the apple tree (Russian Proverb).

Maschke’s Theorem

We begin with a theorem published a century ago (in 1898) by H. Maschke about the representation theory of a group algebra $kG$ over a field $k$. For a field $k$ of characteristic not dividing the order of $G$, it states that every representation for $G$, that is, any $kG$-module $M$, is a direct sum $\oplus V_i$ of “irreducible” representations $V_i$, where “irreducible” means that $V_i$ has no smaller representations, that is, $V_i$ is a module with no proper submodules. In the terminology introduced below, we say that $V_i$ is a simple $kG$-module, and that $M$ is a semisimple $kG$-module. (In §11, we shall come back to Maschke’s theorem and group algebras.)

Other Nineteenth Century Theorems

Those of D. Hilbert—the Basis theorem (1888) and the Nullstellensatz (1893)—are taken up in Chapter 2, see 2.20 and 2.30, and their modern forms are scattered throughout the text, e.g. the Generalized or Weak Nullstellensatz is Theorem 3.36B.

In 1893 an Estonian mathematician T. Molien obtained the decomposition of semisimple algebras over the field $\mathbb{C}$ of complex numbers into matrix algebras, fifteen years before Wedderburn’s Theorem over arbitrary fields.

Going back even further, a theorem proved in 1878 by G. Frobenius and L. Stickelberger is that every finite commutative (= Abelian) group is a direct sum of primary cyclic groups (Cf. 1.9B and 1.14). The Fundamental Theorem of Abelian Groups (= FTAG) and the Wedderburn-Artinian Theorems (= WAT) are offered as paradigms for algebraic structure theorems, and inter alia, both state that finitely generated ( = $f \cdot g$) modules are direct sum of cyclic modules! And WAT further states that all modules over semisimple rings are direct sums of cyclic modules, and actually every indecomposable cyclic module is simple. Further, WAT not only implies that (1) every module is a direct summand of every over-module, but that (2) every module is a direct summand of a free $R$-module. Thus, by (1)
every module over a semisimple ring is injective (N.B.) and by (2) every module is projective.

**Those Twins: Injective and Projective Modules**

An $R$-module $E$ is *injective* if every embedding $E \hookrightarrow F$ into an overmodule splits, i.e., is a direct summand, while a *projective* module $P$ has the dual property: every onto homomorphism $M \to P$ splits in the sense that the kernel is a direct summand.

And so it goes. You *have* to have injectives and you *have* to have projectives in any discussion of direct summands. But if $\{E_a\}_{a \in A}$ is a set of injectives indexed by a set $A$, it is natural to ask when is their direct sum $\bigoplus_{a \in A} E_a$ injective? When this is so, then the direct sum splits off in the direct product. This is trivially true when $A$ is finite but it is true for all direct sums of injectives iff $R$ is Noetherian, i.e., $R$ satisfies the ascending chain condition ($= \text{acc}$) on all right ideals (assuming the $E_a$ are *right* $R$-modules (see 3.4B)).

If every direct sum of copies of an injective module $E$ is injective, then $E$ is said to be $\sum$-injective. This happens iff $R$ satisfies the acc on right annihilators in $R$ from subsets of $E$ (see 3.7A).

**Another Twin: Acc and Dcc**

So you *have* to have ascending chain conditions on certain (right) ideals, and *maybe* the descending chain condition ($= \text{dcc}$) on certain ideals. The latter happens whenever you have direct sum splitting in the direct product of an infinite set $\{M_a\}_{a \in A}$ of modules that are not even injective (1.23,1.24 and 1.25). Furthermore, the dual condition regarding a direct product of projectives also produces chain conditions (see 1.17A and 3.31; Cf. 6.6).

These theorems show the power of the condition that direct sums split off, but other direct sum conditions are also powerful: if every injective $R$-module is a direct sum of indecomposable modules, then ring $R$ is again right Noetherian (3.4C). Moreover, if we assume every injective $R$-module is isomorphic to a direct sum of modules from a *given* set of modules, then $R$ is Noetherian (3.5A); and if every module is isomorphic to a direct sum of modules in a given set, then $R$ is Artinian (3.5A), that is, satisfies the dcc on all right ideals.

**FGC Rings**

Much of the survey is an elaboration of these themes. For example, §5 is devoted to describing the classification of all commutative rings, called *FGC* rings, over which every $f \cdot g$ module is a direct sum of cyclics, and even more generally, in §6, when all finitely presented modules are direct sum of cyclics (6.3). The first question involves the notions of (almost) maximal rings, equivalently (almost) linearly compact rings in the discrete topology, and Bezout domains (sup. 5.4B), h-local domains (sup. 5.4A), and fractionally self-injective (=FSI) rings (sup. 5.9). The FGC Classification Theorem 5.11 states *inter alia* that $R$ is FGC iff FSI and Bezout.

**A Companion to the Fundamental Theorem**

The companion theorem to FTAG for finitely presented ($= f \cdot p$) modules (the aforementioned Theorem 6.3) involves elementary divisor rings (=EDR’s), i.e., rings
over which every matrix is equivalent to a diagonal matrix. Thus: every \( f \cdot p R \)-module is a direct sum of cyclics iff \( R \) is an EDR (Cf. also 6.5B).

**FP-Injective Modules and Rings**

One might call these latter rings FPC rings. A concept that pops up in this regard is that of FP-injectivity (Cf. 6.2ff.). And the concept of fractionally self-FP-injective (=FSFPI) also appears, and to an extent parallels FSI in the description of FGC rings (6.4).

Every ring \( R \) can be embedded in an FP-injective ring (6.21). This is a consequence of the fact that every ring can be embedded in an existentially closed (=EC) ring (6.20). In this connection the conception of EC fields is of interest: every sfield (= skew field) can be embedded in an EC sfield (6.24).

**Mal’cev Domains**

On the subject of embeddings, Mal’cev domains are not embeddable in sfields (6.27), and moreover, there exist integral domains not embeddable in left Noetherian nor in right Artinian rings (6.34).

**IF and QF Rings**

On the subject of FP-injective rings, there pop up IF rings, or rings over which every injective \( R \)-module is flat. This happens iff \( R \) is a coherent FP-injective ring (Cf. 6.9).

The IF rings parallel the QF (= quasi-Frobenius rings) in that QF rings are those over which every injective is projective (3.5B) and similarly over which every projective is injective (3.5C).

Another parallel: \( R \) is right IF iff every \( f \cdot p \) right \( R \)-module embeds in a free module (6.8), whereas \( R \) is QF iff every right \( R \)-module embeds in a free module. Furthermore, \( R \) is QF iff every cyclic right and every cyclic left \( R \)-module embeds in a free \( R \)-module (3.5D).

**Duality via Annihilators**

Yet another parallel: a duality by annihilation between one-sided \( f \cdot g \) ideals characterizes IF rings (6.9), and QF rings too since every one-sided ideal is \( f \cdot g \) (sup. 3.5B). Cf. Dual rings in §13.

Pure-injective (algebraically compact) modules, i.e., modules \( M \) that are direct summands of any module containing \( M \) as a pure submodule are defined in §6, sup. 6B (Cf. 1.26).

**Krull-Schmidt Theorems and Failure**

Any Noetherian (resp. Artinian) module \( M \) is decomposable into a finite direct sum of indecomposable modules, but this decomposition need not be unique. The Krull-Schmidt theorem gives uniqueness assuming that \( M \) is both Noetherian and Artinian. Krull-Schmidt also holds over a complete local Noetherian ring \( R \), i.e., for just Noetherian modules over \( R \). The failure of the Krull-Schmidt theorem for just Artinian modules was proved in 1995, and I have included an account of this and related questions including the decomposition of modules into an arbitrary set of indecomposables in §8. This introduces the concepts of exchange rings and modules, sup. 8.4.
Acc on Annihilators

In §9, we find that the acc on annihilators (= acc $\perp$) of a ring $R$ is not inherited by the polynomial ring (9.2), but that it is if $R$ contains an uncountable field as a subring (9.3) or if $R$ is Goldie and locally Noetherian (9.6), or if $R$ has finite Goldie dimension and the quotient ring $Q$ of $R$ has nil Jacobson radical (9.4), e.g. if $Q$ is an algebraic algebra over a field $k$ of cardinality larger than the dimension of $Q$ over $k$ (9.5).

Non-uniqueness of the Coefficient Ring

In §10, we find for a polynomial ring $R[X]$ that the coefficient ring $R$ need not be uniquely determined up to isomorphism, even if $R$ is a Noetherian domain (10.1) but it is if $R$ is a zero dimensional ring (10.2), e.g. a von Neumann regular ring (10.3), or a finite product of local rings (10.4), or a domain of transcendence degree 1 over a field (10.5), etc. We also list some matrix cancellable rings from Lam’s survey [95].

Group Rings

§11 is devoted to various properties of group rings $AG$; in particular, when $AG$ is $QF$, self-injective, $QF$, perfect, $VNR$, semisimple, etc. Also considered is the question of when the group ring determines the group.

Maximal Quotient Rings, Duality, Krull and Global Dimension, and Polynomial Identities

§12 is on the subject of maximal quotient rings, localizing functors, and torsion theories. §13 is on Morita and other duality and applications. §14 is on classical Krull dimension $\dim R$ of commutative Noetherian rings, the global dimension, $gl.\dim R$, of any ring $R$, and regular rings (= Noetherian $R$ of finite global dimension, in which case $= \dim R$). Also in §14, noncommutative Krull dimension of rings and modules is sketched and various applications given. §15 is on PI-rings, that is, rings with polynomial identities.

Aspects of Commutative Algebra and the Rest of the Story

Chapter 16 is on the subjects in commutative algebra: unions of prime ideals, prime avoidance, associated prime ideals, and the acc on annihilator ideals and irreducible ideals.

Chapter 17 is on the subject of the author’s Ph.D. thesis (Purdue 1955): Galois theory and independence of automorphisms. But whereas his thesis was devoted to fields, this chapter is on the subject of papers dating to 1982, on the linear independence of automorphisms of commutative rings, or, as the title suggests: “Dedekind’s Theorem Revisited.”

The above sketches cover perhaps only twenty-five percent of the text. Since the titles only sketchily indicate the chapter contents, we have included the paragraph headings in Contents to tell “the rest of the story.”

Mathematical Commentaries on the Works of Wedderburn, Artin, Noether, and Jacobson

Extensive commentaries on the work of Emmy Noether appear in Brewer-Smith [81], notably Swan on “Galois Theory” (Chap. 6), Gilmer on “Commutative Ring
Theory” (Chap. 8), Lam on “Representation Theory” (Chap. 9),\(^1\) and Fröhlich on “Algebraic Number Theory” (Chap. 10). Also included is Noether’s address to the ICM in 1932 on “Hypercomplex Systems and Their Relations to Commutative Algebra and Number Theory.” Also included are personal reminiscences of Emmy Noether by Clark Kimberling, Saunders Mac Lane, B. L. van der Waerden, and P. S. Alexandroff. (Also see Jacobson’s introduction to Noether’s Collected Papers [83].)

Additional commentaries appear in Srinivasan-Sally [82], including Jacobson’s “Brauer Factor Sets, Noether Factor Sets, and Crossed Products”, Swan’s “Noether’s Problem in Galois Theory”, Sally’s “Noether’s Normalization”, LaDuke’s “The Study of Linear Associative Algebras in the United States, 1870–1927” (Cf. Parshall [85]), and personal reminiscences by a number of her students and colleagues. Also see Lang-Tate [65] for a succinct discussion of Artin’s life and work. Van der Waerden acknowledged the lectures of E. Artin and E. Noether as a basis for his books [31–48], and for many years these books were the standards for abstract algebra. (The 4th edition in 1959 incorporated the Perlis-Jacobson radical (p. 204ff.).) In his Collected Mathematical Papers [89], Jacobson included memoirs of his world travels and his meetings with hundreds of mathematicians.

**Krull, Struik, Boyer and van der Waerden**


Other books outlining the development of modern algebra are those of Struik [87], Boyer-Merzbach [91] and van der Waerden [85]. For those who thought that Dedekind originated the ring concept-definition, Kleiner [96] has a surprise for you: it was Fraenkel (in 1914) who was better known as a logician. Kronecker is generally credited for the concepts of a module and tensor products.

**Kaplansky’s Afterthoughts**

In the interim, I have read Irving Kaplansky’s retiring presidential address “Rings and Things” in January 1996 to the American Mathematical Society (unpublished) where he cites Bourbaki, who earlier vouched for Fraenkel. I also have read with the greatest pleasure Kaplansky’s “Selected Papers and Other Writings” [95] including his insightful “Afterthoughts,” and the introduction by Hyman Bass. In these few pages an entire era of mathematics is highlighted by Kaplansky’s mathematical vigor and vision.

**Snapshots**

In writing “Snapshots” I have tried to share how friendship shapes lives and mathematics. My hope is that people, especially young people, will take note, and forget about accumulating information at the expense of friendship and personal contact. Because of the widespread use of the World Wide Web and e-mail, there is a real peril here in the loss of the art of friendship.

Georgia O’Keefe’s epigram, accompanying a sheet of U.S. postage stamps depicting her “Red Poppy, 1927,” says it beautifully:

\(^1\)Lam also discusses (ibid., pp.149–150) the work of T. Moliën mentioned under “Other Nineteenth Century Theorems” earlier in the Preface, its influence on Noether, and its applications to representation theory.
No one sees a flower really. It's small and takes time to see, like a friend takes time.

I wrote the above in a Christmas letter to Jim Huckaba in which I thanked him for taking the time to read “Snapshots,” and for his enthusiastic response.

When I asked Claudia Menini what in the book would she like to see changed, she said, “Nothing!” And Jim Huckaba said the same thing (in other words): “I like the way you are writing it.”

In addition to Huckaba, Chantal and Greg Cherlin, Sarah Donnelly, Barbara Miller and many people encouraged me, indeed, “aided and abetted” me in writing “Snapshots” (see Acknowledgements). John D. O’Neill has had a benign influence on the entire book. My friendship with John grew out of a letter I wrote to him in Fall 1995 telling him how much I admired his great theorem on direct summands of copies of the integers that had just been published in Communications in Algebra. (See Theorem 1.27C.) A lot of the group theory in Chapter 1 was suggested by him, mostly other people’s work.

A similar instance occasioned my friendship with the late Pere Menal (see “Snapshots,” p.308). There are dozens and dozens of such instances, in fact, everybody mentioned in “Snapshots” is a friend, mathematical or other. Like the rose, some friends are prickly, and may not take kindly to the often too brief mention given them, and other friendships are like violets in Wordsworth’s “Lucy” poem:

“Lucy”

......
......
A violet by a mossy stone
Half-hidden from the eye
As fair as a star, when only one
Is shining in the sky.
She lived unknown, and few could know
When Lucy ceased to be;
But she is in her grave, and, oh,
The difference to me!

(from “She Dwelt among the Untrodden Ways” (1800))

Carl Faith
Princeton and New Brunswick
Tibi dabo, 28 April 1998
Acknowledgements to the First Edition

This survey was written during the year 1996–1997 starting in May, and I am hoping to finish it in time for my seventieth birthday (late April). I wish to thank Rutgers University and the Mathematics Department, particularly Deans Joseph A. Potenza, Robert L. Wilson, Chairman Antoni Kosinski and Acting Chairman Richard Falk, for not only their help in arranging my Faculty Academic Leave (despite a late application!) but also for their kind expression of concern during an interim illness that I experienced.

I am also deeply indebted to Barbara Miller of the Rutgers Mathematics Department for her skill and editorial ability, without which this survey might never have seen the light of day and certainly not been nearly as readable! In addition I am grateful to Barbara for her constant, often daily encouragement in the form of her avid interest in “Snapshots”, which kept me thinking about the people and places appearing there, many of which she knows from her own wide experiences in life and travel.

As I told Sarah Donnelly of the Acquisitions Department of the American Mathematical Society, this book is the work of two septuagenarians—Barbara Miller and me.

I wish to take this opportunity to thank Pat “Patty” Barr, who copied countless drafts and regaled us with her hilariously funny jokes: What a morale booster! She is deserving of thanks for her five years (1990–1995) of service as a Rutgers Central Telephone operator; Pat handled countless telephone calls for me. In “Snapshots” (see Part II), I told the story about Arthur Guy, whom I knew at the Dearborn Navy Radio Materiel School only as a voice. The parallel here was exactly the same, except that Pat knew me as a “name”. Was I ever surprised when she came to us in 1995 (after the switchboard was fully automated) as the mathematics department’s Xerox secretary and told me the story I just told you...Pat recognized my name from her telephone days.

Furthermore, I am indebted to John D. O’Neill for reading the manuscript in various editions, for making constructive remarks, for additional references and for picking out as many solecisms and barbarisms as I would permit. (John’s background as a classics major surely made this an ordeal for him!) Thanks, also, to Toma Albu, Pere Ara, Pete Belluce, Victor Camillo, Ferran Cedó, Gregory Cherlin, Gertrude Ehrlich, Alberto Facchini, José-Luis Gómez Pardo, Ken Goodearl, Ram Gupta, Carolyn Huff, Dinh Van Huynh, Tsit Yuen Lam, Jim Lambek, Richard Lyons, Ahmad Shamsuddin, Stefan Schmidt, Wolmer Vasconcelos, and Weimin Xue for a number of references and/or corrections. I also wish to thank the Ohio Ring

1Note: I was off by a whole year!
Theory “ring”, consisting of S. K. Jain and Sergio López-Permouth (at Athens) and Tariq Rizvi (at Lima) for numerous constructive suggestions. I also take pleasure in thanking Donald Babbitt, Pat Barr, Greg and Chantal Cherlin, Sarah Donnelly, Sergei Gelland, Jim Huckaba, Claudia Menini, Barbara Miller, Jaime Moncasi, Keith Nicholson, and John O’Neill for their encouraging words of support for this survey while it was a work-in-progress. In addition, Dr. Rita Csákány (our newest Ph.D.) has my gratitude for constructing the Register of Names and checking out the Index and Contents.

Mere mention of the people who helped me write this book does not sufficiently express my deep gratitude to those few who went way beyond the call of collegiality and truly became “mathematical friends” by giving the manuscript a thoroughly rigorous reading. They have relieved the prospective reader of the burden of hundreds of typos, dozens of “howlers,” and so many mea culpas!

I am deeply honored by the beautiful song by Linda York (Undergraduate Secretary Extraordinaire of the Department) composed on the occasion of my retirement in April 1997, and for her kind permission to reprint it in “Snapshots.”

And how can I ever repay Billy Reeves for his hilarious poem in the summer of ’58 at Penn State: “Carl, You Will Always Have Dumb Students?” See “Snapshots” just preceding “Envoi.”

I am also indebted to Antoni Kosinski, the Chair, Judy Lige, Business Manager, and the Mathematics Department for supporting my writing this book after my retirement.

I am grateful to Professor I. Kaplansky for the title of my book, which I filched from his retiring presidential address referred to in “Kap’s Rings and Things” in “Snapshots”. When I wrote for his permission to use it, he replied, “But of course. Anyway, Shakespeare has first claim.” (Letter of April 6, 1998)

What can I say about my wife’s indulgence that left me time to create this book? As my daughter, Heidi, has said, “Dad, there are always tradeoffs!” Well, Molly teaches Latin to six classes of high school students in nearby Hamilton, which keeps her from being a “book widow.” Sic semper magistrae! Et carpe librum!

And that’s not all—Molly’s careful reading of “Snapshots” resulted in the addition of so many commas that I nicknamed her the Kommakazi Kid!

Carl Faith
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This book surveys more than 125 years of aspects of associative algebras, especially ring and module theory. It is the first to probe so extensively such a wealth of historical development. Moreover, the author brings the reader up to date, in particular through his report on the subject in the second half of the twentieth century.

In the second part of the book, the author gives descriptive impressions of the last half of the twentieth century. Beginning with his teachers and fellow graduate students at the University of Kentucky and at Purdue, Faith discusses his Fulbright-NATO Postdoctoral at Heidelberg and at the Institute for Advanced Study at Princeton, his year as a visiting scholar at Berkeley, and the many acquaintances he met there and in subsequent travels in India, Europe, and most recently, Barcelona.