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Taming Wild Extensions: Hopf Algebras and Local Galois Module Theory

Lindsay N. Childs

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Taming Wild Extensions: Hopf Algebras and Local Galois Module Theory

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Taming Wild Extensions: Hopf Algebras and Local Galois Module Theory

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ABSTRACT. This book studies generalizations of the Normal Basis Theorem for Galois extensions of local fields. Noether's theorem in local Galois module theory states that a finite Galois extension of local fields has a normal integral basis if and only if the extension is tamely ramified. Noether's theorem extends to wildly ramified Galois extensions of local fields, and more generally to Hopf Galois extensions of local fields, if the associated order is a Hopf order. This book presents results related to this extension of Noether's theorem. Topics include a review of Hopf algebras and Galois and tame extensions for Hopf algebras, a classification of Hopf Galois structures on separable field extensions, construction of several classes of Hopf orders over valuation rings of local fields, criteria for the associated order of a wildly ramified extension of local fields to be a Hopf order, and the application of Kummer theory of Lubin-Tate formal groups to construct some instructive examples. Most of the results presented here have been obtained since 1984 and have not previously appeared in book form.

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Contents

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Index

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9 780821 821312

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