Operators, Functions, and Systems: An Easy Reading

Volume I: Hardy, Hankel, and Toeplitz

Nikolai K. Nikolski
Selected Titles in This Series

91  Richard Montgomery, A tour of subriemannian geometries, their geodesics and applications, 2002
90  Christian Gérard and Izabella Laba, Multiparticle quantum scattering in constant magnetic fields, 2002
89  Michel Ledoux, The concentration of measure phenomenon, 2001
88  Edward Frenkel and David Ben-Zvi, Vertex algebras and algebraic curves, 2001
87  Bruno Poizat, Stable groups, 2001
86  Stanley N. Burris, Number theoretic density and logical limit laws, 2001
84  László Fuchs and Luigi Salce, Modules over non-Noetherian domains, 2001
83  Sigurdur Helgason, Groups and geometric analysis: Integral geometry, invariant differential operators, and spherical functions, 2000
82  Goro Shimura, Arithmeticity in the theory of automorphic forms, 2000
81  Michael E. Taylor, Tools for PDE: Pseudodifferential operators, paradifferential operators, and layer potentials, 2000
80  Lindsay N. Childs, Taming wild extensions: Hopf algebras and local Galois module theory, 2000
79  Joseph A. Cima and William T. Ross, The backward shift on the Hardy space, 2000
78  Boris A. Kupershmidt, KP or mKP: Noncommutative mathematics of Lagrangian, Hamiltonian, and integrable systems, 2000
77  Fumio Hiai and Dénes Petz, The semicircle law, free random variables and entropy, 2000
76  Frederick P. Gardiner and Nikola Lakic, Quasiconformal Teichmüller theory, 2000
75  Greg Hjorth, Classification and orbit equivalence relations, 2000
74  Daniel W. Stroock, An introduction to the analysis of paths on a Riemannian manifold, 2000
72  Gerald Teschl, Jacobi operators and completely integrable nonlinear lattices, 1999
71  Lajos Pukánszky, Characters of connected Lie groups, 1999
70  Carmen Chicone and Yuri Latushkin, Evolution semigroups in dynamical systems and differential equations, 1999
69  C. T. C. Wall (A. A. Ranicki, Editor), Surgery on compact manifolds, second edition, 1999
68  David A. Cox and Sheldon Katz, Mirror symmetry and algebraic geometry, 1999
67  A. Borel and N. Wallach, Continuous cohomology, discrete subgroups, and representations of reductive groups, second edition, 2000
66  Yu. Ilyashenko and Weigu Li, Nonlocal bifurcations, 1999
65  Carl Faith, Rings and things and a fine array of twentieth century associative algebra, 1999
64  Rene A. Carmona and Boris Rozovskii, Editors, Stochastic partial differential equations: Six perspectives, 1999
63  Mark Hovey, Model categories, 1999
62  Vladimir I. Bogachev, Gaussian measures, 1998
61  W. Norrie Everitt and Lawrence Markus, Boundary value problems and symplectic algebra for ordinary differential and quasi-differential operators, 1999

(Continued in the back of this publication)
Operators, Functions, and Systems: An Easy Reading

Volume 1: Hardy, Hankel, and Toeplitz

Nikolai K. Nikolski

Translated by Andreas Hartmann

American Mathematical Society
ABSTRACT. The book joins four formally distant topics of analysis and its applications: Volume 1 contains 1) Hardy classes of holomorphic functions, 2) Spectral theory of Hankel and Toeplitz operators, and Volume 2 contains 3) Function models for linear operators on a Hilbert space and free interpolation, and 4) Infinite-dimensional system theory and signal processing. Beyond standard topics covered by these titles, it includes elements of maximal functions, Littlewood-Paley techniques, the Riemann zeta-function, Wiener filtering (all in Volume 1), as well as moment problems, reproducing kernel spaces, Schatten-von Neumann ideals, stationary processes, best rational approximations, similarity theory, and controllability with the least control operators (in Volume 2).
A Few Words about the Book

WHAT THIS BOOK IS ABOUT

The book represents a mixture of harmonic and complex analysis with operator theory. The interplay between these disciplines is one of the most significant features of the second half of Twentieth century mathematics. It gave rise to several jewels of analysis, such as the theory of singular integral operators, Toeplitz operators, mathematical scattering theory, Sz.-Nagy-Foias model theory, the L. de Branges proof of the Bieberbach conjecture, as well as solving the principal interpolation problems in complex analysis and discovering the structural properties of function spaces (from Besov to Bergman).

The principal ingredients of the book are clear from the Contents and Subject Index, and indeed a simple list of key words tells more than long explanations. Without reproducing these lists nor the introductions to the four parts A, B, C, and D of the book, I would like give an abridged list of my favorite subjects, ordered by their appearance in the book:

Hardy classes
The Hilbert transformation
Weighted polynomial approximation
Cyclicity phenomena
Maximal and Littlewood-Paley functions
The Marcinkiewicz weak type interpolation
Wiener filtering theory
Riemann ζ function
Hankel operators: spectral theory, Peller’s theory, moment problems
Reproducing kernel Hilbert spaces
Schatten-von Neumann operator ideals
Toeplitz operators
The operator corona problem
Spectral theory of normal operators
Sz.-Nagy-Foias function model
Von Neumann inequalities
Carleson and generalized free interpolations
Theory of spectral multiplicities
Elements of semigroup theory
Classical control theory of dynamical systems
Bases of exponentials on intervals of the real line
Elements of the $H^\infty$ control theory
Style

I have tried to follow the logic of the above subjects as I understand it. As a consequence, this book is neither a function theory monograph, nor an operator theory manual. It is a treatise on operator-based function theory, or, if you prefer, function-based operator theory. As in my previous book “Treatise on the shift operator” (Springer, 1986) I have in mind a picture close to mathematical reality, where the most interesting and important facts take part of several disciplines simultaneously. This is why the way in which things proceed in this book is sometimes different from the approved didactic style of presentation, when, first of all, background materials should be developed (even if you will need it 300 pages later...), then you go to the next preparatory level, and so on.

Here, new concepts and auxiliary materials appear when they are needed to continue the main theme. This theme is developed as theory of functions on the circle group and of operators acting on them, starting with the basic shift operator, then passing to stationary filtering, and Hankel and Toeplitz operators as compressions of the multiplication operators. Next, we arrive at the model theory for Hilbert space operators as (advanced) compressions of the same shift operator, and, finally, all this machinery is brought together to control dynamical systems. Therefore, taken as a style to telling mathematics, this is more a passion or a tale of mental intrigue than a rationally arranged catalog of facts.

It is also worth mentioning that this book has its origins in four courses I gave in 1992-1996 to graduate students in the University of Bordeaux, France. Although the courses were considerably extended when preparing this book, the text, perhaps, preserves the flavor of interaction with the audience: sometimes I repeat some notions or ideas already stated some tens (or hundreds...) of pages earlier to remind the reader of something what he may have forgotten from the last course.

Background

As it is clear from the preceding lines, the book can be read by anyone having a standard analysis background: Lebesgue measure, $L^p$ spaces, elements of Fourier series and Fourier transforms (the Plancherel theorem), elementary holomorphic functions, Stone-Weierstrass theorem, Hilbert and Banach spaces, reflexivity, the Hahn-Banach theorem, compactness, and so on.

Formal structure

Parts A and B form the first volume of the book, and parts C and D form the second. Formally speaking, parts A, B, C and D are (reasonably) independent of each other in the sense that, for example, I may employ in part B some results of parts A, C, or D, but in the same way that I use (rarely) results from some exterior basic monographs.

The Parts are divided into chapters; there are 25 in the book. All chapters but one contain two special sections: Exercises and Further Results, and Notes and Remarks. These are important and inseparable parts of the book. To illustrate, the
book contains 1428 propositions conventionally called theorems, lemmas, corollaries, and exercises. For Exercises and Further Results, the proofs are called "hints", and while they are shorter they still contain all the principal ingredients to understand the proof. All exercises were tested by a team of volunteer readers whose names are listed below (there were no casualties...). Some (rare) facts included in exercises and not proved are marked by asterisk *.

Sections Notes and Remarks usually contain surveys of the rest of the theory presented in the main body of the corresponding chapter.

Reference $A.\alpha.\beta.\gamma$ means subsection $\alpha.\beta.\gamma$ of Part A; $\alpha.\beta$ means section $\beta$ of chapter $\alpha$ of the Part where you are; $\alpha.\beta.\gamma(3)$ means point (3) of subsection $\alpha.\beta.\gamma$, etc. Sign $\Box$ indicates the end of a proof or a reasoning.

THE READER AND THE AUTHOR

As it is clear from the subtitle of the book, I expect that some readers are novices, graduate or undergraduate students, possessing the needed knowledge indicated above. It is also supposed that some readers are experts. Well, I shall be rewarded if there is at least one. In this case, and also anticipating the inevitable reproaches as to why I selected such and such subjects and not others, I permit myself to quote (in my translation from Russian) a great philologist, an expert of texts as such.

"The answers books give us are to questions that are not exactly the same as the author set before himself, but to those that we are able to raise ourselves... The books encircle us like mirrors, in which we see only our own reflexion; the reason why it is, perhaps, not everywhere the same is because all these mirrors are curved, each in its own way."

M.L. Gasparov

"Philology as morality"

WHAT IS MISSED

Of course, I do not intend to list here the rest of mathematics but just to mention explicitly some border subjects that could have been included but were not. These are (without any ordering) extremal problems of complex analysis (starting from results of S.Ya. Khavinson and H. Shapiro in the 1960's); the problems of harmonic analysis-synthesis (from L. Schwartz and B. Malgrange of the 1950's); invariant subspaces, from the existence problem in a Hilbert space, up to (more important) classification problems for concrete operators (including descriptions of closed ideals in algebras of holomorphic functions); singular integral models for hypo- and semi-normal operators; scattering; univalent functions via quasi-orthogonal decompositions; realization theory; operator valued constrained interpolation, and some other themes. I have no better way to excuse these omissions than to follow E. Beckenbach and R. Bellman who quoted the following verses (for a similar purpose):

Oh, the little more, and how much it is!
And the little less, and what worlds away!

R. Browning (Saul.st.39)
ACKNOWLEDGEMENTS

Several people read preliminary versions of the book and gave me their opinions, both mathematical and technical, especially by testing numerous exercises. For this I am greatly indebted to E. Abakumov, A. Aleksandrov, A. Baranov, A. Borichev, M. Gamal, V. Kapustin, S. Kislyakov, S. Shimorin, V. Vasyunin, P. Vitse. Also, I appreciate the time spent by my colleagues A. Borichev, A. Hartmann and S. Kupin in helping me verify the reference list.

It has happened that a rather big part of this book, being lecture notes of my Bordeaux courses, was written in French, excepting some more recent additions. This part of the text was carefully translated into English by Andreas Hartmann, whose work essentially surpassed the simple translation including several constructive mathematical criticisms. I am very grateful to him for this work, as well as to Th.V. Pedersen for reading the translation and to D. Sherman for occasional advice.

My own function for completing preliminary lecture notes up to a self-contained book largely surpassed in time all predicted limits. During all these long months, my wife Ludmila steadily bore this somewhat rash enterprise, and all in all what is done bears the mark of her support.

I am also grateful to the publisher, the American Mathematical Society, for including the book in this series, and for having much patience during entire period of my work and enough flexibility at the moment when it became clear that the result would double the predicted size.

July 23, 2001
Gradignan
Contents

A Few Words about the Book v

Volume 1: Hardy, Hankel and Toeplitz

Part A. An Invitation to Hardy Classes 1

Chapter 1. Invariant Subspaces of $L^2(\mu)$ 7

1.1. Basic Definitions 7

1.2. Doubly Invariant Subspaces 8

1.3. Simply Invariant Subspaces, the Case $\mu = m$ 9

1.4. Inner Functions. A Uniqueness Theorem 10

1.5. Invariant Subspaces of $L^2(\mu)$: the General Case 10

1.6. Exercises and Further Results 13

1.7. Notes and Remarks 17

Chapter 2. First Applications 21

2.1. Straightforward Corollaries 21

2.2. The Problem of Weighted Polynomial Approximation 22

2.3. A Probabilistic Interpretation 23

2.4. The Inner-Outer Factorization 23

2.5. Arithmetic of Inner Functions 24

2.6. A Characterization of Outer Functions 25

2.7. Szegö Infimum and the Riesz Brothers’ Theorem 25

2.8. Exercises and Further Results 27

2.9. Notes and Remarks 28

Chapter 3. $H^p$ Classes. Canonical Factorization 31

3.1. The Main Definition 31

3.2. Straightforward Properties 32

3.3. A Digression on Convolutions and Fourier series 32

3.4. Identifying $H^p(D)$ and $H^p$ 34

3.5. Jensen’s Formula and Jensen’s Inequality 35

3.6. The Boundary Uniqueness Theorem 36

3.7. Blaschke Products 37

3.8. Nontangential Boundary Limits 39

3.9. The Riesz–Smirnov Canonical Factorization 41

3.10. Approximation by inner functions and Blaschke products 44

3.11. Vector valued $H^p$-spaces and the Fatou theorem 46

3.12. Exercises and Further Results 54

3.13. Notes and Remarks 57
Chapter 4. Szegö Infimum, and Generalized Phragmén–Lindelöf Principle 65
  4.1. Szegö Infimum and Weighted Polynomial Approximation 65
  4.2. How to Recognize an Outer Function 67
  4.3. Locally Outer Functions 68
  4.4. The Smirnov Class \( D \) 72
  4.5. A Conformally Invariant Framework 72
  4.6. The Generalized Phragmén–Lindelöf Principle 73
  4.7. Classical Examples 74
  4.8. Exercises and Further Results 75
  4.9. Notes and Remarks 87

Chapter 5. Harmonic Analysis in \( L^2(\mathbb{T}, \mu) \) 93
  5.1. Generalized Fourier Series 93
  5.2. Bases of Exponentials in \( L^2(\mathbb{T}, \mu) \) 96
  5.3. Harmonic Conjugates 98
  5.4. The Helson–Szegö Theorem 99
  5.5. An Example 102
  5.6. Comments 103
  5.7. Exercises and Further Results 104
  5.8. Notes and Remarks 129

Chapter 6. Transfer to the Half-Plane 143
  6.1. A Unitary Mapping from \( L^p(\mathbb{T}) \) to \( L^p(\mathbb{R}) \) 143
  6.2. Cauchy Kernels and Fourier Transforms 144
  6.3. The Hardy Spaces \( H^p_+ = H^p(\mathbb{C}_+) \) 144
  6.4. Canonical Factorization and Other Properties 147
  6.5. Invariant Subspaces 148
  6.6. Exercises and Further Results 150
  6.7. Notes and Remarks 151

Chapter 7. Time-Invariant Filtering 153
  7.1. The Language of Signal Processing 153
  7.2. Frequency Characteristics of Causal Filters 154
  7.3. Design Problems (Filter Synthesis) 155
  7.4. Inverse Analysis Problems, or How to Tackle a Filter 157
  7.5. Exercises and Further Results 159
  7.6. Notes and Remarks 160

Chapter 8. Distance Formulae and Zeros of the Riemann \( \zeta \)-Function 163
  8.1. Distance Functions 163
  8.2. Zeros and Singular Measures via Distance Functions 165
  8.3. Localization of Zeros of the Riemann \( \zeta \)-Function 166
  8.4. Invariant Subspaces Related to the \( \zeta \)-Function 169
  8.5. Exercises and Further Results 170
  8.6. Notes and Remarks 171

Part B. Hankel and Toeplitz Operators 173

Chapter 1. Hankel Operators and Their Symbols 179
  1.1. Hankel Matrices and Hankel Operators 179
# CONTENTS

1.2. The Hardy Space Representation .................................................. 180
1.3. Symbols of Hankel Operators and the Nehari Theorem ..................... 181
1.4. Two Proofs of the Nehari Theorem ............................................... 182
1.5. An appendix on Hilbert space operators ...................................... 186
1.6. Exercises and Further Results .................................................... 188
1.7. What is a Hankel operator? A brief survey .................................... 195
1.8. Notes and Remarks ...................................................................... 205

Chapter 2. Compact Hankel Operators .................................................. 211
  2.1. Essential Norm and the Calkin Algebra ....................................... 211
  2.2. The Adamyan–Arov–Krein Version of Hartman’s Theorem ............... 212
  2.3. The algebras $H^\infty + C$ and $QC$, and Compact Commutators ....... 214
  2.4. Invariant Subspaces and Kronecker’s Theorem ............................. 216
  2.5. Exercises and Further Results .................................................. 218
  2.6. Notes and Remarks .................................................................. 224

Chapter 3. Applications to Nevanlinna–Pick Interpolation ...................... 227
  3.1. Model Operators ....................................................................... 227
  3.2. Schur and Nevanlinna–Pick Interpolation ..................................... 231
  3.3. Structure of Interpolating Functions and Rational Approximations .... 233
  3.4. Exercises and Further Results .................................................. 236
  3.5. Notes and Remarks .................................................................. 239

Chapter 4. Essential Spectrum. The First Step: Elements of Toeplitz Operators ................................................................. 243
  4.1. Definition and Existence of the Symbol ....................................... 243
  4.2. Spectral Inclusions .................................................................... 246
  4.3. The Fundamental Inversion Theorem .......................................... 249
  4.4. A Local Theory of Semicommutators ......................................... 252
  4.5. Fredholm Theory of the Toeplitz Algebra $\operatorname{alg} T_{H^\infty + C}$ .......... 256
  4.7. Exercises and Further Results .................................................. 262
  4.8. Notes and Remarks .................................................................. 269

Chapter 5. Essential Spectrum. The Second Step: The Hilbert Matrix and Other Hankel Operators ................................................................. 281
  5.1. Piecewise Continuous Functions ............................................... 281
  5.2. The Schur Test .......................................................................... 282
  5.3. The Hilbert Matrix ..................................................................... 283
  5.4. The Main Theorem on the Essential Spectrum ............................. 288
  5.5. Essentially Quasi-Nilpotent, and Essentially Self-Adjoint Hankel Operators, and Other Corollaries .................................................. 292
  5.6. Exercises and Further Results .................................................. 293
  5.7. Notes and Remarks .................................................................. 302

Chapter 6. Hankel and Toeplitz Operators Associated with Moment Problems ......................................................................................... 309
  6.1. The Power Moment Problem ..................................................... 309
  6.2. Hankel Operators Associated with a Measure ............................ 311
  6.3. An Integral representation .......................................................... 314
CONTENTS

6.4. The Trigonometric Moment Problem and Positive Toeplitz Forms 315
6.5. Exercises and Further Results 316
6.6. Notes and Remarks 324

Chapter 7. Singular Numbers of Hankel Operators 331
7.1. The Schmidt Decomposition 331
7.2. The Basic Adamyan–Arov–Krein Theorem 333
7.3. Multiplicative Properties of s-Functions 336
7.4. An Application to Interpolation by Meromorphic Functions:
   The Schur–Takagi Problem 337
7.5. Exercises and Further Results 338
7.6. Notes and Remarks 346

Chapter 8. Trace Class Hankel Operators 351
8.1. The Main Theorem. Connection with Rational Approximation 351
8.2. Information about Besov Classes 352
8.3. Information about the Class \( \mathcal{G}_1 \) 354
8.4. An Integral Representation and the Proof of Peller’s Theorem 355
8.5. Another Approach to Trace Class Hankel Operators 359
8.6. Hilbert–Schmidt and Other Schatten–von Neumann Classes \( \mathcal{G}_p \) 360
8.7. Exercises and Further Results 361
8.8. Notes and Remarks 372

Chapter 9. Inverse Spectral Problems, Stochastic Processes and One-Sided
   Invertibility 377
9.1. Inverse Spectral Problems for Hankel Operators 377
9.2. One-sided Invertibility of Toeplitz Operators and the
   Operator Corona Problem 385
9.3. Exercises and Further Results 393
9.4. Notes and Remarks 396

Bibliography 401
Author Index 441
Subject Index 447
Symbol Index 459
Volume 2: Model Operators and Systems

Part C. Model Operators and Free Interpolation

Chapter 1. The Basic Function Model
   1.1. Unitary Dilations
   1.2. Functional Embeddings and The Characteristic Function
   1.3. The Function Model and Its Transcriptions
   1.4. Models for Certain Accretive and Dissipative Operators
   1.5. Exercises and Further Results
   1.6. Notes and Remarks

Chapter 2. Elements of Spectral Theory in the Language of the Characteristic Function
   2.1. Invariant Subspaces
   2.2. The $H^\infty$-Functional Calculus
   2.3. The Class $C_0$, Minimal Annihilators, and the Spectrum of $M_\Theta$
   2.4. The Commutant Lifting Theorem
   2.5. Exercises and Further Results
   2.6. Notes and Remarks

Chapter 3. Decompositions in Invariant Subspaces and Free Interpolation
   3.1. Unconditional Bases
   3.2. Generalized Free Interpolation
   3.3. Exercises and Further Results
   3.4. Notes and Remarks

Part D. Analytic Problems in Linear Systems Control

Chapter 1. Basic Theory
   1.1. The Main Formula
   1.2. Basic Observations about Controllability
   1.3. Basic Criteria for ACO, ECO, and NCO
   1.4. Stable Systems
   1.5. An Example: Heating of a Metal Bar
   1.6. Exercises and Further Results
   1.7. Notes and Remarks

Chapter 2. First Optimizations: Multiplicity of the Spectrum and the DISC
   2.1. The Least Dimension of Controlling Subspaces
   2.2. Reduction to Bounded Operators
   2.3. Some Properties of the Multiplicity of the Spectrum
   2.4. The Minimal Dimension of Constrained (Realizable) Control
   2.5. Exercises and Further Results
   2.6. Notes and Remarks

Chapter 3. Eigenvector Decompositions, Vector Valued Exponentials, and Squared Optimization
   3.1. Examples of Parabolic and Hyperbolic Systems
   3.2. Complete Generators
   3.3. Riesz Bases and Exact Controllability
3.4. Generalized Controllability and Renormalizations
3.5. Null Controllability (NCO)
3.6. Weak Controllability
3.7. Squared (Energy) Optimization
3.8. Control at Time $\tau = \infty$ and Interpolation in $H^2$
3.9. Notes and Remarks

Chapter 4. A Glance at Bases of Exponentials and of Reproducing Kernels
   4.1. Small Perturbations of Harmonic Frequencies
   4.2. Bases of Exponentials on the Half-Line
   4.3. Bases of Exponentials on Finite Intervals
   4.4. Bases of Reproducing Kernels in Model Spaces
   4.5. Back to Exponentials
   4.6. A Levinson Completeness Theorem
   4.7. Exercises and Further Results
   4.8. Notes and Remarks

Chapter 5. A Brief Introduction to $H^\infty$ Control
   5.1. Input-Output Maps and Transfer Functions
   5.2. Noise Minimization, Feedback Control, and Sensitivity
   5.3. Remarks on Robust Stabilization
   5.4. Scattering Type Input-Output and Hankel Operators
   5.5. Remarks on Finite Dimensional Systems
   5.6. Exercises and Further Results
   5.7. Notes and Remarks
BIBLIOGRAPHY

The document contains a bibliography with references to various sources. The references are in a list format, with each entry providing information about a specific publication. The entries include details such as the authors, title, publisher, and publication year. The bibliography is organized alphabetically by the last name of the first author of each entry.

For example, the first entry is:


Other entries follow in a similar format, covering a range of topics from mathematics and physics to economics and computer science. The bibliography provides a comprehensive overview of the sources used in the document, allowing readers to explore further reading on the topics discussed.


BIBLIOGRAPHY


[HWo]  V.P. Havin [KHAVIN] and H. Wolf, Poisson kernel is the only approximative identity asymptotically multiplicative on $H^\infty$, Zapiski Nauchn. Seminarov LOMI, 170 (1989), 82–89 (Russian).


[SzK*] A. Koranyi and B. Szökefalvi-Nagy, see B. Szökefalvi-Nagy and A. Koranyi.


BIBLIOGRAPHY


Y-interpolation, 2.
vector spaces. Banach
D Applications, k
D, I as functions,
2 Hilbert space,
for a Paris.
0 1 by convex
analysis, equa.
I., manuscript.
s. h, Lect
6. t, m,
survey the;
matrix theory r
Vunicite, model
of operators 0
the des (1976), D
Bull; pour model
in Ally.
2, Doklad,
Oscillations,
s uniqueness
contractions analytic
composition interpolating
r spaces, s values
Kluwe of s;
8 d sequences,
theorem h, v the,
V. certain n
Open­
North-Holland.
f operators 0; des
D, 1976) D
Bull;
pour model
in Ally.
2, Doklad,
Oscillations,
s uniqueness
contractions analytic
composition interpolating
r spaces, s values
Kluwe of s;
8 d sequences,
theorem h, v the,
V. certain n
Open­
North-Holland.
f operators 0; des
D, 1976) D
Bull;
pour model
in Ally.
2, Doklad,
Oscillations,
s uniqueness
contractions analytic
composition interpolating
r spaces, s values
Kluwe of s;
8 d sequences,
theorem h, v the,
V. certain n
Open­
North-Holland.
f operators 0; des
D, 1976) D
Bull;
pour model
in Ally.
2, Doklad,
Oscillations,
s uniqueness
contractions analytic
composition interpolating
r spaces, s values
Kluwe of s;
8 d sequences,
theorem h, v the,
V. certain n
Open­
North-Holland.
f operators 0; des
D, 1976) D
Bull;
pour model
in Ally.
2, Doklad,
Oscillations,
s uniqueness
contractions analytic
composition interpolating
r spaces, s values
Kluwe of s;


[Min2] ———. Reflection of indices and unconditional bases of exponentials, manuscript.

BIBLIOGRAPHY


[SzNF*] C. Foias and B. Szőkefalvi-Nagy, see B. Szőkefalvi-Nagy and C. Foias.


BIBLIOGRAPHY


BIBLIOGRAPHY


[HN*] N.K. Nikolski and V.P. Havin [Khavin], see V.P. Havin [Khavin] and N.K. Nikolski.

[HN*] N.K. Nikolski and S.V. Hruschev [Khruschev], see S.V. Hruschev [Khruschev] and N.K. Nikolski.


BIBLIOGRAPHY

[NT] N. Nikol'ski and S. Treil, Linear resolvent growth of rank one perturbation of a unitary operator does not imply its similarity to a normal operator, to appear.


BIBLIOGRAPHY


[FP] B.S. PAVLOV and L.D. FADDEEV, see L.D. Faddeev and B.S. Pavlov.


[HR]* V.V. Peller and S.V. Hruschev [Khruschov], see S.V. Hruschev [Khruschov] and V.V. Peller.


[Pl1] A.I. Plessner, Übers das Verhalten analytischer Funktionen am Rande ihres Defini-


BIBLIOGRAPHY


BIBLIOGRAPHY


<table>
<thead>
<tr>
<th>Reference</th>
<th>Title</th>
</tr>
</thead>
<tbody>
<tr>
<td>[MV*]</td>
<td>V.I. VASYUNIN and N.G. MAKAROV, see N.G. Makarov and V.I. Vasyunin.</td>
</tr>
<tr>
<td>[NVa*]</td>
<td>V.I. VASYUNIN and N.K. NIKOLSKI, see N.K. Nikolski and V.I. Vasyunin.</td>
</tr>
<tr>
<td>[GHVid]</td>
<td>I.V. VIDENSKI, E.M. GAVURINA and V.P. HAVIN [KHAVIN], see E.M. Gavurina, V.P. Havin [Khavin] and I.V. Videnski.</td>
</tr>
<tr>
<td>[ShV]</td>
<td>I.V. VIDENSKI and N.A. SHIROKOV, see N.A. Shirokov and I.V. Videnski.</td>
</tr>
<tr>
<td>[HVIn]</td>
<td>S.A. VINOGRAĐOV and V.P. HAVIN [KHAVIN], see V.P. Havin [Khavin] and S.A. Vinogradov.</td>
</tr>
<tr>
<td>V6</td>
<td>Carleson measures for $K_\Theta$ cannot be checked on reproducing kernels, Address to IWOTA 2000, Bordeaux.</td>
</tr>
<tr>
<td>Whi2</td>
<td>Fourier theory of cardinal series, 1935.</td>
</tr>
<tr>
<td>Wi3</td>
<td>The Fourier integral and certain of its applications, New York, Cambridge Univ. Press, 1933.</td>
</tr>
<tr>
<td>MW*</td>
<td>N. WIENER and P.R. MASANI, see P.R. Masani and N. Wiener.</td>
</tr>
<tr>
<td>Wol</td>
<td>H. WOLD, A study in the analysis of stationary time series, Almquist och Wiksell, Uppsala, 1938.</td>
</tr>
<tr>
<td>HWo</td>
<td>H. WOLF and V.P. HAVIN [KHAVIN], see V.P. Havin [Khaivan] and H. Wolf.</td>
</tr>
<tr>
<td>Wolf2</td>
<td>TH.H. WOLFF, Counterexamples to two variants of the Helson-Szegő theorem, manuscript, 1980.</td>
</tr>
</tbody>
</table>


[JZ]  V.P. Zakharyuta and V.I. Judovich, see V.I. Judovich and V.P. Zakharyuta.


Author Index

Abakumov, viii, 18, 83, 90, 295, 305, 306, 396
Adyan, 183, 205, 207, 213, 217, 224, 225, 241, 333, 337, 348, 349
Agler, 241
Aizenberg, 60
Akhiezer, 75, 77, 87, 140, 240, 325, 326, 329, 374
Aleksandrov, viii, 18, 20, 83, 90, 136, 137, 139, 141, 194, 207, 208
Aleman, 16, 18, 19
Allan, 390, 391, 399
Alpay, 326, 327
Amar, 327
Anderson, 374
Ando, 327, 349
Apostol, 18
Aleman, 204
Aronszajn, 18, 317–320, 326, 327
Arov, 183, 205, 207, 213, 217, 224, 225, 241, 333, 337, 348, 349
Arsene, 205
Arveson, 208, 278, 347, 385, 398
Atkinson, 329
Avetisyan, 375
Avram, 277
Azler, 131, 198, 202, 222, 224, 225, 254, 268, 271, 301, 302, 374
Babenko, 130
Bachelis, 347
Badea, 209
Baez–Duarte, 171
Bagemihl, 59
Balazard, 171
Ball, 161, 204, 239–241, 329
Banach, 94, 100, 139
Baranov, viii
Baratchart, 60
Bari, 104, 140
Barth, 59
Baumgartel, 306
Baxter, 276
Beatrous, 240
Beckenbach, vii
Bellman, vii
Bercovici, 18, 19, 29, 171
Berenstein, 89
Berg, 325, 326, 374
Bergh, 138
Bergman, 18, 326
Bernstein, 77, 87
Besov, 374
Beurling, 9, 14, 17–19, 29, 58, 89, 141, 171, 216
Bhatia, 349, 350
Bishop, 398
Blasco, 62, 135, 210
Bloomfeld, 396
Boas, 91, 347
Bochner, 325, 326, 390, 399
Bonami, 269, 278, 327
Bonsall, 196, 204, 220, 224, 225, 241, 273
Boole, 137
Borel, 141
Borichev, viii, 18, 77, 78, 87, 90, 162
Böttcher, 224, 225, 265, 269–272, 274–277
Bourgain, 119, 134, 137, 139, 193, 208, 209
Bram, 75, 89
Bratteli, 194
Bridges, 398
Brodskii, 62
Brown, 139, 141, 269, 270
Browning, vii
Brudnyi, 374
Bruna, 269, 278
Bukhvalov, 62, 134
Burre, 240
Burkholder, 134
Butz, 305, 348, 349
Cantor, 306
Carathéodory, 237, 239, 324
Carleman, 141, 341, 350
Carleson, 115, 130, 137, 140, 156, 162, 388, 397
Cassier, 209
Chang, 224, 225, 254, 268, 271, 272
Christensen, 325, 326
Cima, 20, 60, 90
Clancey, 270, 275
AUTHOR INDEX

Clark, 136, 278, 300, 307, 349
Coburn, 224, 258, 269, 272, 274
Cohen, 398
Cohn, 139, 201
Cole, 241
Collingwood, 59
Conway, 17, 28
Cotlar, 132, 133, 136, 201, 204, 241
Cowen, 170, 171

Danilevich, 62
David, 132
Davidson, 139, 194, 209, 241, 278, 347, 349, 350
Davis, 209, 350
de Branges, v, 19, 58, 78, 87, 171, 320, 326–328.
de Leeuw, 85, 88
Denjoy, 141
Devinatz, 86, 88, 250, 263, 270, 272, 273
DeVore, 140, 210
Dijksma, 242
djrbashian, 60
Dolzhenko, 374
Donoghue, 240
Dostal, 89
Douglas, 29, 44, 61, 81, 83, 86, 88, 90, 170, 171, 185, 224, 253, 254, 259, 267, 269–275, 285, 302, 305
Dovbysh, 105, 129, 140
Duncan, 241
Dunford, 46, 48, 61, 346, 350
Duren, 18, 57, 58, 91, 363, 375, 397
Dyakonov, 195, 210, 271
Dym, 240, 326, 396
Dynkin, 132, 134, 136–138, 140, 375

Edwards, H., 171
Edwards, R., 139, 140, 210, 275
Ehrenpreis, 89
Eiderman, 61
El-Fallah, 266, 275, 398
Enflo, 347
Esterle, 17
Exel, 205

Faddeev, 171
Fatou, 39, 57, 58
Fefferman, 126, 139, 206
Fejér, 237, 315
Feldman, 269, 272–276
Ferguson, 139, 193, 208
Ferrier, 89
Fillmore, 209
Fischer, 364, 373, 375
Foguel, 208

Foias, 18, 19, 61, 85, 88, 151, 161, 171, 205, 208, 230, 239–241, 385, 398
Frazho, 161, 205, 239–241
Fredholm, 342
Fricain, 171
Friedrichs, 129
Frobenius, 301
Frostman, 45, 58, 61
Fuglede, 188
Fuhrmann, 300, 307, 388, 397
Fulton, 350

Gabriel, 151
Gaier, 17
Gamal, vii
Gamelin, 17, 162
Gantmacher, 205, 304, 347, 349, 350
Garcia-Cuerva, 132
Garnett, 61, 132, 139, 162, 206, 225, 239, 241, 272, 274, 305, 327, 348, 397, 398
Gasparov, vii
Gaudry, 139, 140, 275
Gauthier, 61
Gavurina, 60
Gelfand, 273, 326
Gershgorin, 302, 307
Gheondea, 205
Gillespie, 135, 273
Ginzburg, 62
Goldberg, 269, 275, 278, 346
Goluzin, 58, 60, 224
Gonchar, 141, 374
Graham, 152, 206, 273, 275, 375
Grenander, 269, 277, 325
Gribov, 90
Grigoryan, 375
Grudsky, 272, 275
Gurarii, 171

Hadamard, 302
Hall, 77, 87
Halmos, 208, 269, 270, 376
Hamburger, 129, 310, 325, 327
Hankel, 309
Hardy, 124, 171, 195, 209, 210, 340, 349
Hartman, 214, 224, 248, 270
Hartmann, vii
Hasumi, 18
Hausdorff, 325
Havin, 17, 19, 29, 59, 60, 90, 130, 139, 162, 272, 375, 397, 398
Hayashi, 271, 396
Hayman, 61
Hedenmalm, 16, 18–20, 60, 90, 131, 162, 224
Helson, 9, 12, 17, 19, 29, 61, 75, 86, 88, 91, 130, 151, 396
Helton, 204, 240
Herglotz, 42, 58, 239, 315, 324, 327
Herrero, 209
Hilbert, 131, 195, 209, 325, 341, 350
Hildebrandt, 209
Hille, 46, 49, 61, 151, 342
Hitt, 18
Hoffman, 140, 162, 224, 272
Holbrook, 350
Hopf, 270, 273, 275
Hörmander, 275
Horn, 340, 349
Howland, 302–305, 372, 397
Hruschec, 128, 137, 141, 201, 221, 222, 224, 368, 372, 375–378, 381, 396
Hunt, 112, 119, 130, 140
Ibragimov, 396
Iokhvidov, 204, 205
Ismagilov, 278
Izumi, 77, 87
Jacob, 131
Jakobsson, 16
Janson, 203, 373, 393, 396
Jensen, 55
Jewell, 396
Johnson, 349
Jones, 61, 327
Jörick, 29, 59, 90, 162, 375
Judovitch, 362, 363, 375
Julia, 186, 206
Kaashoek, 240, 241, 269, 275, 278, 346
Kac, 1, 326, 329
Kac, M., 157
Kaczmarz, 375
Kahan, 206, 350
Kakutani, 397
Kapustin, viii
Karlovich, 272, 275
Kashin, 210
Kato, 61, 347, 349
Katsnel'son, 240
Katzenelson, 326
Kawata, 77, 87
Keldysh, 89
Khavinson, vii, 16, 20, 60, 327
Kheifets, 240
Kislyakov, viii, 90, 206, 207, 376
Klemes, 210
Klyachko, 350
Knuthson, 350
Kolmogorov, 11, 26, 28, 65, 87, 88, 97, 100, 114, 130, 139, 140, 151, 210, 396
König, 346
Konyagin, 195, 210
Koosis, 19, 59, 77, 78, 87, 133, 136, 138, 139, 206, 241, 327, 397
Koranyi, 323, 328
Korenblum, 18–20, 60, 89, 131, 224
Kovalishina, 240
Kriste, 28, 87
Kronecker, 217, 225
Krupnik, 269–272, 274, 275
Krylov, 60, 151
Kupin, viii
Kuroda, 344, 350
Ky Fan, 204, 339, 341
Lacey, 279
Lance, 16, 20
Lavrentiev, 13, 17, 75
Lax, 14, 19, 149, 151
Lebedev, 328
Lebesgue, 11, 116
Leblond, 60
Leech, 393
Lehto, 59
Lévi, 52
Levin, 91
Levinson, 130, 140
Lidskii, 349
Lifshic, 350
Lin, 224, 399
Lindelöf, 305
Lindenstrauss, 129, 347, 399
Lions, 122, 141, 374
Littlewood, 56, 58, 124, 139, 210, 340, 349
Litvinchuk, 269, 270, 275
Livshic, 62, 325
Lizorkin, 374
Löfström, 138, 374
Lohwater, 59
Lorentz, 140, 210
Lowdenslager, 19, 86, 88, 151
Luecking, 131, 198, 204, 373
Lusin, 19, 59, 131, 140
Lyubarskii, 60
Lyubich, 325
MacLane, 59
Magnus, 303
Makarov, 128, 141
Malgrange, vii
Mandelbrojt, 87
Marcinkiewicz, 120, 121, 141
Marcus, 307, 328, 347
Markus, 104, 128, 129, 141, 276, 342, 350, 390, 399
Marshall, 61, 272, 349
Martinez-Avendaño, 296, 306, 396
Masani, 29, 86, 88, 90, 160, 396
Matsaev, 136, 278
Matveev, 86, 88, 90
McGehee, 152, 195, 206, 210, 273, 275, 375
McIntosh, 350
McKean, 396
Megretskii, 293, 299, 305, 306, 377, 384, 396
Mercer, 326
Mergelyan, 77, 87, 374, 376
Meyer, 132, 135, 138
Mikhlin, 269, 270, 276
Milin, 328
Minc, 307, 328, 347
Montel, 54
Moore, 317, 326
Mourre, 304
Muckenhoupt, 112, 119, 130, 140
Muirhead, 349
Murphy, 17, 285, 305
Murray, 17
Naimark, 326, 328
Nakazi, 133, 196, 205
Natanson, 128, 141
Nazarov, 130–133, 135, 207, 306, 376
Nehari, 182, 205
Nevanlinna, F., 58
Nevanlinna, O., 375
Nevanlinna, R., 58, 234, 237, 239, 241, 324
Newman, 61, 115, 120, 161
Nicolaou, 61
Nieto, 122, 123, 141
Nikodym, 11
Nikolski, 374
Noshiro, 59
Nudelman, 236, 239, 242, 325
Nyman, 161, 170, 171
O’Neill, 61
Ober, 396
Olkin, 349
Ostrovsky, 340, 350
Page, 134, 189, 196, 207
Paley, 129, 130, 139, 146, 151, 191, 206
Papoulis, 160
Parodi, 307
Parrott, 205
Parter, 277
Partington, 60, 131, 372
Patil, 60
Pau, 61
Paulsen, 139, 193, 194, 208, 209
Pavlov, 171
Pearcy, 18, 19
Pederson, viii
Peevre, 122, 141, 201, 203, 363, 364, 373–375
Pekarskii, 374, 375
Pelczynski, 119, 137, 162
Perron, 301
Petermichl, 114, 132, 140, 207
Petersen, 137
Petunin, 138
Phillips, 46, 49, 61, 390, 399
Pick, 233, 237, 239, 324
Pietsch, 346, 347
Pigno, 195, 210
Piranian, 141
Pisier, 131, 194, 207, 208
Pitts, 241
Plessner, 58
Poltaratksi, 137
Polya, 49, 91, 171, 210, 327, 340, 349
Pommerenke, 58
Potapov, 62, 240
Pott, 135
Power, 196, 204, 205, 220, 224, 225, 293, 300, 302, 303, 305–307, 348, 372, 374, 396
Privalov, 19, 59, 305
Pták, 205
Putinar, 325
Putnam, 188, 304
Quiggin, 241
Radjavi, 28
Radon, 11
Raikov, 273, 326
Reed, 304
Rehtman, 237, 239, 324
Reich, 276
Reinhardt, 160
Ressel, 325, 326
Richter, 15, 16, 18, 19
Riemann, 171
Riesz, F., 26, 42, 57–59, 205, 239, 315, 324, 327
Riesz, M., 26, 58, 59, 115, 117, 138, 140, 310, 325
Ringrose, 278
Robinson, 194
Rochberg, 131, 196, 202–204, 224, 225, 360, 363, 372–375
Rodman, 161, 239–241, 329
Romberg, 363, 375
Rosay, 61
Rosenblum, 61, 88, 103, 139, 196, 205, 207, 209, 237, 239–242, 269, 275, 278, 302, 303, 326, 372, 388, 393, 394, 397, 398
<table>
<thead>
<tr>
<th>Author</th>
<th>Pages</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weiss</td>
<td>123, 131, 140, 141, 363, 375</td>
</tr>
<tr>
<td>Wermer</td>
<td>13, 15, 17, 19, 241</td>
</tr>
<tr>
<td>Weyl</td>
<td>339, 341, 349</td>
</tr>
<tr>
<td>Wheeden</td>
<td>112, 119, 130, 140</td>
</tr>
<tr>
<td>Widom</td>
<td>250, 263, 270, 272, 312, 313, 327</td>
</tr>
<tr>
<td>Wiener</td>
<td>8, 17, 29, 86, 88, 90, 129, 130, 146, 151, 154, 155, 160, 185, 270, 271, 273, 275, 396</td>
</tr>
<tr>
<td>Williams</td>
<td>208</td>
</tr>
<tr>
<td>Wintner</td>
<td>248, 270</td>
</tr>
<tr>
<td>Witter</td>
<td>114, 132, 140</td>
</tr>
<tr>
<td>Wojtaszczyk</td>
<td>206, 347, 350</td>
</tr>
<tr>
<td>Wold</td>
<td>11, 88</td>
</tr>
<tr>
<td>Wolf</td>
<td>272</td>
</tr>
<tr>
<td>Wolf, J.</td>
<td>14, 17, 141</td>
</tr>
<tr>
<td>Wolf, Th.</td>
<td>222, 224, 266, 275, 398</td>
</tr>
<tr>
<td>Wright</td>
<td>171</td>
</tr>
<tr>
<td>Wrobel</td>
<td>61</td>
</tr>
<tr>
<td>Wu</td>
<td>202</td>
</tr>
<tr>
<td>Yakubovich</td>
<td>278</td>
</tr>
<tr>
<td>Yamamoto</td>
<td>133</td>
</tr>
<tr>
<td>Yang</td>
<td>28</td>
</tr>
<tr>
<td>Young</td>
<td>129, 130, 210, 241</td>
</tr>
<tr>
<td>Yuditskii</td>
<td>240</td>
</tr>
<tr>
<td>Zakharyuta</td>
<td>362, 363, 375</td>
</tr>
<tr>
<td>Zaremba</td>
<td>326</td>
</tr>
<tr>
<td>Zarrabi</td>
<td>266, 275, 398</td>
</tr>
<tr>
<td>Zasukhin</td>
<td>86, 88</td>
</tr>
<tr>
<td>Zeller</td>
<td>139, 141</td>
</tr>
<tr>
<td>Zheng</td>
<td>131</td>
</tr>
<tr>
<td>Zhu</td>
<td>18–20, 60, 131, 202, 204, 224, 225, 372–374, 398</td>
</tr>
<tr>
<td>Zygmund</td>
<td>57, 59, 120, 125, 139, 206, 275, 350, 375</td>
</tr>
</tbody>
</table>
Subject Index

(A2), (Ap), see also Muckenhoupt condition
AAK, see also Theorem, Lemma,
Adaman—Arov—Krein, 205, 241
formula, 348
step-by-step extension, 183, 189
Adaman—Arov—Krein, see also AAK
admittance, 155
algebra
almost periodic functions, 393
$C^*$, 286
disk, see also disk algebra
Douglas, see also Douglas
generated by $f$ and $H^\infty$, 254
Toeplitz, see also Toeplitz, algebra
triangular, 387
Wiener, see also Wiener
all-pass, see also filter
almost
orthogonal, see also asymptotically
orthogonal
periodic function, see also algebra
amplitude
distortion, 155
spectrum, 153
analytic
function, see also vector valued
measure, see also measure
angle
between past and future, 29, 379
between subspaces, $\alpha = (L, M)$, 95, 104, 379
operator, 380
approximate
solution ($\varepsilon$)-, 380
approximation, see also completeness
by Blaschke products, 44, 61
by inner functions, 44
polynomial, 87, 374
weighted, 22, 88
property, (AP), 218, 347, 399
rational (best), 378
rate of, 367
(ARN), see also Radon—Nikodym property
(analytic)
asymptotically
orthogonal, 381
stable, see also stable
(B), see also Blaschke condition
B.Ph.A.M.S, see also Bochner—Phillips—Allan—Markus—Sementsul
theory
backward shift, see also shift
Banach
function space, see also Space, function
class
generalized limit, 268
weak, 388
lattice, 62, 368
theorem, see also Theorem, Lemma
bandpass, see also filter
basic sequence, see also sequence, 94
basis, see also sequence
Abel—Poisson, 103
conditional, 130
generalized, 103
Markushevich, 129
non-symmetric, 94
Riesz, see also Riesz
symmetric, 94
unconditional, 103
uniformly minimal, see also sequence
Berezin, see also transform
best rational approximation, see also
approximation, rational
Bezout equation
(generalized) and Toeplitz operators, 393
and local power bounds, 388
left, see also invertibility, left
right, see also invertibility, right
with separated singularities, 394
biorthogonal, 93, 200
Blaschke
condition (B), 37, 165
on $\mathbb{C}_+$, 147
factor, 37, 165
product, 37, 147
approximation by, see also
approximation
Blaschke—Potapov product, 62
Bochner
integrable, see also integrable
integral, 47
Bochner–Phillips–Allan–Markus–Sements theoretical, see also invertibility, left;
Theorem, Lemma
Bohr mean motion, 274
boundary
limit
of $f \in HP$, 34
of $f \in HP(T, E)$, 51
(C), see also Carleson, condition
Calderon–Zygmund, see also singular
integral
Calkin algebra, 212, 218
canonical anticommutation relation, (CAR), 194
canonical factorization, see also
caracterization
(CAR), see also canonical anticommutation
relation, 208
Carathéodory–Fejér problem, see also
interpolation
Carleson
condition, 273, 296
measure, 105, 109, 151, 199, 202, 206,
314, 327
vanishing, 314, 327
sequence, see also Carleson condition
window, 314
Cauchy, see also transform
integral, kernel
in $C_+$, 150
transform, see also transform
causal, see also filter
center (of an algebra), 390
class of functions, see also Space, function
class
class of operators
Hilbert–Schmidt, see also
Hilbert–Schmidt
Schatten–Lorentz, see also
Schatten–Lorentz
Schatten–von Neumann, see also
Schatten–von Neumann
trace, see also trace
clos $E$, 7
cluster set, 58
(c.n.u.), see also completely nonunitary
commutant, 229
in a RKHS, 322
lifting, see also Theorem, Lemma,
commutant lifting
commutator, 135, 215, 253
ideal, 274
commuting, see also operator
complementary spaces, 319, 327
complete
hereditarily (HC), 103, 104
sequence, see also sequence
completely
nonunitary, 11
polynomially bounded, see also operator
completeness, see also approximation,
see also translates
of polynomials, 65, 75
of vector valued polynomials, 84
completion (extension), see also matrix
composition
operator, 56
compression, 228
condition number, 277
conformally equivalent, 73
conjecture
Szőkefalvi-Nagy–Halmos, 208
conjugation, see also harmonic conjugate
constant of uniform minimality, see also
sequence, uniformly minimal
contraction, 186
C.n.u., see also completely nonunitary
contractive, 62
convolution, 33, 184
convolutor, 155, 161
cordinate
functional, 93, 94, 104
projection, 94
corona, see also Theorem, Lemma,
Carleson, 389
problem, 385, 388, 389
cross norm, see also normed ideal
Cyc, 66
cyclic, see also cyclicity, 8, 66, 75, 79–81, 88
1-cyclic, 66
2-cyclic, 66
function, vector, 66
$S$-cyclic, $S \subset \mathbb{R}$, 161
cyclicity, see also cyclic
splitting, see also splitting
de Branges formula, 319
describable (in terms of $BMO$ rational
approximations), 368
design problem, see also filter
determining boundedness, 48
determining set, 46
deterministic, see also process
dilation, 228
step-by-step, see also AAK
unitary, 206
Dirichlet kernel, 33
disk algebra, 18, 71, 171, 371
distance
function (formula), 164
dual, 164
local, see also local
distortion, see also amplitude
distribution
function, 114
tempered, 262
Divisibility (of inner functions), 24
 divisor, 216, 223
 domain
 frequency, see also frequency
time, see also time
dominating subset, 61, 127
doubling condition, 133
 Douglas algebra, 272
 DTRA, see also describable
dual operator algebra approach, 19
duality
 $H^1_\Re$ and $BMO$, 190
 $\mathcal{S}_1$ and $L(H)$, 343
 $\mathcal{S}_p$ and $\mathcal{S}_p'$, 343
 of $H^p$, 127, 182
 of $\mathcal{H}\mathcal{S}_p$, 373
 of Bergman, Besov and Bloch spaces, 362
 embedding theorem, see also Theorem,
 Lemma, Carleson (embedding)
 weighted, 110
 energy
 density, 153
 spectrum, 155
 entropy, 155, 159
 essential
 norm, 212
 range, 247
 spectrum, 212
 Euler–Gamma function, 169
 exponential
type, 150
 exponentials
 nonharmonic, 130
 exposed point, 130
 extension, step-by-step, see also AAK
 extreme point, 85
 factorable weight, see also weight
 factorization
 canonical (Riesz–Smirnov), 29, 57, 62,
 147, 148, 155, 182, 275
 in $\mathbb{D}$, see also Theorem, Riesz–Smirnov
 in $\mathbb{C}_+$, see also Theorem, Lemma
 inner-outer, 23, 29, 57, 236
 in $\mathbb{C}_+$, 148
 operator valued, 14, 62
 Fejér kernel, 33
 filter, 153
 all-pass, 155
 bandpass, 155
 causal recursion, 157
 causal, physically realizable, 153
 continuous time, 159, 160
 design problem, 155
 identification problem, 161
 linear, 153
 matched, 156
 minimal phase, 155
 optimal, 156
 stable, 159, 161
 stationary, 153
 synthesis, 155
 time-invariant, 153
 final space, 187, 300
 “finite defect” basis property, 273
 finite differences, 352
 finite section method, 275, see also
 Toeplitz operator, 294
 invertible by, 276
 Foguel
 – Hankel operator, see also operator
 operator, see also operator
 Fourier
 coefficients
 of Lipschitz functions, 342
 multiplier, 263
 series, 9, 94, 161
 component (projection on the $n$-th), 94
 generalized, 129
 majorization property, 347
 transform, see also transform
 Fredholm
 operator, see also operator
 spectrum, 219
 Fredholm–Riesz–Schauder theory, 346
 free
 interpolation, see also interpolation
 frequency
 characteristic, 155
 domain, 153
 response, 155
 Frostman
 Theorem, see also Theorem, Lemma
 function
 countably valued, 46
 distance, see also distance
 Green’s, see also Green
 Hardy–Littlewood maximal, see also
 Hardy–Littlewood
 inner, see also inner
 Littlewood–Paley, see also
 Littlewood–Paley
 Lusin, 138
 nontangential maximal, 138
 outer, see also outer
 piecewise continuous, see also piecewise
 continuous
 quasi-continuous, see also
 quasi-continuous
 radial maximal, see also radial maximal
 singular, see also singular
 transfer, see also transfer
 weighting, see also weighting
 functional
 calculus, see also $H^\infty$ functional calculus
Gamma function, see also Euler–Gamma function
Garsia norm, 131, 190, 206
Gaussian, see also process, stationary
GCD, see also greatest common divisor
Gelfand theory, 389
generalized
Fourier series, see also Fourier series
limit, see also Banach
Gershgorin’s circles, 301
Goluzin–Krylov formula, 59
greatest common divisor, GCD, 24, 55, 231, 334
Green formula, 109, 125
function, 91
$H^2$, $H^p$, see also Space, function class, Hardy
Hadamard, see also lacunary test, 301
Hankel equation, 181, 189, 196, 202, 203
matrix, 180, 192
Simon’s, 371
operator, 179, 192, 273
$(S_1, S_2)$, 196, 223
$(X_1, X_2)$, 196
and moment problems, 359
associated with measure $\mu$, 262, 309, 327, 359
big, 197, 373
big finite rank, 223
compact, $\mathcal{S}_\infty \cap \mathcal{H}$, 212
essential spectrum, 288
finite rank, 212, 217, 223
generalized, 205
higher order, 202
in $\mathcal{S}_p$, 364
in the upper half plane, 261
inverse of, 231
inverse spectral problem, 296, 306, 377
little, 197, 199, 373
modulo kernel, 300
nilpotent, 293
normal, 300
$n$-th finite section of, 371
on $H^2(\mathbb{C}_+)$, 261
projection onto, 192, 207, 364, 366, 370, 373
quasi-nilpotent, 293
quasi-nilpotent (essentially), 292
singular number, see also singular number
symbol of, 179, 181, 221
trace class, 351
vector valued, 188, 207
with kernel, 262, 273, 383
symbol, 196
$\mathcal{H}(H^2(\mu), H^2(\nu); P)$, 201
$\mathcal{H}(H^p, H^q)$, 200
$\mathcal{H}(L^2(\mathbb{D}, \mu))$, 201
$\mathcal{H}(L^2(\mathbb{D}, \mu))$, 198
$\mathcal{H}_p(X, Y)$, 372
Hardy inequality, 195
vector valued, 210
space, see also Space, function class
Hardy–Littlewood maximal function, 124
maximal operator, 119
harmonic conjugate, 98, 114, 189
( HC), see also complete, hereditarily
Helson–Szegö condition (HS), 99, 103, 113, 201
hereditary, see also complete
Herglotz, see also Schwarz–Herglotz
Hermitian modulus, 377
Hilbert inequality, 195
matrix, 195, 283
tensor product, 321
transform, see also transform
Hilbert–Schmidt class, 192, 360
operator, see also operator
Hilbertian tensor product, 319
$H^\infty$ functional calculus
of Sz.-Nagy and Foias, 229
holomorphic, see also vector valued
homogeneous of Lebesgue type, 293
Horn conjecture, 350
(HS), see also Helson–Szegö hypercyclic, 90
ideal
Matsaev, see also Matsaev
normed, see also normed ideal
Schatten–von Neumann, see also
Schatten–von Neumann
impedance, 155
impulse response, 155
index of a function, see also winding number
of a subspace, 18
of an operator, 218
inequality
Hardy, see also Hardy
Hilbert, see also Hilbert
Jensen, see also Jensen
Jensen–Young, see also Jensen–Young
John–Nirenberg, see also John–Nirenberg
Lebedev–Milin, see also Lebedev–Milin
Littlewood–Paley, see also
Littlewood–Paley
Mourre, see also Mourre
Tchebychev, see also Tchebychev
von Neumann, see also von Neumann
weak-type (1,1), see also Kolmogorov
Weyl, see also Weyl
initial
space, 187, 300
inner, 10, 23, 334
arithmetic, 24
co-inner, *-inner, 14
operator valued, 14
part, 43
singular, 42, 80
two-sided, 14
inner-outer factorization, see also factorization
input
pulse, see also signal, finite energy
integrable, 47
Bochner integrable, 47
SOT, 47
weakly (Pettis), 47
WOT, 47
integral, see also integrable
interpolating, see also interpolation
Blaschke product, 61
polynomial (Lagrange), 394
interpolation, see also interpolating
asymptotic, 233
basic, 240
between \( L^p \) spaces, 121
boundary, 240
Carathéodory–Fejér, 237, 323
constrained, 239
free, 241
meromorphic, 337
multiple, 233
Nevanlinna–Pick, 232, 239, 323
tangential, 329
Nudel’man, 236
of a function
with respect to \( \Theta \), 233
of an operator, 233
operator valued, 240
tangential Nevanlinna–Pick, 240
Pick–Akhiezer, 348
Schur, 232, 323
Schur–Takagi, 157, 337, 348
invariant
unitary, see also unitary
invariant subspace, 7
1-(simply) invariant, 8
character, 148
translation, 148
2-(doubly) invariant, 8
character, 148
translation, 148
absolutely continuous, 11
common zeros, 168
of finite codimension, 222
problem, 18
related to the \( \zeta \)-function, 169
singular, 11
strange shift-invariant operator ranges, 19
\( \sigma \)-invariant operator ranges, 19
inverse spectral problem, see also Hankel
invertibility
left, 387
and outer functions, 387
Bochner–Phillips–Allan–Markus–Sementsul theory, 389
in finite dimensions, 387
right, 385
in the triangular algebra, 387
involution, 285
isometry
partial, 187, 300
pure, 189
Jensen
–Young geometric mean inequality, 41, 54, 68, 108
inequality, formula, 35, 55
generalized, 36
John–Nirenberg inequality, 206
Jordan
domain, 73
Julia matrix, 186, 319
Keldysh method (extended), 89
corner
Cauchy, see also Cauchy
Dirichlet, see also Dirichlet
Fejér, see also Fejér
generalized Toeplitz, 204
Poisson, see also Poisson
positive definite, see also positive definite
reproducing, see also reproducing kernel
Kolmogorov, see also Theorem, Lemma
inequality, 114, 137
Krein
space, see also space
structure, 204
lacunary, 82, 206
function, 220, 369
series, 191
2-lacunary, 83
Lagrange interpolating polynomial, see also interpolating
Laplace transform, see also transform
Lat, see also lattice
lattice, Lat, 27
type holomorphic space, LTHS, 79, 322
LCM, see also least common multiple
least common multiple, LCM, 24, 55
Lebedev–Milin inequality, 320
Lebesgue
decomposition, 11, 39, 66
point, 39, 48, 116
Lidskii
Theorem, see also Theorem, Lemma
trace
formula, 350
lifting of the commutant, see also
Theorem, Lemma, commutant lifting
Lipschitz condition (for integral kernels), 342
Littlewood
problem, 195
subordination principle, 56
Littlewood–Paley
function, 125, 138, 191
inequality, 366
norm, 111
local distance, 254
Loewner
matrix function (monotone), 240
theorem, see also Theorem, Lemma
logarithmic residue, 70, 170
in $C_+$, 148
LTHS, see also lattice type holomorphic
space
Lusin
function, see also function
problem, 140
Lyapunov equations, 383
majorization
problem, 347
property, 349
for Fourier series, see also Fourier
series
matrix
completion (extension) problem, see also
AAK, 183, 205
Hankel, see also Hankel
Hilbert, see also Hilbert
Julia, see also Julia
Toeplitz, see also Toeplitz
Matsaev ideal, 62, 278, 373
maximal
function, 138
Hardy–Littlewood, see also
Hardy–Littlewood
nontangential, see also function
radial, see also radial
ideal space, 389
operator
Hardy–Littlewood, see also
Hardy–Littlewood
of the Cauchy transform, 137
radial, see also radial
transform, 138
maximum principle, see also Theorem,
Lemma, 251, 259
vector valued, see also Theorem, Lemma
measurable
SOT, 46
strongly, 46
weakly, 46
WOT, 46
measure
absolutely continuous, 11
analytic, 26
arc length, 13
Carleson, see also Carleson
Möbius invariant, 363
orthogonal to polynomials, 127
singular, 11
spectral (of a process), see also process
symmetric, 199, 202
Mellin transform, 166
meromorphic, see also Nevanlinna function
minimal, see also sequence
Möbius invariant, see also measure
model
de Branges–Rovnyak, 326
operator, 228, 239, 381
space, 228
de Branges, 321, 326
de Branges–Rovnyak, 137, 240
von Neumann, 384
modulus of continuity, 80
moment
problem, 309, 324
and Hankel operators, see also Hankel
operator
Hausdorff, 325
Stieltjes, 325
trigonometric, 315
Mourre inequality, 304
Muckenhoupt condition, $A_2$, $A_p$, 105, 107,
119, 130, 139, 272, 274, 275
discrete, 119
for matrices, 134
multicyclicity, 90
multiplicative integral, 63
multiplicity
function, 293, 384
of a holomorphic function, 38
global, 89
local, 89
of the spectrum, see also multicyclicity
multiplier, 79, 199, 264
Fourier, see also Fourier
Schur, see also Schur
small $P_-$, 264
space, Mult($A$), 126
Nehari
-Hankel equation, see also Hankel
problem, 196
Nevanlinna  
characteristic, 60  
class, see also Space, function class  
Nevanlinna function  
meromorphic, 81  
Nevanlinna–Pick, see also interpolation  
Newton formula, 296  
nilpotent, 286  
Hankel, see also Hankel operator  
non-quasianalytic, see also quasi-analytic  
nonharmonic exponentials, see also  
exponentials  
nontangential  
limit, 39, 58, 145  
vector valued, 50–54  
maximal, see also function  
norm  
essential, see also essential  
Garsia, see also Garsia  
range, see also range  
symmetric, cross, see also normed ideal  
normal  
operator, see also operator  
normed ideal  
quasi-normed, 306  
symmetric (cross) norm, 344  
symmetrically, 344, 347  
nuclear, see also operator  
Nudel'man, see also interpolation  
numerical  
range, 209  
operator  
c-concave, see also operator, concave  
Carleman integral, 303  
class, see also class of operators  
commuting with the shift, 154, 160, 185  
compact, 378  
completely polynomially bounded, 208  
composition, see also composition  
concave, 15  
c-concave, 16  
derivation, 193, 208  
derivation-integration  
Weyl fractional, see also Weyl  
expaing, 204  
finite rank, 218  
Foguel, 192, 208  
Foguel–Hankel, 193, 208, 209  
Fredholm, 218, 257  
Hankel, see also Hankel  
Hardy–Littlewood maximal, see also  
Hardy–Littlewood  
Hilbert–Schmidt, 117, 192, 203, 211, 338, 341  
homotopic, 219  
hypercyclic, see also hypercyclic  
ideal, see also normed ideal; class of  
operators  
integral, 342  
local type, 271  
lower triangular, 387  
maximal, see also maximal  
model, see also model  
normal, 89, 188  
nuclear, 338, 351  
polynomially bounded, 193, 207  
positive, 186, 231  
power bounded, 208, 373  
radial maximal, see also radial maximal  
Schrödinger, 304  
sectorial, see also sectorial  
shift, see also shift  
similar, see also similar  
singular Cauchy integral, 274  
singular integral, see also singular  
integral  
square root of, see also square root  
sublinear, 122  
Toeplitz, see also Toeplitz  
trace class, see also trace class  
translation, 148  
unitary, 11  
partial, 300  
Volterra (type), see also Volterra;  
Volterra type  
Volterra Hankel, 294  
weak-type, 112, 114, 344, 365  
Wiener–Hopf, 262, 273  
Wold–Kolmogorov type, see also  
Wold–Kolmogorov  
Wolff’s diagonal, 14  
orthogonal subspaces, 95  
outer, 15, 17, 23, 43, 67, 72, 79, 201, 250, 277  
at a point, 68  
co-outer, *-outer, 15  
exposed, rigid, 396  
in Ω, 73  
in C+ , 147  
locally, 68  
part, 43  
Paley–Wiener  
condition, 160  
theorem, see also Theorem, Lemma  
parametrization, 241, 348  
partial  
isometry, see also isometry  
PDK, see also positive definite kernel  
Pettis, see also integrable  
phase  
delay, 155  
minimal, see also filter  
Phragmén–Lindelöf  
majorant, 73  
principle, 29, 91
principle (generalized), see also
Theorem, Lemma
Pick body, 241
piecewise continuous, 262, 281
Poisson, 80
integral, formula, 39
on $\mathbb{C}_+$, 147
kernel, 33
on $\mathbb{R}$, 316
polar
decomposition, 187
polynomially
bounded, see also operator
convex hull, 28
polynomials
analytic, $\mathcal{P}ol_+$, 9
trigonometric, $\mathcal{P}ol$, 8
positive definite
function, 317
kernel, 317
subordinated, 317
prediction
$n$-step, 23, 379
one-step, 23, 88
principal value integral, 116
problem
Carathéodory–Fejér, 277
design, see also filter
filter identification, see also filter
invariant subspace, see also invariant
subspace
inverse spectral, see also Hankel
Littlewood, see also Littlewood
Lusin, see also Lusin
majorization, see also majorization
moment, see also moment
Nehari, see also Nehari
Nevanlinna–Pick, see also interpolation,
Nevanlinna–Pick
Nudel’man, see also interpolation,
Nudel’man
Riemann–Hilbert, see also
Riemann–Hilbert
Schur, see also interpolation, Schur, 277
Schur–Takagi, see also interpolation,
Schur–Takagi
process
deterministic, 23
future of, 23, 379
non-deterministic, 23, 379
past of, 23, 379
regular, 88, 379
completely, 379
spectral measure, 23
stationary, 28, 379
gaussian, 378
stochastic, 23
projection
Bergman, 203, 362
on the $n$-th component of a Fourier
series, see also Fourier series
onto Hankel operators, see also Hankel
operator
onto Toeplitz operators, see also Toeplitz
operators
Riesz, 97, 99, 114, 158, 181, 200
of a measure, 137
on $L^2(T, E)$, 189
skew, 95
pseudo
-continuous, 81
pseudohyperbolic disk, 169, 198, 360
pulse input, see also quasi signal, finite energy
QC, see also quasi-continuous
quasi
-analytic, 87
non-quasianalytic weight, 162
-continuous function, 215, 221, 252
-normed ideal, see also normed ideal
radial maximal
function, 124
operator, 119, 125
Radon–Nikodym
derivative, 11, 66
property
analytic (ARN), 62
theorem, see also Theorem, Lemma
range
essential, see also essential
norm, 318
reducing, 8, 13, 22, 188
regularizer, 219, 257, 259, 267
removable singularity property, 223
reproducing kernel, 164, 190, 216, 221, 222,
228, 317
Hilbert space, RKHS, see also Space,
functions class
normalized, 294, 381
of $H^2(\mathbb{C}^+)$, 166
thesis, RKT, 131, 204, 224, 246, 398
for Toeplitz operator, 398
residue, logarithmic, see also logarithmic
resolvent, 249
map, 237
Riemann
$\zeta$-function, 163
functional equation, 169
hypothesis, 163
Riemann–Hilbert problem, 274
Riesz
basis, 296
property, 273
projection, see also projection
RKHS, see also Space, function class,
reproducing kernel Hilbert space
<table>
<thead>
<tr>
<th>Subject</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>RKT, see also reproducing kernel thesis</td>
<td>146</td>
</tr>
<tr>
<td>“rolling a disk”, 146</td>
<td></td>
</tr>
<tr>
<td>(\mathcal{R})-subset, 29</td>
<td></td>
</tr>
<tr>
<td>running mean, 159</td>
<td></td>
</tr>
<tr>
<td>Schatten–Lorentz class (\mathcal{S}_{pq})</td>
<td>372</td>
</tr>
<tr>
<td>Schatten–von Neumann class (\mathcal{S}_p)</td>
<td>207, 211, 360</td>
</tr>
<tr>
<td>Schmidt</td>
<td></td>
</tr>
<tr>
<td>decomposition, 331</td>
<td></td>
</tr>
<tr>
<td>of normal operators, 349</td>
<td></td>
</tr>
<tr>
<td>pair, 333</td>
<td></td>
</tr>
<tr>
<td>vector, 333</td>
<td></td>
</tr>
<tr>
<td>Schur, see also interpolation</td>
<td></td>
</tr>
<tr>
<td>multiplier, 202, 373</td>
<td></td>
</tr>
<tr>
<td>procedure, 241</td>
<td></td>
</tr>
<tr>
<td>product (of matrices), 319</td>
<td></td>
</tr>
<tr>
<td>test, see also Theorem, Lemma, Schur test,</td>
<td></td>
</tr>
<tr>
<td>see also Vinogradov–Senichkin, 191, 192,</td>
<td></td>
</tr>
<tr>
<td>282, 301, 362</td>
<td></td>
</tr>
<tr>
<td>Schwarz–Herglotz kernel, 41</td>
<td></td>
</tr>
<tr>
<td>outer, 58</td>
<td></td>
</tr>
<tr>
<td>in (\mathbb{C}_+), 147</td>
<td></td>
</tr>
<tr>
<td>transform, 137</td>
<td></td>
</tr>
<tr>
<td>section, see also Toeplitz operator</td>
<td></td>
</tr>
<tr>
<td>sectorial, 248</td>
<td></td>
</tr>
<tr>
<td>semicommutator, 131, 253</td>
<td></td>
</tr>
<tr>
<td>separated, see also sparse</td>
<td></td>
</tr>
<tr>
<td>singularities, see also Bezout equation</td>
<td></td>
</tr>
<tr>
<td>separating, 48</td>
<td></td>
</tr>
<tr>
<td>sequence</td>
<td></td>
</tr>
<tr>
<td>basic, see also basic</td>
<td></td>
</tr>
<tr>
<td>complete, 104</td>
<td></td>
</tr>
<tr>
<td>(\Lambda(p)), 90</td>
<td></td>
</tr>
<tr>
<td>minimal, 93, 104, 200</td>
<td></td>
</tr>
<tr>
<td>stationary, 23, 28</td>
<td></td>
</tr>
<tr>
<td>unconditional, see also Riesz, sequence</td>
<td></td>
</tr>
<tr>
<td>uniformly minimal, 93</td>
<td></td>
</tr>
<tr>
<td>constant of uniform minimality, 139</td>
<td></td>
</tr>
<tr>
<td>shape of a drum, 157</td>
<td></td>
</tr>
<tr>
<td>shift, 5, 7, 28, 154, 180, 192, 196, 222,</td>
<td></td>
</tr>
<tr>
<td>228, 387</td>
<td></td>
</tr>
<tr>
<td>backward, 81, 90, 181, 192, 222, 228, 237</td>
<td></td>
</tr>
<tr>
<td>periodic, 371</td>
<td></td>
</tr>
<tr>
<td>simple, 197</td>
<td></td>
</tr>
<tr>
<td>signal, 153</td>
<td></td>
</tr>
<tr>
<td>finite energy, 153</td>
<td></td>
</tr>
<tr>
<td>input, output, 153</td>
<td></td>
</tr>
<tr>
<td>plus-, 153</td>
<td></td>
</tr>
<tr>
<td>similar, see also similarity, 193</td>
<td></td>
</tr>
<tr>
<td>similarity, see also similar</td>
<td></td>
</tr>
<tr>
<td>to a contraction, 193, 207</td>
<td></td>
</tr>
<tr>
<td>to a unitary, 207</td>
<td></td>
</tr>
<tr>
<td>singular</td>
<td></td>
</tr>
<tr>
<td>inner, see also inner</td>
<td></td>
</tr>
<tr>
<td>integral operator, 117, 131</td>
<td></td>
</tr>
<tr>
<td>Calderon–Zygmund, 133</td>
<td></td>
</tr>
<tr>
<td>number, 211, 224, 333, 338, 351, 378</td>
<td></td>
</tr>
<tr>
<td>of a Hankel operator, 331</td>
<td></td>
</tr>
<tr>
<td>singularities, see also Bezout equation</td>
<td></td>
</tr>
<tr>
<td>SIO, see also singular integral operator</td>
<td></td>
</tr>
<tr>
<td>(s)-number, see also singular number</td>
<td></td>
</tr>
<tr>
<td>SOT, see also measurable</td>
<td></td>
</tr>
<tr>
<td>Space, function class</td>
<td></td>
</tr>
<tr>
<td>analytic functions (vector valued), 48</td>
<td></td>
</tr>
<tr>
<td>Banach function, 88, 196, 223</td>
<td></td>
</tr>
<tr>
<td>Bergman, 18, 89, 131, 197, 202, 225, 320</td>
<td></td>
</tr>
<tr>
<td>(p)-Bergman, 197</td>
<td></td>
</tr>
<tr>
<td>Peller’s form, 366</td>
<td></td>
</tr>
<tr>
<td>weighted, 16, 80, 361</td>
<td></td>
</tr>
<tr>
<td>Besov class, 352, 369</td>
<td></td>
</tr>
<tr>
<td>analytic, 354, 361</td>
<td></td>
</tr>
<tr>
<td>Bloch, 198, 361</td>
<td></td>
</tr>
<tr>
<td>Bounded Mean Oscillation, (BMO), 189, 206</td>
<td></td>
</tr>
<tr>
<td>Bounded Mean Oscillation, analytic, (BMO_A), 139, 189, 206</td>
<td></td>
</tr>
<tr>
<td>derivative space ((H^p)^*), 126</td>
<td></td>
</tr>
<tr>
<td>Dirichlet, 131, 198, 202, 320, 361</td>
<td></td>
</tr>
<tr>
<td>(-)-type, 16</td>
<td></td>
</tr>
<tr>
<td>disk algebra, see also disk algebra</td>
<td></td>
</tr>
<tr>
<td>Hardy, 9, 32</td>
<td></td>
</tr>
<tr>
<td>on an annulus, 18</td>
<td></td>
</tr>
<tr>
<td>on the right half-plane, 166</td>
<td></td>
</tr>
<tr>
<td>on the upper half-plane, 145</td>
<td></td>
</tr>
<tr>
<td>operator valued, 189</td>
<td></td>
</tr>
<tr>
<td>real (H^1_{\Re}), 190</td>
<td></td>
</tr>
<tr>
<td>vector valued, 46, 188</td>
<td></td>
</tr>
<tr>
<td>weighted, 137, 201, 274, 322</td>
<td></td>
</tr>
<tr>
<td>with respect to measure (\mu), 22</td>
<td></td>
</tr>
<tr>
<td>(H^\infty + C), 214</td>
<td></td>
</tr>
<tr>
<td>Krein, 204</td>
<td></td>
</tr>
<tr>
<td>(L^p) vector valued, 47</td>
<td></td>
</tr>
<tr>
<td>Nevanlinna, Nev, 54, 72</td>
<td></td>
</tr>
<tr>
<td>(QC), see also quasi-continuous</td>
<td></td>
</tr>
<tr>
<td>reproducing kernel Hilbert space, RKHS, 240, 317, 362</td>
<td></td>
</tr>
<tr>
<td>(\Lambda(p)), 90</td>
<td></td>
</tr>
<tr>
<td>minimal, 93, 104, 200</td>
<td></td>
</tr>
<tr>
<td>stationary, 23, 28</td>
<td></td>
</tr>
<tr>
<td>unconditional, see also Riesz, sequence</td>
<td></td>
</tr>
<tr>
<td>uniformly minimal, 93</td>
<td></td>
</tr>
<tr>
<td>constant of uniform minimality, 139</td>
<td></td>
</tr>
<tr>
<td>shape of a drum, 157</td>
<td></td>
</tr>
<tr>
<td>shift, 5, 7, 28, 154, 180, 192, 196, 222,</td>
<td></td>
</tr>
<tr>
<td>228, 387</td>
<td></td>
</tr>
<tr>
<td>backward, 81, 90, 181, 192, 222, 228, 237</td>
<td></td>
</tr>
<tr>
<td>periodic, 371</td>
<td></td>
</tr>
<tr>
<td>simple, 197</td>
<td></td>
</tr>
<tr>
<td>signal, 153</td>
<td></td>
</tr>
<tr>
<td>finite energy, 153</td>
<td></td>
</tr>
<tr>
<td>input, output, 153</td>
<td></td>
</tr>
<tr>
<td>plus-, 153</td>
<td></td>
</tr>
<tr>
<td>similar, see also similarity, 193</td>
<td></td>
</tr>
<tr>
<td>similarity, see also similar</td>
<td></td>
</tr>
<tr>
<td>to a contraction, 193, 207</td>
<td></td>
</tr>
<tr>
<td>to a unitary, 207</td>
<td></td>
</tr>
<tr>
<td>spectral measure, 384</td>
<td></td>
</tr>
<tr>
<td>multiplicity, see also multicyclicity</td>
<td></td>
</tr>
<tr>
<td>function, see also multiplicity function</td>
<td></td>
</tr>
<tr>
<td>radius, 187</td>
<td></td>
</tr>
<tr>
<td>of (T_p), 246</td>
<td></td>
</tr>
<tr>
<td>synthesis, 103</td>
<td></td>
</tr>
</tbody>
</table>
spectrum, 27, 187, 246
amplitude, see also amplitude
continuous, 247
ergy, 155
essential, see also essential, 246
left, 247
right, 247
Fredholm, see also Fredholm
left, 247
limit distribution (of finite sections), 277, 305
of an inner function, see also spectrum of
an operator-valued function
of an operator valued function, 71
point, 249
right, 247
with respect to $\Gamma_n[1]$, 304
splitting
of cyclicity, 89
of subspaces, 27, 28
splitting (of subspaces), 27
square root of a positive operator, 186
stable, see also filter
asymptotically, 383
rank problem, 397
stationary, see also process
filter, see also filter
sequence, see also sequence
step-by-step extension, see also AAK
stochastic
matrix, 349
process, see also process
Stolz angle, 39, 127, 138, 145
strictly normed, 50
strong $H^2$-function, 14
subordination principle, see also Littlewood
summation method, 103
Abel–Poisson, 103
$s$-vector, see also Schmidt vector
symbol, 269
matrix, 275
of a Hankel operator, see also Hankel
of a Toeplitz operator, see also Toeplitz
symmetric norm, see also normed ideal
symplectic matrix, 294, 371
Szego
condition, 135
uniform, 136
infimum, 23, 28, 65
vector valued, 86
Tauberian, see also Theorem, Lemma
Tchebyshev inequality, 121
tensor product, see also Hilbert
test, see also Schur; Vinogradov–Senichkin
Theorem, Lemma
Adyan–Arov–Krein, see also AAK,
213, 217, 333, 337
Agler, 241
All Stationary Filters, 158
Allan, 391
anonymous, 253
Arveson–Sz.-Nagy–Foias–Schubert, 385
Axler–Chang–Sarason–Volberg, 131, 254
$B$ valued $C_A$ functions, 392
$B$ valued $H^\infty$ functions, 391
B.Ph.A.M.S., see also invertibility, left,
see also Theorem, Lemma, Bochner–Phillips–Allan–Markus–Sementsul
Banach, 94, 220
Beurling, 10, 164, 216, 335, 337
Beurling–Helson, 9, 149, 161, 169, 170
Beurling–Lax (vector valued), 14, 204
Bochner, 326
Bochner–Phillips–Allan–Markus–Sementsul,
390
Bohr, 393
Bonsall–Power, 131
boundary uniqueness, 36, 59
on $C_+$, 147
Bourgain, 208
Bourgain’s Lemma, 208
Bram, 75, 89
Burkholder–Gundy–Silverstein, 138
Canonical factorization
in $D$, see also Theorem,
Riesz–Smirnov, 165
in $C_+$, 147
Carleson (corona), 388, 394
Carleson (embedding), 105, 130, 145, 151, 312
Carleson (free interpolation), 241
Causal Filters, 158
Coburn, 258, 270, 275, 293, 382
Coifman–Rochberg, 360
commutant lifting, 229, 230, 381, 386
compact operators, 378
Cotlar–Sadosky, 136
Devinnatz–Widom, 250
Dijksma, 242
dominated convergence (vector valued), 47
Douglas, 185, 259
Douglas–Rudin, 44
Douglas–Sarason, 254, 255
embedding, see also Theorem, Lemma,
Carleson
Fabry gap, 90
Fatou, 39, 46, 127, 145
vector valued, 50
Fefferman–Stein, 126, 208
Fejér–Riesz, 315
Frostman, 45
Fubini (vector valued), 47
Fuglede–Putnam, 188
Gelfand–Pettis, 46
Generalized Maximum Principle, 73
Generalized Phragmén–Lindelöf
Principle, 73
Hamburger, 310
Hartman, 214, 220, 224, 253, 292, 327
Hartman–Wintner, 248
Helson, 12
Helson–Szegö, 99
Herglotz, 42
Hrushev–Peller, 381
Hunt–Muckenhoupt–Wheeden, 132
Jackson–Bernstein, 352, 374
Julia, 186
Kolmogorov, 97
Krein–Shmulian, 49
Kronecker, 217, 225, 369, 382
Lavrentiev, 13, 75
Lax, 149
Lax–Halmos, 386
Lidskii, 349
Lindelöf, 140, 285, 292
Loewner, 240, 326
Magnus, 287
Marcinkiewicz, 121, 138, 344
Matsaev, 278
Maximum Principle
generalized, 72
vector/operator valued, 50
Mean Value, 14
Megretskii–Peller–Treil, 306, 384
Montel, 183
Muirhead, 349
Naimark, 286
Natanson, 141
Nazarov–Volberg, 133
Nehari, 182, 217, 230, 327, 369
Nevanlinna, 234
Nyman, 170
operator valued $H^\infty$ functions, 392
Ostrovskii, 340
Page, 224
Paley–Wiener, 146, 150, 262
Feller, 352, 361
Perron–Frobenius, 301
Phragmén–Lindelöf (generalized),
see also Theorem, Lemma,
Generalized Phragmén–Lindelöf; and
Phragmén–Lindelöf principle
Pick, 233
Plessner, 58
Power, 288
Radon–Nikodym, 11
Riesz Brothers, 26, 29, 35, 75, 157
in $\mathbb{R}$, 151
Riesz, M., 310
Riesz–Herglotz, 315, 316, 328
Riesz–Markov, 311
Riesz–Smirnov, 42, see also factorization,
canonical; Theorem, Lemma,
canonical factorization
Riesz–Thorin, 138
Rosenblum, 103
Rouché, 260, 295
Rudin–Carleson, 156, 161
Sarason, 229, 255
Sarason–Sz-Nagy–Foias, 230
Schmidt, 331
Schur, 232, 234, 238
Schur test, see also Schur test, 282
Sidon, 207
Singular Inner Functions, 42
Smirnov, 23, 67
Smirnov–Kolmogorov, 100
Spectral mapping, 187
Stechkin, 264
Stone–Weierstrass, 187
Structure of Outer Functions, 43, 158
Szegö–Kolmogorov, 26, 65
vector valued, 84
Tauberian, 152, 161
Treil, 378
Uniqueness theorem, see also Theorem,
Lemma, boundary uniqueness, 10,
37, 60, 159, 258
for vector valued functions, 170,
see also Theorem, Lemma –
Maximum Principle, vector/operator
valued
Vitali Covering, 124
Vitse, 392
von Neumann spectral, 123, 235, 384
Weierstrass, 214
Widom, 270, 312, 313
Wiener, see also Theorem, Lemma,
Tauberian, 8, 17, 149, 154, 155, 161,
185, 245, 264
continuous time, 159
Wold–Kolmogorov, see also
Wold–Kolmogorov, 11, 86, 189
time domain, 153
Toeplitz
algebra, 256, 301
matrix, 232, 238, 243
Simon’s, 371
operator, 221, 243, 273, 385
aggregated, 257
finite section method, see also finite
section method, invertible by, 275
in the upper half plane, 261
$n$-th section of, 238, 371
on $l^p(\mathbb{Z}_+)$, 264
on $F^p(\mathbb{Z}_+)$, 264
projection onto, 268
resolvent growth, 265
symbol of, 244
total, 95
trace, 48, 338, 355
class \( \mathcal{S} \), 211, 338, 351
formula
\( \text{Lidskii, see also Lidskii} \)
\( \text{Lifshic–Krein, 350} \)
transfer function, 155
transform
\( \text{Berezin, 131, 224, 246, 373} \)
\( \text{Cauchy} \)
\( \text{normalized, 136} \)
of a measure, 316
\( \text{Fourier, see also Fourier, 144, 185} \)
\( \text{Hilbert, 98, 114, 115, 189} \)
discrete, 117, 264
two-weighted estimates, 133
weighted estimates, see also Theorem, Lemma,
\( \text{Hunt–Muckenhoupt–Wheeden} \)
\( \text{Laplace, 262} \)
\( \text{Mellin, see also Mellin} \)
\( \text{Schwarz–Herglotz, see also Schwarz–Herglotz} \)
translates
\( \text{completeness, 149, 152, 161} \)
\( \text{integer (Z-cyclicity), 162} \)
translation, see also operator
triangular, see also operator, lower triangular
type (of an entire function), 150
\( \text{UMD, see also entire function} \)
unconditional
basis, see also basis
martingale differences (UMD), 134
sequence, see also Riesz, sequence
uniformly
\( \text{minimal, see also sequence; basis} \)
unimodular, 234
uniqueness, see also Theorem, Lemma,
\( \text{uniqueness and boundary uniqueness} \)
unitary
\( \text{invariants, 384} \)
operator, see also operator

variation, 26
vector valued
function
\( \text{analytic, holomorphic, 48} \)
\( \text{integrable, see also integrable} \)
\( \text{measurable, see also measurable} \)
\( \text{weakly holomorphic, 48} \)
\( \text{\( H^p \)-spaces, see also Space, function class, Hardy} \)
\( \text{Vinogradov–Senichkin test, see also Schur test, 106, 131, 302} \)
\( \text{Vitali covering, see also Theorem, Lemma} \)
voltage ratio, 155

von Neumann
\( \text{inequality, 193} \)
\( \text{spectral theorem, see also Theorem, Lemma} \)
wandering subspace, 10, 189
weak-\( F \) equality, 237
weak-type, see also operator
weight
\( \text{Bernstein, 87} \)
\( \text{factorable, 86, 88} \)
\( \text{summable, 134} \)
weighted estimates, see also transform, Hilbert
weighting function, 155
Weyl
\( \text{fractional derivative, 372} \)
\( \text{fractional integration-derivation operator, 201, 376} \)
\( \text{inequality, 339, 341, 347} \)
white noise, 156
Wiener
\( \text{algebra, 264, 367, 370, 398} \)
\( \text{\( B \) valued, 392} \)
Wiener–Hopf
\( \text{factorization, 270} \)
\( \text{method, 270} \)
\( \text{operator, see also operator} \)
\( \text{winding number, 258} \)
WK-type, see also Wold–Kolmogorov type operator
Wold–Kolmogorov
\( \text{-type operator, 15} \)
\( \text{decomposition, see also Theorem, Lemma, 15} \)
Wolff–Denjoy series, 141
WOT, see also measurable
\( \text{(Wtt), see also Theorem, Lemma, Tauberian} \)
zero divisor, 38, 55
zero multiplicity, 38
zero, localization of, 166, 171
Symbol Index

\[ (L, M), 95 \]
\[ (x, g)_{\text{col}}, 319 \]
\[ <, 317 \]
\[ A \leq B, 185 \]
\[ A^{1/2}, 186 \]
\[ A^#, 212 \]
\[ E^-, 164 \]
\[ L \perp M, 95 \]
\[ (F_+, f), 215 \]
\[ (T_f, T_g), 253 \]
\[ (T_f, T_g), 253 \]
\[ \Gamma, 179 \]
\[ [t], 167 \]
\[ \|a\|^{-1}, 390 \]
\[ \|a\|_\ell, 192 \]
\[ \|a\|_{\text{Mult}}, 79 \]
\[ \|a\|_\ell, 206 \]
\[ \|a\|_{\text{VMO}}, 220 \]
\[ \|a\|_{\text{VMO}}, 220 \]
\[ \|a\|_{\text{ess}}, 212 \]
\[ \|\phi\|_{\ell}, 198 \]
\[ \Theta, \Theta, 11 \]
\[ \otimes, 390 \]
\[ |A|, 187 \]
\[ \varphi^*, 253 \]
\[ f * g, 32 \]
\[ A(1/\omega), 80 \]
\[ (A_2), (A_p), 119, 274 \]
\[ A_{\text{ao}}, 354 \]
\[ \text{alg}(X), 254 \]
\[ \text{alg} T_{H \approx C}, 258 \]
\[ \text{alg} T_X, 256 \]
\[ \text{alg} T_{\ell \approx (\Gamma)}, 256 \]
\[ B, 198 \]
\[ BDA_p, 198 \]
\[ b_\lambda, 37 \]
\[ BMO, 189 \]
\[ BMO_{\text{A}}, 189 \]
\[ B_p, 361 \]
\[ B_p, 352 \]
\[ B_p, 354 \]
\[ B_1 = B_1, 352 \]
\[ B_{pq}, 352 \]
\[ C_{A}, 71, 171 \]
\[ \text{Cap}(f), 83 \]
\[ \mathbb{C}_+, 143 \]
\[ \mathbb{C}_-, 150 \]
\[ \mathbb{C}^+, 163, 166 \]
\[ \text{clos}, 7 \]
\[ C_\mu, 136, 316 \]
\[ \text{Com} T_X, 274 \]
\[ \text{conv}(A), 100 \]
\[ c(X, \delta), cH(X, \delta), 265 \]
\[ \text{Cyc}(T), 66 \]
\[ \text{Cyc}(X), 79 \]
\[ D = \{ z \in \mathbb{C} : |z| < 1 \}, 27 \]
\[ \mathcal{D}, 72, 90 \]
\[ D_{H \approx C}, 241 \]
\[ d_A(x), 384 \]
\[ D_A(\zeta), 241 \]
\[ d_E, 164 \]
\[ \text{deg}(\Theta), 333 \]
\[ \delta, 203, 204 \]
\[ \Delta_t, 352 \]
\[ \mathcal{D}(f), 256 \]
\[ d_\lambda, 167 \]
\[ \text{dist}, 22 \]
\[ d_\lambda, 363 \]
\[ d_\lambda(f, g), 254 \]
\[ d_\lambda(f, M), 254 \]
\[ \mathcal{D}^p(\mu), 198 \]
\[ E_f, 8 \]
\[ \bar{E}, 150, 235 \]
\[ E_-, E_+, E_{\text{ess}}, 23 \]
\[ E_F, 165 \]
\[ \mathcal{F}, \mathcal{F}^{-1}, 144 \]
\[ f_{\text{inn}}, f_{\text{out}}, 23 \]
\[ F, 166 \]
\[ \Phi, 154, 159 \]
\[ f(M_\theta), 229 \]
\[ \text{Fred}(X, Y), 218 \]
\[ F(X, Y), 218 \]
\[ F_n(H, K), 331 \]
\[ [\Gamma], 179 \]
\[ \Gamma, 169 \]
\[ \Gamma_{\psi}, 261 \]
\[ \overline{\Gamma}, 262 \]
\begin{align*}
\Gamma[\mu] & , 309 \\
\Gamma_\varphi & , 252 \\
\text{GCD} & , 24 \\
\text{GLIM} & , 268 \\
H^0 & , 264 \\
\mathcal{H}\text{Rank} = \mathcal{H} = \mathcal{H}(H^2, H^2) & , 192 \\
H_\varphi & , 181 \\
\tilde{H} & , 261 \\
\mathcal{H}_n & , 212 \\
\mathcal{H}_n(X_1, X_2) & , 203 \\
\mathcal{H}_n & , 212 \\
\mathcal{H}_\infty & , 212 \\
\mathcal{H}(S_1, S_2; P_-) & , 196 \\
\mathcal{H}(X_1, X_2; P_-) & , 196 \\
\mathcal{H}(X_1, X_2) & , 196 \\
H^2 & , 9 \\
H^2 - & 181 \\
H^2(E), H^2(E); H^2(T, E) & , 188 \\
H^2(\mu), H^2(\mu) & , 22, 79 \\
H^2(C^+) & , 166 \\
(H^p)^r & , 126 \\
H^p & , 145 \\
H^p(D) & , 31 \\
H^p(D, X) & , 50 \\
H^\infty + C & , 213, 214 \\
H^\infty(F \rightarrow E) & , 189 \\
H^\infty[f] & , 254 \\
H_{\varepsilon} & , 132 \\
\text{Hol}(D) & , 31 \\
\text{Hol}(\Omega) & , 38 \\
\text{Hol}(\Omega, X) & , 48 \\
\text{id}(f_1, \ldots, f_n) & , 389 \\
\text{id}(E) & , 18 \\
\text{Ind}(\varphi) & , 258 \\
\text{ind} T & , 218 \\
\mathcal{J} f(z) = \mathcal{J} f(\overline{z}) & , 181 \\
\mathcal{J}_0 & , 261 \\
K_{cb} & , 208 \\
k_\lambda & , 164 \\
K_{\Theta} & , 216, 222 \\
\lambda_f(t) & , 114 \\
\Lambda(p) & , 90 \\
\text{Lat} & , 27 \\
\text{LCM} & , 24 \\
L^2(E), L^2(\mu, F) & , 13 \\
L^2(T, E) & , 188 \\
L^2, L^2(\mu) & , 7 \\
L^p(\Omega, X) & , 47 \\
L^p(\mathbb{R}, w) & , 75 \\
L^p(w) & , 80, 81 \\
L^\infty(F \rightarrow E) & , 189 \\
l^2 & , 9 \\
l^2(\mathbb{Z}_+, w^2) & , 16 \\
l^2(w_n) \rightleftharpoons & , 198 \\
l^p_A & , 225 \\
L_{\varphi} & , 191 \\
\text{LIM}(A) & , 61 \\
\text{Lin} & , 8 \\
(LIP(\overline{\frac{1}{p} - \frac{1}{q}}))_+ & , 201 \\
\text{LTHS} & , 79, 89 \\
\mathfrak{M}(T) & , 34 \\
\{M_\Theta\}' & , 229 \\
|\mu| & , 26 \\
\text{Mult}(X) & , 79, 126, 264 \\
\text{mult}(X) & , 264 \\
\omega_\mu(t) & , 80 \\
P_+ & , 97 \\
P_{L|M} & , 95 \\
\text{PC} & , 281 \\
\text{pch} & , 28 \\
\Pi((e_n)_{n \geq 0}) & , 276 \\
P_n & , 94 \\
\mathcal{P}_{\Theta} & , 8 \\
\mathcal{P}_{\Theta^+} & , 214 \\
\mathcal{P}_{\Theta^+} & , 9 \\
P_+ & , 228 \\
\text{QC} & , 215 \\
\mathcal{R} & , 29 \\
r(A) & , 187 \\
\text{Range}(\varphi) & , 247 \\
R_\mu & , 249 \\
R_X & , 192, 208 \\
\mathcal{R}_{\xi} & , 285 \\
S & , 7 \\
\sigma(T) & , 27 \\
S^* & , 90, 181 \\
\mathcal{S}_2, \mathcal{S}_p & , 192, 203, 207, 211 \\
\mathcal{S}_2 H^\infty & , 203 \\
\mathcal{S}_{\infty} & , 211 \\
\mathcal{S}_M & , 278 \\
\text{sl}(f, x) & , 281 \\
\sigma(A) & , 187 \\
\sigma^1(T) & , 247 \\
\sigma^*(T) & , 247 \\
\sigma_c(T) & , 247 \\
\sigma_{\text{ess}}(A) & , 212 \\
\sigma_{\text{ess}}(T) & , 247 \\
\sigma_{\text{ess}}(T) & , 247 \\
\sigma_{\text{ess}}(T) & , 247 \\
\sigma_{\text{ess}}(T) & , 247 \\
\sigma_p(T) & , 249 \\
\text{SM} & , 373 \\
\text{s}_{\text{n}}(A) & , 211 \\
\text{SOT} & , 46 
\end{align*}
span, 8
Sym $\mathcal{H}(X_1, X_2; P_-)$, 196
Sym $\mathcal{H} = \text{Sym } \mathcal{H}(X_1, X_2)$, 196
Sym$(T)$, 257

$T = \{z \in \mathbb{C} : |z| = 1\}$, 7
$\tau_s$, 148
$T_\varphi$, 244
$\tilde{T}_\psi$, 261
$\Theta$, 10
$\theta_E$, 164
Trace, 338, 351, 355
Traj, 306
$\mathcal{T}(X)$, 256

$U_2$, 143

Var, 264
$V_{MO}$, 220
$V_{MO,A}$, 220
$V_X$, 192

$\mathcal{W}$, 161
$\text{WOT}$, 46
$W_\psi$, 262

$X(\Lambda)$, 206

$\zeta$, 163
Selected Titles in This Series

(Continued from the front of this publication)

60 Iain Raeburn and Dana P. Williams, Morita equivalence and continuous-trace C*-algebras, 1998
59 Paul Howard and Jean E. Rubin, Consequences of the axiom of choice, 1998
57 Marc Levine, Mixed motives, 1998
56 Leonid I. Korogodski and Yan S. Soibelman, Algebras of functions on quantum groups: Part I, 1998
55 J. Scott Carter and Masahico Saito, Knotted surfaces and their diagrams, 1998
54 Casper Goffman, Togo Nishiura, and Daniel Waterman, Homeomorphisms in analysis, 1997
53 Andreas Kriegl and Peter W. Michor, The convenient setting of global analysis, 1997
52 V. A. Kozlov, V. G. Maz'ya, and J. Rossmann, Elliptic boundary value problems in domains with point singularities, 1997
50 Jon Aaronson, An introduction to infinite ergodic theory, 1997
49 R. E. Showalter, Monotone operators in Banach space and nonlinear partial differential equations, 1997
48 Paul-Jean Cahen and Jean-Luc Chabert, Integer-valued polynomials, 1997
47 A. D. Elmendorf, I. Kriz, M. A. Mandell, and J. P. May (with an appendix by M. Cole), Rings, modules, and algebras in stable homotopy theory, 1997
46 Stephen Lipscomb, Symmetric inverse semigroups, 1996
45 George M. Bergman and Adam O. Hausknecht, Cogroups and co-rings in categories of associative rings, 1996
44 J. Amorós, M. Burger, K. Corlette, D. Kotschick, and D. Toledo, Fundamental groups of compact Kähler manifolds, 1996
43 James E. Humphreys, Conjugacy classes in semisimple algebraic groups, 1995
42 Ralph Freese, Jaroslav Ježek, and J. B. Nation, Free lattices, 1995
41 Hal L. Smith, Monotone dynamical systems: an introduction to the theory of competitive and cooperative systems, 1995
40.4 Daniel Gorenstein, Richard Lyons, and Ronald Solomon, The classification of the finite simple groups, number 4, 1999
40.3 Daniel Gorenstein, Richard Lyons, and Ronald Solomon, The classification of the finite simple groups, number 3, 1998
40.2 Daniel Gorenstein, Richard Lyons, and Ronald Solomon, The classification of the finite simple groups, number 2, 1995
40.1 Daniel Gorenstein, Richard Lyons, and Ronald Solomon, The classification of the finite simple groups, number 1, 1994
39 Sigurdur Helgason, Geometric analysis on symmetric spaces, 1994
38 Guy David and Stephen Semmes, Analysis of and on uniformly rectifiable sets, 1993
37 Leonard Lewin, Editor, Structural properties of polylogarithms, 1991
36 John B. Conway, The theory of subnormal operators, 1991
35 Shreeram S. Abhyankar, Algebraic geometry for scientists and engineers, 1990
34 Victor Isakov, Inverse source problems, 1990

For a complete list of titles in this series, visit the AMS Bookstore at www.ams.org/bookstore/.
Together with the companion volume by the same author, Operators, Functions, and Systems: An Easy Reading, Volume 2: Model Operators and Systems, Mathematical Surveys and Monographs, Vol. 93, AMS, 2002, this unique work combines four major topics of modern analysis and its applications:

A. Hardy classes of holomorphic functions,
B. Spectral theory of Hankel and Toeplitz operators,
C. Function models for linear operators and free interpolations, and
D. Infinite-dimensional system theory and signal processing.

This volume contains Parts A and B.

*Hardy classes of holomorphic functions* is known to be the most powerful tool in complex analysis for a variety of applications, starting with Fourier series, through the Riemann ζ-function, all the way to Wiener’s theory of signal processing.

*Spectral theory of Hankel and Toeplitz operators* becomes the supporting pillar for a large part of harmonic and complex analysis and for many of their applications. In this book, moment problems, Nevanlinna-Pick and Carathéodory interpolation, and the best rational approximations are considered to illustrate the power of Hankel and Toeplitz operators.

The book is geared toward a wide audience of readers, from graduate students to professional mathematicians, interested in operator theory and functions of a complex variable. The two volumes develop an elementary approach while retaining an expert level that can be applied in advanced analysis and selected applications.