The Classification of Quasithin Groups

I. Structure of Strongly Quasithin $\mathcal{K}$-groups

Michael Aschbacher
Stephen D. Smith

American Mathematical Society
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To Pam and Judy
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Preface

Abstract. Around 1980, G. Mason announced the classification of the quasithin finite simple groups of characteristic 2-type in which all proper simple sections are known; but he neither completed nor published his work. We provide a proof of a stronger theorem classifying those groups, which is independent of Mason. In particular we close this gap in the proof of the classification of the finite simple groups. We also prove a corollary classifying quasithin groups of even type: providing a bridge to the program of Gorenstein, Lyons, and Solomon; their program seeks to produce a new, simplified proof of the classification of the finite simple groups.

The classification of the quasithin simple groups of even characteristic can be thought of as roughly one fourth of the classification of the finite simple groups. The two volumes in this series provide the first proof that each group in this class is a known simple group. This result closes a gap in the classification of the finite simple groups which has existed for over twenty years.

In addition the series is part of an ongoing effort to reorganize and simplify the original proof of the classification of the finite simple groups, and to write the proof down carefully in a relatively short number of pages (e.g., less than ten thousand). The effort includes the “GLS” series of Gorenstein, Lyons, and Solomon, which at the moment consists of the five volumes [GLS94]–[GLS02], but it also includes smaller projects such as [Asc94] and [BG94].

A detailed discussion of these matters appears in the introductions to each of the two volumes in our series. Roughly speaking, the first volume consists of fairly general results on finite groups (with emphasis on quasithin groups) which serve as the foundation for the classification of the quasithin groups. The second volume consists of a proof that the groups listed in our Main Theorem are the simple quasithin groups of even characteristic, all of whose proper simple sections are known simple groups.

We would be remiss if we did not acknowledge the assistance of a number of people:

During the many years we have worked on this project, each of us visited and benefited from the hospitality of many universities and faculties, whose assistance we gratefully acknowledge.

In particular, we would like to thank Ulrich Meierfrankenfeld for calling our attention to Stellmacher’s $g^r_c$-Lemma, and stating it in the form we use heavily as our Theorem D.1.5. Ulrich also read portions of the manuscript and suggested various simplifications.

We thank Robert Guralnick and Gunter Malle, whose work in [GM02] and [GM04] establishes important results on representations of finite simple groups related to failure of factorization, some of which have been unpublished for years. They relieved us of the need to prove those results; we thank them for providing
us with prepublication copies of their work, and also for reading over the parts of our work which apply their work.

Similarly we would like to thank Richard Lyons and Ronald Solomon, who read over and helped improve our final chapter, which proceeds under the hypothesis of the GLS series.

We also thank the University of Florida group theory seminar (including Chat Ho and Peter Sin) for reading other parts of the manuscript, correcting various errors, and suggesting simplifications.

Most importantly, we would like to thank John Thompson for reading large portions of the two volumes and suggesting numerous improvements and simplifications. The authors, and indeed the finite group theory community, owe him a great debt of gratitude for his selfless work benefiting us all.

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Around 1980, G. Mason announced the classification of a certain subclass of an important class of finite simple groups known as “quasithin groups”. The classification of the finite simple groups depends upon a proof that there are no unexpected groups in this subclass. Unfortunately Mason neither completed nor published his work. In the Main Theorem of this two-part book (Volumes 111 and 112 of the AMS Mathematical Surveys and Monographs series) the authors provide a proof of a stronger theorem classifying a larger class of groups, which is independent of Mason’s arguments. In particular, this allows the authors to close this last remaining gap in the proof of the classification of all finite simple groups.

An important corollary of the Main Theorem provides a bridge to the program of Gorenstein, Lyons, and Solomon (AMS Mathematical Surveys and Monographs, Volume 40) which seeks to give a new, simplified proof of the classification of the finite simple groups.

Part II of the work (Volume 112) contains the proof of the Main Theorem, and the proof of the corollary classifying quasithin groups of even type.

Part I (the current volume) contains results which are used in the proof of the Main Theorem. Some of the results are known and fairly general, but their proofs are scattered throughout the literature; others are more specialized and are proved here for the first time.