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Preface

This work began as a research paper intended to show how the convergence of nonlinear semigroups associated with a sequence of Markov processes implied the large deviation principle for the sequence. We expected the result to be of little utility for specific applications, since classical convergence results for nonlinear semigroups involve hypotheses that are very difficult to verify, at least using classical methods. We should have recognized at the beginning that the modern theory of viscosity solutions provides the tools needed to overcome the classical difficulties. Once we did recognize that convergence of the nonlinear semigroups could be verified, the method evolved into a unified treatment of large deviation results for Markov processes, and the research "paper" steadily grew into the current volume.

There are many approaches to large deviations for Markov processes, but this book focuses on just one. Our general title reflects both the presentation in Part 1 of the theory of large deviations based on the large deviation analogue of the compactness theory for weak convergence, material that is the foundation of several of the approaches, and by the generality of the semigroup methods for Markov processes.

The goal of Part 2 is to develop an approach for proving large deviations, in the context of metric-space-valued Markov processes, using convergence of generators in much the same spirit as for weak convergence (e.g. Ethier and Kurtz [**36**]). This approach complements the usual method that relies on asymptotic estimates obtained through Girsanov transformations.

The usefulness of the method is best illustrated through examples, and Part 3 contains a range of concrete examples.

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Notation

(1) (E, r). A complete, separable metric space.
(2) $\mathcal{B}(E)$. The σ-algebra of all Borel subsets of E.
(3) $A \subset M(E) \times M(E)$. An operator identified with its graph as a subset in $M(E) \times M(E)$.
(4) $B(E)$. The space of bounded, Borel measurable functions. Endowed with the norm $\|f\| = \sup_{x \in E} |f(x)|$, $(B(E), \|\cdot\|)$ is a Banach space.
(5) $B_{loc}(E)$. The space of locally bounded, Borel measurable functions, that is, functions in $M(E)$ that are bounded on each compact.
(6) $B_\epsilon(x) = \{y \in E : r(x, y) < \epsilon\}$. The ball of radius $\epsilon > 0$ and center $x \in E$.
(7) *buc*-convergence, *buc*-approximable, *buc*-closure, closed and dense, and *buc*-lim, See Definition A.6.
(8) $C(E)$. The space of continuous functions on E.
(9) $C_b(E) = C(E) \cap B(E)$.
(10) $C(E, \overline{R})$. The collection of functions that are continuous as mappings from E into $\overline{R}$ with the natural topology on $\overline{R}$.
(11) $C_c(E)$. For E locally compact, the functions that are continuous and have compact support.
(12) $C^k(\mathcal{O})$, for $\mathcal{O} \subset R^d$ open and $k = 1, 2, \ldots, \infty$. The space of functions whose derivatives up to kth order are continuous in $\mathcal{O}$.
(13) $C_c^k(\mathcal{O}) = C^k(\mathcal{O}) \cap C_c(\mathcal{O})$.
(14) $C^{k,\alpha}(\mathcal{O})$, for $\mathcal{O} \subset R^d$ open, $k = 1, 2, \ldots$, and $\alpha \in (0, 1]$. The space of functions $f \in C^k(\mathcal{O})$ satisfying

$$\|f\|_{k,\alpha} = \sup_{0 \le \beta \le k} \sup_{\mathcal{O}} \left| \frac{\partial^\beta f}{\partial x^\beta} \right| + \sup_{|\beta| = k} \left[\frac{\partial^\beta f}{\partial x^\beta} \right]_\alpha < \infty,$$

where

$$\Big[f \Big]_\alpha = \sup_{x \neq y} \frac{|f(x) - f(y)|}{|x - y|^\alpha}, \quad \alpha \in (0, 1].$$

(15) $C_{loc}^{k,\alpha}(\mathcal{O})$. The space of functions $f \in C(\mathcal{O})$ such that $f|_D \in C^{k,\alpha}(D)$ for every bounded open subset $D \subset \mathcal{O}$.
(16) $C_E[0, \infty)$. The space of E-valued, continuous functions on $[0, \infty)$.
(17) $\widehat{C}(U)$, for U locally compact. The space of continuous functions vanishing at infinity.
(18) $\mathcal{D}(A) = \{f : \exists (f, g) \in A\}$. The domain of an operator A.
(19) $\mathcal{D}^+(A) = \{f \in \mathcal{D}(A), f > 0\}$.
(20) $\mathcal{D}^{++}(A) = \{f \in \mathcal{D}(A), \inf_{y \in E} f(y) > 0\}$.
(21) $D_E[0, \infty)$. The space of E-valued, cadlag (right continuous with left limit) functions on $[0, \infty)$ with the Skorohod topology, unless another topology is specified. (See Ethier and Kurtz [**36**], Chapter 3).

(22) $\mathcal{D}(\mathcal{O})$, for $\mathcal{O} \subset R^d$ open. The space $C_c^\infty(\mathcal{O})$ with the topology giving the space of Schwartz test functions. (See D.1).
(23) $\mathcal{D}'(\mathcal{O})$. The space of continuous linear functionals on $\mathcal{D}(\mathcal{O})$, that is, the space of Schwartz distributions.
(24) g^* (respectively g_*). The upper semicontinuous (resp. lower semicontinuous) regularization of a function g on a metric space (E, r). The definition is given by (6.2) (resp. (6.3)).
(25) $\limsup_{n\to\infty} G_n$ and $\liminf_{n\to\infty} G_n$ for a sequence of sets G_n. Definition 2.4 in Section 2.3.
(26) $H_\rho^k(R^d)$, for $\rho \in \mathcal{P}(R^d)$. A weighted Sobolev space. See Appendix D.5.
(27) $K(E) \subset C_b(E)$. The collection of nonnegative, bounded, continuous functions.
(28) $K_0(E) \subset K(E)$. The collection of strictly positive, bounded, continuous functions.
(29) $K_1(E) \subset K_0(E)$. The collection of bounded continuous functions satisfying $\inf_{x\in E} f(x) > 0$.
(30) $M(E)$. The R-valued, Borel measurable functions on E.
(31) $M^u(E)$. The space of $f \in M(E)$ that are bounded above.
(32) $M^l(E)$. The space of $f \in M(E)$ that are bounded below.
(33) $M(E, \overline{R})$. The space of Borel measurable functions with values in $\overline{R}$ and $f(x) \in R$ for at least one $x \in E$.
(34) $M_E[0, \infty)$. The space of E-valued measurable functions on $[0, \infty)$.
(35) $M^{d\times d}$. The space of $d \times d$ matrices.
(36) $M^u(E, \overline{R}) \subset M(E, \overline{R})$ (respectively, $C^u(E, \overline{R}) \subset C(E, \overline{R})$). The collection of Borel measurable (respectively continuous) functions that are bounded above (that is, $f \in M^u(E, \overline{R})$ implies $\sup_{x\in E} f(x) < \infty$).
(37) $M^l(E, \overline{R}) \subset M(E, \overline{R})$ (respectively, $C^l(E, \overline{R}) \subset C(E, \overline{R})$). The collection of Borel measurable (respectively continuous) functions that are bounded below.
(38) $\mathcal{M}(E)$. The space of (positive) Borel measures on E.
(39) $\mathcal{M}_f(E)$. The space of finite (positive) Borel measures on E.
(40) $\mathcal{M}_m(U)$, U a metric space. The collection of $\mu \in \mathcal{M}(U \times [0, \infty))$ satisfying $\mu(U \times [0, t]) = t$ for all $t \geq 0$.
(41) $\mathcal{M}_m^T(U)$ $(T > 0)$. The collection of $\mu \in \mathcal{M}(U \times [0, T])$ satisfying $\mu(U \times [0, t]) = t$ for all $0 \leq t \leq T$.
(42) $\mathcal{P}(E) \subset \mathcal{M}_f(E)$. The space of probability measures on E.
(43) $\overline{R} = [-\infty, \infty]$.
(44) $\mathcal{R}(A) = \{g : \exists (f, g) \in A\}$. The range of an operator A.
(45) $T\#\rho = \gamma$. $\gamma \in \mathcal{P}(E)$ is the push-forward (Definition D.1) of $\rho \in \mathcal{P}(E)$ by the map T.

Bibliography

1. Adams, Robert A. *Sobolev spaces. Pure and Applied Mathematics*, Vol. 65. Academic Press, New York-London, 1975.
2. Aldous, David. Stopping times and tightness. *Ann. Probab.* **6** (1984), 335-340.
3. Ambrosio, Luigi, Gigli, Nicola and Savaré, Giuseppe. *Gradient flows in metric spaces and in the space of probability measures.* Lectures in Mathematics ETH Zrich. Birkhuser Verlag, Basel, (2005).
4. Anderson, Robert F. and Orey, Steven. Small random perturbation of dynamical systems with reflecting boundary. *Nagoya Math. J.* **60** (1976), 189-216.
5. Attouch, Hédy. *Variational Convergence for Functions and Operators*, Pitman Advanced Publishing Program, Boston, London, 1984.
6. Baldi, Paolo. Large deviations for diffusion processes with homogenization and applications. *Ann. Probab.* **19** (1991), 509-524.
7. Barles, G, and Perthame, B. Exit time problems in optimal control and vanishing viscosity method. *SIAM J. Control Optim.* **26** (1988), 1133-1148.
8. Barles, G, and Perthame, B. Comparison results in Dirichlet type first-order Hamilton-Jacobi Equations. *Appl. Math. Optim.* **21** (1990) 21-44.
9. Barles, G, and Souganidis, Panagiotis.E. Convergence of approximation schemes for fully nonlinear second order equations. *Asymptotic Analysis.* **4** (1991) 271-283.
10. Bertini, L., Landim, C. and Olla, S. Derivation of Cahn-Hilliard equations from Ginzburg-Landau models. *J. Stat. Phys.* 88, 365-381, (1997)
11. Borovkov, Alexandr. A. Boundary-value problems for random walks and large deviations in function spaces. *Theory Probab. Appl.* **12** (1967) 575-595.
12. Buck, R. Creighton. Bounded continuous functions on a locally compact space. *Michigan Math. J.* **5** (1958), 95-104.
13. Chen, Hong and Yao, David D. *Fundamentals of Queueing Networks* Springer, New York, 2001.
14. Cooper, J. B. The strict topology and spaces with mixed topologies. *Proc. Amer. Math. Soc.* **30** (1971),583-592.
15. Cordero-Erausquin, Dario; Gangbo, Wilfrid; and Houdré, Christian. Inequalities for generalized entropy and optimal transportation. *Recent advances in the theory and applications of mass transport*, 73–94, *Contemp. Math.*, **353**, Amerocan Mathematical Society, Providence, RI, 2004
16. Cramér, H. Sur un nouveau théoreme-limite de la théorie des probabilités. *Acta. Sci. et Ind.* 736 (1938), 5-23.
17. Crandall, Michael G., Ishii, Hitoshi and Lions, Pierre-Louis. User's Guide to Viscosity Solutions of Second Order Partial Differential Equations. *Bulletin A. M. S., N. S.* **27** (1992), 1-67.
18. Crandall, Michael G. and Liggett, Thomas M. Generation of Semigroups of Nonlinear Transfomations on General Banach Spaces. *Amer. J. Math.* **93** (1971), 265-298.
19. Crandall, Michael G. and Lions, Pierre-Louis. Viscosity solutions of Hamilton-Jacobi Equations. *Trans. A. M. S.* **277** (1983), 1-42.
20. Crandall, Michael G. and Lions, Pierre-Louis. Hamilton-Jacobi Equations in Infinite Dimensions, Part VI: Nonlinear A and Tataru's Method Refined. *Evolution equations, Control theory and Biomathematics (Han Sur Lasse, 1991) 51-89* Lecture Notes in Pure and Applied Math.155. Dekker, New York (1994).
21. Crandall, Michael G. and Lions, Pierre-Louis. Hamilton-Jacobi Equations in Infinite Dimensions, PART I *J. Funct. Anal.* **62** (1985) 3. 379-396. PART II *J. Funct. Anal.* **65** (1986) 3.

368-405. PART III *J. Funct. Anal.* **68** (1986) 2. 214-247. PART IV *J. Funct. Anal.* **90** (1990) 2. 237-283. PART V *J. Funct. Anal.* **97** (1991) 2. 417-465. PART VII *J. Funct. Anal.* **125** (1994) 1. 111-148.

22. Dawson, Donald. A. and Gärtner, Jürgen. Large Deviations from the McKean-Vlasov limit for weakly interacting diffusions. *Stochastics* **20**. 247-308, 1987.
23. Dawson, Donald. A. and Găртner, J. Large deviations, free energy functional and quasi-potential for a mean field model of interacting diffusions. *Memoirs of the American Mathematics Society* **Vol 78**, No. 398, March 1989.
24. de Acosta, Alejandro. Exponential Tightness and Projective Systems in Large Deviation Theory. *Festschrift for Lucien Le Cam. Springer, New York.* 143-156, 1997.
25. de Acosta, Alejandro. Large deviations for vector-valued Lévy processes. *Stochastic Process. Appl.* **51** (1994), 75-115.
26. de Acosta, Alejandro. A general non-convex large deviation result with applications to stochastic equations. *Probab. Theory Relat. Fields* 118 (2000), 483-521.
27. de Acosta, Alejandro. A general non-convex large deviation result II. *Ann. Probab.* (to appear)
28. Dellacherie, Claude and Meyer, Paul-André. *Probabilities and Potential*, (North-Holland mathematics studies, 29; Translation of Probabilitiés et potentiel.) North-Holland Publishing, New York, 1978.
29. Dembo, Amir and Zeitouni, Ofer. *Large Deviations Techniques and Applications*, Jones and Bartlett Publishers, Boston, 1993.
30. Deuschel, Jean-Dominique and Stroock, Daniel W. *Large Deviations.* Academic Press, Boston, 1989.
31. Diestel, J.; Uhl, J. J., Jr. The Radon-Nikodym theorem for Banach space valued measures. *Rocky Mountain J. Math.* **6** (1976), 1-46.
32. Donsker, Monroe D. and Varadhan, S. R. S. On a variational formula for the principal eigenvalue for operators with maximum principle *Proc. Nat. Acad. Sci. USA* **72** No.3 (1975), 780-783.
33. Donsker, Monroe D. and Varadhan, S. R. S. Asymptotic evaluation of Markov process expectations for large time, I,II,III *Comm. Pure Appl. Math.* textbf27 (1975), 1-47, **28** (1975), 279-301, **29** (1976), 389-461.
34. Doss, Halim and Priouret, Pierre. Petites perturbations de systemes dynamiques avec reflexion. *Seminar on probability, XVII*, 353–370, Lecture *Notes in Math.*, 986, Springer, Berlin, 1983.
35. Dupuis Paul and Ellis, Richard S. *A Weak Convergence Approach to the Theory of Large Deviations.* Wiley, New York, 1997.
36. Ethier, Stewart N. and Kurtz, Thomas G. *Markov Processes*, John Wiley and Sons, New York, 1986.
37. Evans, Lawrence C. The perturbed test function method for viscosity solutions of nonlinear PDE. *Proc. Roy. Soc. Edinburgh Sect A.* **111** (1989), 359-375.
38. Evans, Lawrence. *Partial Differential Equations.* Graduate Studies in Mathematics, Volume 19, American Mathematical Society. Providence, Rhode Island 1998.
39. Evans, Lawrence and Gariepy, Ronald. *Measure theory and fine properties of functions.* Studies in Advanced Mathematics, CRC Press, London 1992.
40. Evans, Lawrence C. and Ishii, Hitoshi. A PDE approach to some asymptotic problems concerning random differential equations with small noise intensities. *Ann. Inst. H. Poincaré Anal. Non Linéaire.* **2** (1985), 1-20.
41. Feng, Jin. Martingale problems for large deviations of Markov processes. *Stochastic Process. Appl.* **81** (1999), no. 2, 165–216.
42. Feng, Jin. Large deviation for a stochastic Cahn-Hilliard equation. *Methods of functional analysis and topology***9** (2003), no. 4, 333-356.
43. Feng, Jin and Katsoulakis, Markos A Hamilton-Jacobi theory for controlled gradient flows in infinite dimensions. *Submitted* 2003.
44. Fleming, Wendell. H. Exit probabilities and optimal stochastic control. *Applied Math. Optimiz.* **4** 329-346, 1978.
45. Fleming, Wendell H. A stochastic control approach to some large deviation problems. *Recent mathematical methods in dynamic programmming (Rome 1984). Lecture Notes in Math.* **1119**, Springer, Berlin-New York, 1985. 52-66.

46. Fleming, Wendell H. and Soner, H. Mete. Asymptotic expansions for Markov processes with Lévy generators. *Appl. Math. Optim.* **19** (1989), 203-223.
47. Fleming, Wendell H. and Soner, H.Mete. *Controlled Markov processes and viscosity solutions.* Springer-Verlag, New York, 1991.
48. Fleming, Wendell H. and Souganidis, Panagiotis E. PDE-viscosity solution approach to some problems of large deviations. *Ann. Scuola Norm. Sup. Pisa Cl. Sci.* **13** (1986), 171-192.
49. Freidlin, Mark. I. Fluctuations in dynamical systems with averaging. *Soviet Math. Dokl.* **17** (1976), 104-108.
50. Freidlin, Mark. I. The averaging principle and theorems on large deviations. *Russion Math. Surveys* **33** (1978), 117-176.
51. Freidlin, Mark I., and Sowers, Richard B. A comparison of homogenization and large deviations, with applications to wavefront propagation. *Stochastic Process. Appl.* **82** (1999), 23-52.
52. Freidlin,Mark I. and Wentzell, Alexander D. *Random perturbations of dynamical systems.* Second Edition, Springer-Verlag, New York, 1998.
53. Garcia, Jorge. An extension of the contraction principle. *Journal of Theoretical Probability* **17**., no. 2, (2004), 403-434.
54. Gilbarg, David and Trudinger, Neil S. *Elliptic Partial Differential Equations of Second Order.* Classics in Mathematics, Springer-Verlag, New York, 1998 edition.
55. Giles, Robin. A generalization of the strict topology. *Trans. Amer. Math. Soc.* **161** (1971), 467-474.
56. Guillin, A. Averaging principle of SDE with small diffusion: moderate deviations. *Ann. Probab.* **31** (2003), 413-443.
57. Gulinsky, O. V.; Veretennikov, A. Yu. *Large deviations for discrete-time processes with averaging.* VSP, Utrecht, 1993.
58. Graham, Carl. McKean-Vlasov Itô-Skorohod equations and nonlinear diffusions with discrete jump sets. *Stochastic Process. Appl.* **40** (1992), 69-82.
59. Hoffmann-Jorgensen, J., A generalization of the strict topology *Math. Scand.* **30** (1972), 313-323.
60. Ishii, Hitoshi. On uniqueness and existence of viscosity solutions of fully nonlinear second-order elliptic PDE's. *Comm. Pure Appl. Math.* **42** (1989), 14-45.
61. Ishii, Hitoshi and Lions, Pierre-Louis. Viscosity solutions of fully nonlinear second-order elliptic partial differential equations. *J. Differential Equations* **83** (1990), 26-78.
62. Jakubowski, Adam. On the Skorohod topology. *Ann. Inst. H. Poincaré B* **22** (1986), 263-285.
63. Jordan, Richard, Kinderlehrer, David and Otto, Felix. The variational formulation of the Fokker-Planck equation. *SIAM J. Math. Anal.* **Vol. 29** 1998, No.1, 1-17.
64. Karlin, Samuel. Positive operators *J. Math. Mech.* **8** (1959), 907-937.
65. Kelley, John, *General Topology.* Springer, New York, 1975 (Reprint of the 1955 ed. published by Van Nostrand).
66. Khas'minskii, R. Z. A limit theorem for the solutions of differential equations with random right-hand sides. *Theory Probab. Appl.* **11** (1966) 390-406.
67. Kontoyiannis, Ioannis. and Meyn, Sean. P. Spectral theory and limit theorems for geometrically ergodic Markov processes. *Ann. Appl. Probability* **13** (2003), no.1, 3 04-362.
68. Kontoyiannis, Ioannis. and Meyn, Sean. P. Large deviations asymptotics and the spectral theory of multiplicatively regular Markov processes. *Preprint* 2003.
69. Kurtz, Thomas G. Extensions of Trotter's operator semigroup approximation theorems. *J. Functional Analysis* **3** (1969), 354-375.
70. Kurtz, Thomas G. A general theorem on the convergence of operator semigroups. *Trans. Amer. Math. Soc.* **148** (1970), 23-32.
71. Kurtz, Thomas G. A limit theorem for perturbed operator semigroups with applications to random evolutions *J. Funct. Anal.* **12** (1973), 55-67.
72. Kurtz, Thomas G., Convergence of sequences of semigroups of nonlinear operators with an application to gas kinetics. *Trans. Amer. Math. Soc.* **186**. 259–272, 1973.
73. Kurtz, Thomas G. Semigroups of conditioned shifts and approximation of Markov processes. *Ann. Probab.* **3** (1975), 618-642.
74. Kurtz, Thomas G. A variational formula for the growth rate of a positive operator semigroup *SIAM J. Math. Anal.* **10** (1979), 112-117.

75. Kurtz, Thomas G. Martingale problems for controlled processes *Springer Lecture Notes in Control and Information Sciences* **91**. 75-90, 1987.
76. Kurtz, Thomas G. and Protter, Philip. Weak convergence of stochastic integrals and differential equations II: Infinite dimensional case. *Probabilistic Models for Nonlinear Partial Differential Equations (Montecatini Terme, 1995)* 197–285. *Lecture Notes in Math* 1627, Springer, Berlin, 1996.
77. Kurtz, Thomas G.and Stockbridge, Richard H. Stationary solutions and forward equations for controlled and singular martingale problems. *Electron. J. Probab.* **6** (2001), no. 15, 52 pp
78. Kurtz, Thomas G. and Xiong, Jie. Particle representations for a class of nonlinear SPDEs. *Stochastic Process. Appl.* **83** (1999), no. 1, 103–126.
79. Kushner, Harold J., *Approximation and Weak Convergence Methods for Random Processes* MIT Press, Cambridge, MA, 1984.
80. Lions, Pierre-Louis. Neumann type boundary conditions for Hamilton-Jacobi equations. *Duke Math. J.* **52** (1985), 793-820.
81. Lions, Pierre-Louis, and Sznitman, Alain-Sol. Stochastic differential equations with reflecting boundary conditions. *Comm. Pure Appl. Math.* **37** (1984), 511-537.
82. Lynch, James. and Sethuraman, Jayaram. Large deviations for processes with independent increments. *Ann. Probab.* **15** (1987), 610-627.
83. McCann, J. Robert. Existence and uniqueness of monotone measure-preserving maps. *Duke Math. J.* **Vol. 80** 1995, No.2, 309-323.
84. McCann, J. Robert. A convexity principle for interacting gases. *Advances in Mathematics* **128**, 1997, 153-179.
85. Méléard, Sylvie. Asymptotic behaviour of some interacting particle systems; McKean-Vlasov and Boltzmann models. *Probabilistic models for nonlinear partial differential equations (Montecatini Terme, 1995)*, 42–95, *Lecture Notes in Math.*, **1627**, Springer, Berlin, 1996.
86. Miyadera, Isao. *Nonlinear Semigroups*, Translations of Mathematical Monographs, **109**. AMS, 1991.
87. Mogulskii, A. A. Large deviations for trajectories of multi-dimensional random walks. *Theory Probab. Appl.* **21** (1976) 300-315.
88. Mogulskii, A. A. Large deviations for processes with independent increments. *Ann. Probab.* **21** (1993) 202-215.
89. O'Brien, George L. Sequences of capcities with connections to large-deviation theory. *J. Theoret. Probab.* **9** (1996), 19-35.
90. O'Brien, George L. and Vervaat, Wim. Capacities, large deviations and loglog laws. *Stable processes and related topics* (Ithaca, NY, 1990), 43–83, *Progr. Probab.*, **25**, Birkhuser Boston, Boston, MA, 1991.
91. O'Brien, George L. and Vervaat, Wim. Compactness in the theory of large deviations. *Stochastic Process. Appl.* **57** (1995), 1-10.
92. Otto, Felix. The geometry of dissipative evolution equations: the porous medium equation. *Comm. P.D.Es* **26** (2001), no. 1-2, 101-174.
93. Papanicolaou, George C. and Varadhan, S. R. S. A limit theorem with strong mixing in Banach space and two applications to stochastic differential equations. *Comm. Pure Appl. Math.* **26** (1973), 497-524.
94. Pinsky, Mark A. *Lectures on random evolution.* World Scientific Publishing Co., Inc., River Edge, NJ, 1991.
95. Pinsky, Ross G. *Positive Harmonic Functions and Diffusions.* Cambridge Studies in Advanced Mathematics, Vol 45. Cambridge University Press, 1995.
96. Protter, Philip *Stochastic integration and differential equations. A new approach.* Applications of Mathematics, **21**. Springer-Verlag, Berlin, 1990.
97. Puhalskii, Anatolii. On functional principle of large deviations. *New Trends in Probability and Statistics. Vol. 1 (Bakuriani, 1990)* 198-219, VSP, Utrecht, 1991.
98. Puhalskii, Anatolii. The method of stochastic exponentials for large deviations. *Stochastic Process. Appl.* **54** (1994), 45-70.
99. Puhalskii, Anatolii. Large deviations of semimartingales: a maxingale problem approach. I. Limits as solutions to a maxingale problem. *Stochastic and Stochastic Reports* **61** (1997) 141-243. II. Uniqueness for the maxingale problem. *Stochastic and Stochastic Reports* **68** (1999) 65-143.

100. Puhalskii, Anatolii. *Large deviations and Idempotent Probability.* Chapman and Hall/CRC, 2001, New York.
101. Rachev, Svetlozar T. and Rüschendorf, Ludger. *Mass transportation Problems, Vol I: Theory.* Springer-Verlag, New York, 1998.
102. Reed, Michael; Simon, Barry. *Methods of modern mathematical physics. I. Functional analysis.* Second edition. Academic Press, Inc. [Harcourt Brace Jovanovich, Publishers], New York, 1980.
103. Russell, Raymond. *The large deviations of random time-changes.* PhD Thesis, University of Dublin, 1997.
104. Rockafellar, R. Tyrrell. *Convex analysis.* Princeton University Press, Princeton, NJ, 1970.
105. Sato, K. On the generators of non-negative contraction semi-groups in Banach lattices. *J. Math. Soc. Japan* **20** No.3, 1968, 423-436.
106. Schied, Alexander. Criteria for exponential tightness in path spaces. *Unpublished preprint.* 1995.
107. Schilder, Michael. Some asymptotic formulae for Wiener integrals *Trans. Amer. Math. Soc.* **125** (1966), 63-85.
108. Sheu, Shuenn Jyi. Stochastic control and exit probabilities of jump processes. *SIAM J. Control Optim.* **23** (1985), 306-328.
109. Sheu, Shuenn-Jyi. Stochastic control and exit probabilities of jump processes. *Springer Lecture Notes in Control and Information Sciences* **91**, 75-90, 1987.
110. Sinestrari, Eugenio. Accretive Differential Operators. *Bollettino U.M.I.*, **13**-B, (5), 1976, 19-31.
111. Sion, Maurice. On general minimax theorems. *Pacific J. Math.* **8** (1959), 171-176.
112. Sowers, Richard. Large deviations for a reaction-diffusion equation with non-Gaussian perturbations *Annals of Probability* 20 (1992) 504-537.
113. Spohn, H. *Large Scale Dynamics of Interacting Particles.* Texts and Monographs in Physics, Springer, (1991)
114. Stroock, Daniel W. Diffusion processes associated with Levy generators. *Z. Wahrsch. verw. Gebiete* **32** (1975), 209-244.
115. Stroock, Daniel W. *An introduction to the theory of large deviations.* Springer-Verlag, Berlin, 1984.
116. Stroock, Daniel W. and Varadhan, S. R. S. *Multidimensional diffusion processes.* Grundlehren der Mathematischen Wissenschaften, 233. Springer-Verlag, Berlin-New York, 1979.
117. Tataru, Daniel. Viscosity solutions of Hamilton-Jacobi equations with unbounded nonlinear terms. *Journal of Mathematical Anlaysis and Applications* **163** (1992), 345-392.
118. Trotter, H. F. Approximation of semi-groups of operators. *Pacific J. Math.* **8** (1958), 887-919.
119. Varadhan, S. R. S. Asymptotic probabilities and differential equations. *Comm. Pure Appl. Math.* **19** (1966), 261-286.
120. Veretennikov, A. Yu. On large deviations in averaging principle for stochastic differential equations with periodic coefficients. I. *Probability theory and mathematical statistics*, Vol. II (Vilnius, 1989), 542-551, "Mokslas", Vilnius, 1990.
121. Veretennikov, A. Yu. *On large deviations in the averaging principle for stochastic differential equations with periodic coefficients. II. Math. USSR-Izv.* **39** (1992), no. 1, 677–701
122. Veretennikov, A. Yu. On large deviations in the averaging principle for stochastic difference equations on a torus. *Proc. Steklov Inst. Math.* **202** (1994), 27-33.
123. Veretennikov, A. Yu. On large deviations in averaging principle for systems of stochastic differential equation with unbounded coefficients. *Probability theory and mathematical statistics (Vilnius, 1993)*, 735-742, TEV, Vilnius, 1994.
124. Veretennikov, A. Yu. On large deviations in the averaging principle for Markov processes (compact case, discrete time). *Probability theory and mathematical statistics (St. Petersburg, 1993)*, 235-240, Gordon and Breach, Amsterdam, 1996.
125. Veretennikov, A. Yu. On large deviations for stochastic differential equations with small diffusion and averaging. *Theory Probab. Appl.* **43** (1998), 335–337.
126. Veretennikov, A. Yu. On large deviations for SDEs with small diffusion and averaging. *Stochastic Process. Appl.* **89** (2000), 69-79.
127. Villani, Cedric. *Topics in optimal transportation.* American Mathematical Society, Providence, Rhode Island. Graduate Studies in Mathematics, Vol 58. 2003.

128. Wentzell, Alexander. D. Rough limit theorems on large deviations for Markov stochastic processes. I, II, III. *Theory Probab. Appl.* **21** (1976), 227-242, **24** (1979), 675-672, **27** (1982), 215-234.
129. Wentzell, Alexander. D. *Limit Theorems on Large Deviations for Markov Stochastic Processes.* Kluwer, Dordrecht. (1990)
130. Wheeler, Robert F., A survey of Baire measures and strict topologies. *Exposition. Math.* **1** (1983), 97-190.

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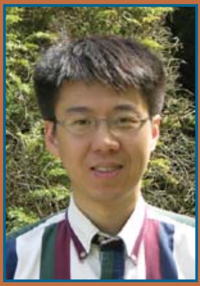

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