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Quantum Field Theory

A Tourist Guide for Mathematicians

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Preface

This book is an attempt to present the rudiments of quantum field theory in general and quantum electrodynamics in particular, *as actually practiced by physicists for the purpose of understanding the behavior of subatomic particles*, in a way that will be comprehensible to mathematicians.

It is, therefore, *not* an attempt to develop quantum field theory in a mathematically rigorous fashion. Sixty years after the growth of quantum electrodynamics (QED) and forty years after the discovery of the other gauge field theories on which the current understanding of the fundamental interactions of physics is based, putting these theories on a sound mathematical foundation remains an outstanding open problem — one of the Millennium prize problems, in fact (see [**67**]). I have no idea how to solve this problem. In this book, then, I give mathematically precise definitions and arguments when they are available and proceed on a more informal level when they are not, taking some care to be honest about where the problems lie. Moreover, I do not hesitate to use the informal language of distributions, with its blurring of the distinction between functions and generalized functions, when that is the easiest and clearest way to present the ideas (as it often is).

So: why would a self-respecting mathematician risk the scorn of his peers by undertaking a project of such dubious propriety, and why would he expect any of them to read the result?

In spite of its mathematical incompleteness, quantum field theory has been an enormous success for physics. It has yielded profound advances in our understanding of how the universe works at the submicroscopic level, and QED in particular has stood up to extremely stringent experimental tests of its validity. Anyone with an interest in the physical sciences must be curious about these achievements, and it is not hard to obtain information about them at the level of, say, *Scientific American* articles. In such popular accounts, one finds that (1) interaction processes are described pictorially by diagrams that represent particles colliding, being emitted and absorbed, and being created and destroyed, although the relevance of these diagrams to actual computations is usually not explained; (2) some of the lines in these diagrams represent real particles, but others represent some shadowy entities called "virtual particles" that cannot be observed although their effects can be measured; (3) quantum field theories are plagued with infinities that must be systematically subtracted off to yield meaningful answers; (4) in spite of the impression given by (1)–(3) that one has blundered into some sort of twilight zone, these ingredients can be combined to yield precise answers that agree exquisitely with experiment. (For example, the theoretical and experimental values of the magnetic moment of the electron agree to within one part in 10^{10}, which is like determining the distance from the Empire State Building to the Eiffel Tower to within a millimeter.)

People with mathematical training are entitled to ask for a deeper and more quantitative understanding of what is going on here. They may feel optimistic about attaining it from their experience with the older areas of fundamental physics that have proved very congenial to mathematical study: the differential equations of classical mechanics, the geometry of Hamiltonian mechanics, and the functional analysis of quantum mechanics. But when they attempt to learn quantum field theory, they are likely to feel that they have run up against a solid wall. There are several reasons for this.

In the first place, quantum field theory is *hard*. A mathematician is no more likely to be able to pick up a text on quantum fields such as Peskin and Schroeder [**89**] and understand its contents on a first reading than a physicist hoping to do the same with, say, Hartshorne's *Algebraic Geometry*. At the deep conceptual level, the absence of firm mathematical foundations gives a warning that some struggle is to be expected. Moreover, quantum field theory draws on ideas and techniques from many different areas of physics and mathematics. (Despite the fact that subatomic particles behave in ways that seem completely bizarre from the human perspective, our understanding of that behavior is built to a remarkable extent on classical physics!) At the more pedestrian level, the fact that the universe seems to be made out of vectors and spinors rather than scalars means that even the simplest calculations tend to involve a certain amount of algebraic messiness that increases the effort needed to understand the essential points. And at the mosquito-bite level of annoyance, there are numerous factors of -1, i, and 2π that are easy to misplace, as well as numerous disagreements among different authors as to how to arrange various normalization constants.

But there is another difficulty of a more cultural and linguistic nature: physics texts are usually written by physicists for physicists. They speak a different dialect, use different notation, emphasize different points, and worry about different things than mathematicians do, and this makes their books hard for mathematicians to read. (Physicists have exactly the same complaint about mathematics books!) In the mathematically better established areas of physics, there are books written from a more mathematical perspective that help to solve this problem, but the lack of a completely rigorous theory has largely prevented such books from being written about quantum field theory.

There have been some attempts at cross-cultural communication. Mathematical interest in theoretical physics was rekindled in the 1980s, after a period in which the long marriage of the two subjects seemed to be disintegrating, when ideas from gauge field theory turned out to have striking applications in differential geometry. But the gauge fields of interest to the geometers are not quantum fields at all, but rather their "classical" (unquantized) analogues, so the mathematicians were not forced to come to grips with quantum issues. More recently, motivated by the development of string theory, in 1996–97 a special year in quantum field theory at the Institute for Advanced Study brought together a group of eminent mathematicians and physicists to learn from each other, and it resulted in the two-volume collection of expository essays *Quantum Fields and Strings* [**21**]. These books contain a lot of interesting material, but as an introduction to quantum fields for ordinary mortals they leave a lot to be desired. One drawback is that the multiple authorships do not lead to a consistent and cohesively structured development of the subject. Another is that the physics is mostly on a rather formal and abstract level; the down-to-earth calculations that lead to experimentally verifiable results are given scant attention. Actually, I would suggest that the reader might study *Quantum Fields and Strings* more profitably *after* reading the present book, as the real focus there is on more advanced topics.

There is another book about quantum fields written by a mathematician, Ticciati's *Quantum Field Theory for Mathematicians* [**121**]. In its general purpose it has some similarity to the present book, but in its organization, scope, and style it is quite different. It turned out not to be the book I needed in order to understand the subject, but it may be a useful reference for others.

The foregoing paragraphs should explain why I thought there was a gap in the literature that needed filling. Now I shall say a few words about what this book does to fill it.

First of all, what are the prerequisites? On the mathematical side, the reader needs to be familiar with the basics of Fourier analysis, distributions (generalized functions), and linear operators on Hilbert spaces, together with a couple of more advanced results in the latter subject — most notably, the spectral theorem. This material can all be found in the union of Folland [**48**] and Reed and Simon [**94**], for example. In addition, a little Lie theory is needed now and then, mostly in the context of the specific groups of space-time symmetries, but in a more general way in the last chapter; Hall [**62**] is a good reference for this. The language of differential geometry is employed only in a few places that can safely be skimmed by readers who are not fluent in it. On the physical side, the reader should have some familiarity with the Hamiltonian and Lagrangian versions of classical mechanics, as well as special relativity, the Maxwell theory of electromagnetism, and basic quantum mechanics. The relevant material is summarized in Chapters 2 and 3, but these brief accounts are meant for review and reference rather than as texts for the novice.

As I mentioned earlier, quantum field theory is built on a very broad base of earlier physics, so the first four chapters of this book are devoted to setting the stage. Chapter 5 introduces free fields, which are already mathematically quite nontrivial although physically uninteresting. The aim here is not only to present the rigorous mathematical construction but also to introduce the more informal way of treating such objects that is common in the physics literature, which offers both practical and conceptual advantages once one gets used to it. The plunge into the deep waters of interacting field theory takes place in Chapter 6, which along with Chapter 7 on renormalization contains most of the really hard work in the book. I use some imagery derived from the Faust legend to describe the necessary departures from mathematical rectitude; its significance is meant to be purely literary rather than theological. Chapter 8 sketches the attractive alternative approach to quantum fields through Feynman's sum-over-histories view of quantum mechanics, and the final chapter presents the rudiments of gauge field theory, skirting most of the quantum issues but managing to derive some very interesting physics nonetheless.

There are several ways to get from the starting line to the goal of calculating quantities with direct physical meaning such as scattering cross-sections. The path I follow here, essentially the one pioneered by Dyson [**25**], [**26**], is to start with free fields, apply perturbation theory to arrive at the integrals associated to Feynman diagrams, and renormalize as necessary. This has the advantages of directness and of minimizing the amount of time spent dealing with mathematically ill-defined objects. Its drawback is that it tethers one to perturbation theory, whereas nonperturbative arguments would be more satisfying in some situations. Physicists may also object to it on the grounds that free fields, although mathematically meaningful, are physically fictitious.

The problem with interacting fields, on the other hand, is exactly the reverse. Hence, although some might prefer to give them a more prominent role, I sequester them in the last section of Chapter 6, where the mathematical soundness of the narrative reaches its nadir, and do not use them at all in Chapter 7 except for a couple of passing mentions. Their credibility is somewhat enhanced, however, by the arguments in Chapter 8 using functional integrals, which are also mathematically ill-defined but intuitively more accessible and seductively close to honest mathematics. Some physicists like to use functional integrals as the principal route to the main results, but despite their appeal, I find them a bit too much like sorcery to be relied on until one already knows where one is going.

This book is meant to be only an introduction to quantum field theory, and it focuses on the goal of explaining actual physical phenomena rather than studying formal structures for their own sake. This means that I have largely (though not entirely) resisted the temptation to pursue mathematical issues when they do not add to the illumination of the physics, and also that I have nothing to say about the more speculative areas of present-day theoretical physics such as supersymmetry and string theory. Even within these restrictions, there are many important topics that are mentioned only briefly or omitted entirely — most notably, the renormalization group. My hope is that this book will better prepare those who wish to go further to tackle the physics literature. References to sources where further information can be obtained on various topics are scattered throughout the book. Here, however, I wish to draw the reader's attention to three physics books whose quality of writing I find exceptional.

First, everyone with any interest in quantum electrodynamics should treat themselves to a perusal of Feynman's *QED* [**38**], an amazingly fine piece of popular exposition. On a much more sophisticated level, but still with a high ratio of physical insight to technical detail, Zee's *Quantum Field Theory in a Nutshell* [**138**] makes very good reading. (Both of these books adopt the functional integral approach.) And finally, for a full-dress treatment of the subject, Weinberg's *The Quantum Theory of Fields* [**131**], [**132**], [**133**] is the sort of book for which the overworked adjective "magisterial" is truly appropriate. Weinberg does not aim for a mathematician's level of rigor, but he has a mathematician's respect for careful reasoning and for appropriate levels of generality, and his approach has influenced mine considerably. I will warn the reader, however, that Weinberg's notation is at variance with standard usage in some respects. Most notably, he takes the Lorentz metric (which he denotes by $\eta^{\mu\nu}$) to have signature $-+++$ rather than the usual $+---$, and since he wants his Dirac matrices γ^μ to satisfy $\{\gamma^\mu,\gamma^\nu\} = 2\eta^{\mu\nu}$, what he calls γ^μ is what most people call $-i\gamma^\mu$.[1]

I call this book a tourist guide for mathematicians. This is meant to give the impression not that it is easy reading (it's not) but that the intended audience consists of people who approach physics as tourists approach a foreign country, as a place to enjoy and learn from but not to settle in permanently. It is also meant to

[1]There is yet a third convention for defining Dirac matrices, found in Sakurai [**103**] among other places.

free me and my readers from guilt about omitting various important but technical topics, viewing others from a point of view that physicists may find perverse, failing to acquire a scholarly knowledge of the literature, and skipping the gruesome details of certain necessary but boring calculations.

I wish to state emphatically that I am a tourist in the realm of physics myself. I hope that my foreigner's perceptions do not do violence to the native culture and that my lack of expertise has not led to the perpetration of many outright falsehoods. Given what usually happens when physicists write about mathematics, however, I dare not hope that there are none. Corrections will be gratefully received at `folland@math.washington.edu` and recorded on a web page accessible from `www.math.washington.edu/~folland/Homepage/index.html`. (*Note added for the second printing*: Numerous small misprints and other errors have been corrected for this printing, and two items have been added to the bibliography. As a result, the page breaks are different in a few places, and many references have been renumbered.) The American Mathematical Society will also host a web page for this book, the URL for which can be found on the back cover above the barcode.

Acknowledgments. I am grateful to the students and colleagues who sat through the course I offered in 2001 in which I made my rather inept first attempt to put this material together. Several physicists, particularly David Boulware, have patiently answered many questions for me, and they are not to blame if their answers have become distorted in passing through my brain. Finally, an unnamed referee provided several helpful suggestions and useful references.

The Feynman diagrams in this book were created with JaxoDraw, available at `jaxodraw.sourceforge.net/sitemap.html`.

Gerald B. Folland
Seattle, April 2008

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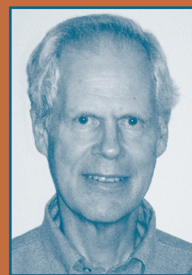
Quantum field theory has been a great success for physics, but it is difficult for mathematicians to learn because it is mathematically incomplete. Folland, who is a mathematician, has spent considerable time digesting the physical theory and sorting out the mathematical issues in it. Fortunately for mathematicians, Folland is a gifted expositor.



The purpose of this book is to present the elements of quantum field theory, with the goal of understanding the behavior of elementary particles rather than building formal mathematical structures, in a form that will be comprehensible to mathematicians. Rigorous definitions and arguments are presented as far as they are available, but the text proceeds on a more informal level when necessary, with due care in identifying the difficulties.

The book begins with a review of classical physics and quantum mechanics, then proceeds through the construction of free quantum fields to the perturbation-theoretic development of interacting field theory and renormalization theory, with emphasis on quantum electrodynamics. The final two chapters present the functional integral approach and the elements of gauge field theory, including the Salam–Weinberg model of electromagnetic and weak interactions.

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