Maximum Principles and Sharp Constants for Solutions of Elliptic and Parabolic Systems

Gershon Kresin
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American Mathematical Society
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List of Symbols

Point Sets

\( \Omega \) domain in \( \mathbb{R}^n \) with closure \( \overline{\Omega} \) and boundary \( \partial \Omega \) 1

\( Q_T \) cylinder \( \Omega \times (0, T] \) in the space \( \mathbb{R}^{n+1} \) 1

\( \Gamma_T \) parabolic boundary of the cylinder \( Q_T \), where

\[ \Gamma_T = \{(x, t) \in \partial Q_T : 0 \leq t < T \} \]

\( \mathbb{R}_+ \) upper half-space \( \{x = (x_1, \ldots, x_n) : x_n > 0 \} \) in \( \mathbb{R}^n \)

\( \mathbb{R}_+^{n+1} \) layer \( \mathbb{R}^n \times (0, T] \) in \( \mathbb{R}^{n+1} \)

\( S^{n-1} \) unit sphere in \( \mathbb{C}^n \) centered at 0 16

\( S_{n-1}^- \) lower hemisphere \( \{x \in \mathbb{R}^n : |x| = 1, x_n < 0 \} \) 24

\( \mathbb{B}_r \) ball in \( \mathbb{R}^n \) with radius \( r \) centered at 0 27

\( \mathbb{R}^n_+(\nu) \) half-space \( \{x \in \mathbb{R}^n : (x, \nu) > 0 \} \), where

\( \nu \) is a unit \( n \)-dimensional vector 27

\( S^{n-1} \) unit sphere in \( \mathbb{R}^n \) centered at 0 27

\( \mathbb{B} \) unit ball in \( \mathbb{R}^n \) centered at 0 34

\( \mathbb{B}_r(y) \) ball in \( \mathbb{R}^n \) with radius \( r \) centered at \( y \) 44

\( \mathbb{R}^n_+(O) \) tangent space to \( \partial \Omega \) at a point \( O \in \partial \Omega \) 77

\( \mathbb{D} \) disk \( |z| < 1 \) in the complex plane \( \mathbb{C} \) 148

\( \mathbb{C}_+ \) upper half-plane of the complex plane \( \mathbb{C} \) 149

\( \mathcal{B} \) Borel set in \( \mathbb{R}^n \) with interior \( \text{int} \mathcal{B} \), closure \( \overline{\mathcal{B}} \), boundary \( \partial \mathcal{B} \) and complement \( \mathcal{C} \mathcal{B} = \mathbb{R}^n \backslash \mathcal{B} \) 154

\( \partial^*E \) reduced boundary of a set \( E \subset \mathbb{R}^n \) 157

\( \Pi_T \) set \( \mathcal{D} \times (0, T] \), where \( \mathcal{D} \) is either a

bounded domain in \( \mathbb{R}^n \) or \( \mathcal{D} = \mathbb{R}^n \) 205

\( S_T \) cylindrical surface, where \( S_T = \partial \Omega \times (0, T] \) 237

\( \mathcal{B} \) unit ball of a generalized norm 252

\( S^{m-1} \) unit sphere of the generalized norm 253

Vectors

\( e_\sigma \) \( n \)-dimensional unit vector joining the origin to \( \sigma \in \mathbb{S}^{n-1} \) 56

\( e_{xy} \) \( n \)-dimensional unit vector joining the point \( x \) to point \( y \) 56

\( e_i \) unit vector of the \( i \)-th coordinate axis 60

\( r_{xy} \) \( n \)-dimensional vector joining the point \( x \) to point \( y \)

with the length \( r_{xy} \) 154

Conv \( S \) convex hull of a vector set \( S \) 255

Span \( S \) linear span of a vector set \( S \) 256

Set Functions

\( \omega(x, \mathcal{B}) \) solid angle at which a set \( \mathcal{B} \subset \partial \mathbb{R}_+^n \) is seen from \( x \in \mathbb{R}^n_+ \) 24

\( H_k \) \( k \)-dimensional Hausdorff measure in \( \mathbb{R}^n \) 154

\( \text{mes}_n \) Lebesque measure in \( \mathbb{R}^n \) 154

\( P(\mathcal{B}) \) perimeter of the set \( \mathcal{B} \) in the sense of Caccioppoli and De Giorgi 155

\( \omega_D(p, \mathcal{B}) \) solid angle at which the set \( \mathcal{B} \cap \partial \mathcal{D} \) is seen from \( p \in \mathbb{R}^n \) 156

\( \Psi_D(p, \mathcal{B}) \) matrix-valued set function 161
Functions

\begin{align*}
\Gamma(\alpha) & \quad \text{Gamma-function} \\
E(k) & \quad \text{complete elliptic integral of the second kind} \\
B(\alpha, \beta) & \quad \text{Beta-function} \\
J_\nu(x) & \quad \text{Bessel function of the first kind} \\
F(\alpha, \beta; \gamma, x) & \quad \text{hypergeometric Gauss function} \\
K(k) & \quad \text{complete elliptic integral of the first kind} \\
D(k) & \quad \text{complete elliptic integral} \\
W(q) & \quad \text{vector-valued double layer potential} \\
W_\nu^{(n)}(q) & \quad \text{vector-valued elastic/hydrodynamic double layer potential} \\
E(\varphi, k) & \quad \text{elliptic integral of the second kind} \\
\chi_B & \quad \text{characteristic function of a Borel set } B
\end{align*}

Spaces

\begin{align*}
X & \quad \text{locally compact Hausdorff space} \\
[C_v(X)]^n & \quad \text{space of continuous } n\text{-component real vector-valued functions on } X \text{ which vanish at infinity} \\
[C_v(X)]^n & \quad \text{space of continuous } n\text{-component complex vector-valued functions on } X \text{ which vanish at infinity} \\
(\mathcal{X}, \mathcal{A}, \mu) & \quad \text{space with a measure} \\
[L^p(\mathcal{X}, \mathcal{A}, \mu)]^n & \quad \text{Lebesgue space of } n\text{-component functions on } (\mathcal{X}, \mathcal{A}, \mu) \\
[L^p(\mathcal{X}, \mathcal{A}, \mu)]^n & \quad \text{Lebesgue space of complex } n\text{-component functions on } (\mathcal{X}, \mathcal{A}, \mu) \\
\mathbb{R}^n & \quad n\text{-dimensional Euclidean space} \\
\mathbb{C}^n & \quad n\text{-dimensional unitary space} \\
[B_b(X)]^n & \quad \text{space of real vector-valued functions with } n \text{ components which are Borel and bounded on } X \\
[B_b(X)]^n & \quad \text{space of complex vector-valued functions with } n \text{ components which are Borel and bounded on } X \\
[C_b(X)]^n & \quad \text{space of real vector-valued functions with } n \text{ components which are continuous and bounded on } X \\
[C_b(X)]^n & \quad \text{space of complex vector-valued functions with } n \text{ components which are continuous and bounded on } X \\
\mathcal{M}_{\mathbb{R}}(\mathcal{B}_X) & \quad \text{space of all finite signed regular Borel measures on } \sigma\text{-algebra } \mathcal{B}_X \text{ of Borel subsets of } X \\
\mathcal{M}_{\mathbb{C}}(\mathcal{B}_X) & \quad \text{space of all finite complex regular Borel measures on } \sigma\text{-algebra } \mathcal{B}_X \text{ of Borel subsets of } X \\
[C(K)]^m & \quad \text{space of real continuous vector-valued functions with } m \text{ components on a compact } K \text{ in } \mathbb{R}^k \\
[C(K)]^m & \quad \text{space of complex continuous vector-valued functions with } m \text{ components on a compact } K \text{ in } \mathbb{R}^k \\
[C^k(\Omega)]^m & \quad \text{space of real } m\text{-component vector-valued functions with continuous derivatives up to order } k \text{ in } \Omega
\end{align*}
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tr>
<td>$[C^k(\Omega)]^m$</td>
<td>space of complex $m$-component vector-valued functions with continuous derivatives up to order $k$ in $\Omega$</td>
</tr>
<tr>
<td>$[\mathcal{C}^k(\Omega)]^m$</td>
<td>space of complex $m$-component vector-valued functions with continuous derivatives up to order $k$ in $\Omega$</td>
</tr>
<tr>
<td>$[\mathcal{C}^{k,\alpha}(\overline{\Omega})]^{m \times m}$</td>
<td>space of real $(m \times m)$-matrix-valued functions whose elements have continuous derivatives up to order $k$ and satisfy the Hölder condition with exponent $\alpha$ on $\overline{\Omega}$</td>
</tr>
<tr>
<td>$[C^{k,\alpha}(\overline{\Omega})]^{m \times m}$</td>
<td>space of complex $(m \times m)$-matrix-valued functions whose elements have continuous derivatives up to order $k$ and satisfy the Hölder condition with exponent $\alpha$ on $\overline{\Omega}$</td>
</tr>
<tr>
<td>$C^k_0(G)$</td>
<td>space of real functions with continuous derivatives up to order $k$ with compact support in $G$</td>
</tr>
<tr>
<td>$C_0(G)$</td>
<td>space of real continuous functions with compact support in $G$</td>
</tr>
<tr>
<td>$[W^1_p(\Omega)]^m$</td>
<td>Sobolev space of $m$-component vector-valued functions on $\Omega$ with each component in $W^1_p(\Omega)$</td>
</tr>
<tr>
<td>$[W^l_p(\Omega)]^m$</td>
<td>Sobolev space of $m$-component vector-valued functions on $\Omega$ with each component in $W^l_p(\Omega)$</td>
</tr>
<tr>
<td>$[L^p(G)]^m$</td>
<td>space of real vector-valued functions $u = (u_1, \ldots, u_m)$ for which $</td>
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<tr>
<td>$h^p(\mathbb{R}^n_+)$</td>
<td>Hardy space of harmonic functions on $\mathbb{R}^n_+$ which can be represented as the Poisson integral</td>
</tr>
<tr>
<td>$h^p(\mathbb{B})$</td>
<td>Hardy space of harmonic functions on $\mathbb{B}$ which can be represented as the Poisson integral</td>
</tr>
<tr>
<td>$BV(\mathbb{R}^n)$</td>
<td>space of locally integrable functions on $\mathbb{R}^n$ whose gradients (in the distributional sense) are finite vector-valued charges on $\mathbb{R}^n$</td>
</tr>
<tr>
<td>$[C^{(k,1)}(\Pi_T)]^m$</td>
<td>space of real $m$-component vector-valued functions on $\Pi_T$ whose derivatives with respect to $x$ up to order $k$ and first derivative with respect to $t$ are continuous</td>
</tr>
<tr>
<td>$[C^k_b(\mathbb{R}^n)]^m$</td>
<td>space of real $m$-component vector-valued functions on $\mathbb{R}^n$ with continuous and bounded derivatives up to order $k$ which satisfy the uniform Hölder condition with exponent $\alpha$</td>
</tr>
<tr>
<td>$[C^k_b,\alpha/2\ell(\mathbb{R}^{n+1}_T)]^m$</td>
<td>space of real $m$-component vector-valued functions with derivatives up to order $k$ with respect to $x$ which are bounded in $\mathbb{R}^{n+1}_T$ and satisfy the uniform Hölder condition with exponent $\alpha$ with respect to the parabolic distance</td>
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\( [\mathcal{C}(\mathcal{Q}_T)]^m \) \space of \( m \)-component vector-valued functions from 
\( [\mathcal{C}(\mathcal{Q}_T)]^m \) \space vanishing on \( \mathcal{S}_T \)

**Operators**

\[
\begin{align*}
D_x & \quad (\partial/\partial x_1, \ldots, \partial/\partial x_n) \\
\mathfrak{A}_0(D_x) & \quad \text{principal homogeneous part of the operator } \mathfrak{A}(D_x) \\
F[\cdot] & \quad \text{Fourier transform} \\
\Delta & \quad \text{Laplace operator} \\
\text{grad} & \quad \text{gradient} \\
\text{div} & \quad \text{divergence} \\
\mathfrak{C}_0(D_x) & \quad \text{principal homogeneous part of the operator } \mathfrak{C}(D_x) \\
\mathfrak{A}(x, D_x) & \quad \text{linear differential operator of the second order} \\
& \quad \text{with real } (m \times m)\text{-matrix-valued coefficients} \\
& \quad \text{defined on } \overline{\Omega} \\
\mathfrak{A}_0(x, D_x) & \quad \text{principal homogeneous part of the operator } \mathfrak{A}(x, D_x) \\
\mathfrak{A}(D_x) & \quad \text{linear differential operator of the second order} \\
& \quad \text{whose coefficients are real constant} \\
& \quad (m \times m)\text{-matrices} \\
[A, B] & \quad \text{commutator of operators } A \text{ and } B \\
\mathfrak{C}(x, D_x) & \quad \text{linear differential operator of the second order} \\
& \quad \text{with complex } (m \times m)\text{-matrix-valued coefficients} \\
& \quad \text{defined on } \overline{\Omega} \\
\mathfrak{C}(D_x) & \quad \text{linear differential operator of the second order} \\
& \quad \text{whose coefficients are complex constant} \\
& \quad (m \times m)\text{-matrices} \\
D_x^\beta & \quad \partial^{(\beta_1)}/\partial x_1^{\beta_1} \ldots \partial x_n^{\beta_n}, \text{ where } \beta = (\beta_1, \ldots, \beta_n) \\
P(D_x) & \quad \text{elliptic operator of order } 2\ell \\
& \quad \text{with constant complex coefficients} \\
P_0(D_x) & \quad \text{principal homogeneous part of } P(D_x) \\
\Delta^2 & \quad \text{biharmonic operator} \\
F^{-1}[\cdot] & \quad \text{inverse Fourier transform} \\
T_n^{(n)} & \quad \text{matrix-valued integral operator generated} \\
& \quad \text{by the vector-valued elastic/hydrodynamic} \\
& \quad \text{double layer potential } W^{(n)}(q) \\
\mathfrak{P}(x, t, D_x) & \quad \text{linear differential operator of order } 2\ell \\
& \quad \text{with real } (m \times m)\text{-matrix-valued coefficients} \\
& \quad \text{defined on } \Pi_T \\
\mathfrak{P}_0(x, t, D_x) & \quad \text{principal homogeneous part of the operator} \\
\mathfrak{P}(x, t, D_x) & \quad \text{defined on } \Pi_T \\
\mathfrak{S}(x, t, D_x) & \quad \text{linear differential operator of order } 2\ell \\
& \quad \text{with complex } (m \times m)\text{-matrix-valued coefficients} \\
& \quad \text{defined on } \Pi_T
\end{align*}
\]
**LIST OF SYMBOLS**

- $\mathcal{L}_0(x,t,D_x)$ principal homogeneous part of the operator $\mathcal{L}(x,t,D_x)$ 215
- $\mathfrak{A}_0(x,t,D_x)$ principal homogeneous part of the operator $\mathfrak{A}(x,t,D_x)$ 256
- $\mathfrak{A}(x,t,D_x)$ linear differential operator of the second order with real $(m \times m)$-matrix-valued coefficients defined on $\Pi_T$ 280

**Other Symbols**

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<td>$\nabla_k u$</td>
<td>gradient of order $k$ of a function $u$</td>
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<tr>
<td>$\nabla u$</td>
<td>gradient of $u$</td>
<td>3</td>
</tr>
<tr>
<td>$\omega_n$</td>
<td>area of the unit sphere in $\mathbb{R}^n$</td>
<td>3</td>
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<tr>
<td>$\mathcal{B}_X$</td>
<td>$\sigma$-algebra of Borel subsets of $X$</td>
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</tr>
<tr>
<td>$</td>
<td>\cdot</td>
<td>$</td>
</tr>
<tr>
<td>$(\cdot,\cdot)$</td>
<td>inner product of vectors in Euclidean or unitary space</td>
<td>10</td>
</tr>
<tr>
<td>$</td>
<td>\cdot</td>
<td>$</td>
</tr>
<tr>
<td>$\Re$</td>
<td>real part</td>
<td>16, 148</td>
</tr>
<tr>
<td>$\Im$</td>
<td>imaginary part</td>
<td>16, 149</td>
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<tr>
<td>$\delta_{ij}$</td>
<td>Kronecker delta</td>
<td>64</td>
</tr>
<tr>
<td>$|\cdot|_p$</td>
<td>the norm in $L^p$-space</td>
<td>79, 131</td>
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<tr>
<td>$\partial u/\partial \ell$</td>
<td>derivative of $u$ in the direction of a unit vector $\ell$</td>
<td>105</td>
</tr>
<tr>
<td>$\Lambda_p(u)$</td>
<td>best approximation of a function $u$ on $\mathbb{S}^{n-1}$ by a constant in the norm of $L_p(\mathbb{S}^{n-1})$</td>
<td>108</td>
</tr>
<tr>
<td>$d_x$</td>
<td>distance from the point $x$ to the boundary of domain</td>
<td>116</td>
</tr>
<tr>
<td>$\mathcal{Q}_D(u;x)$</td>
<td>characteristics of bounded or semibounded functions in a domain $D \subset \mathbb{R}^n$</td>
<td>116</td>
</tr>
<tr>
<td>$\text{osc}_D(u)$</td>
<td>oscillation of a function $u$ on a domain $D \subset \mathbb{R}^n$</td>
<td>148</td>
</tr>
<tr>
<td>$\text{ess}|L|$</td>
<td>essential norm of a linear bounded operator $L$ acting on a Banach space $\mathfrak{B}$</td>
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<tr>
<td>$v_n$</td>
<td>volume of the unit ball in $\mathbb{R}^n$</td>
<td>154</td>
</tr>
<tr>
<td>$\text{dist}(p,\mathcal{B})$</td>
<td>distance from a point $p$ to a set $\mathcal{B}$</td>
<td>154</td>
</tr>
<tr>
<td>$R(L)$</td>
<td>Fredholm radius of a linear bounded operator $L$ acting on a Banach space $\mathfrak{B}$</td>
<td>197</td>
</tr>
<tr>
<td>$C_L$</td>
<td>continuity degree of a linear bounded operator $L$ acting on a Banach space $\mathfrak{B}$</td>
<td>198</td>
</tr>
<tr>
<td>$d[(x,t),(x',t')]$</td>
<td>parabolic distance between the points $(x,t)$ and $(x',t')$ in $\mathbb{R}^{n+1}$</td>
<td>203</td>
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<tr>
<td>$</td>
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<td>$</td>
<td>\cdot</td>
<td>_s$</td>
</tr>
<tr>
<td>$[[\mathbf{v}_1,\ldots,\mathbf{v}_m]]$</td>
<td>$(m \times m)$-matrix whose columns are $m$-component vectors $\mathbf{v}_1,\ldots,\mathbf{v}_m$</td>
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<td>$\emptyset$</td>
<td>empty set</td>
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The main goal of this book is to present results pertaining to various versions of the maximum principle for elliptic and parabolic systems of arbitrary order. In particular, the authors present necessary and sufficient conditions for validity of the classical maximum modulus principles for systems of second order and obtain sharp constants in inequalities of Miranda-Agmon type and in many other inequalities of a similar nature. Somewhat related to this topic are explicit formulas for the norms and the essential norms of boundary integral operators. The proofs are based on a unified approach using, on one hand, representations of the norms of matrix-valued integral operators whose target spaces are linear and finite dimensional, and, on the other hand, on solving certain finite dimensional optimization problems.

This book reflects results obtained by the authors, and can be useful to research mathematicians and graduate students interested in partial differential equations.