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Unitary Representations of Groups, Duals, and Characters

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Foreword

This is an expository book on unitary representations of topological groups, and of several dual spaces, which are spaces of such representations up to some equivalence. The most important notions are defined for topological groups, but special attention is paid to the case of discrete groups.

The unitary dual of a group G is the space of equivalence classes of its irreducible unitary representations; it is both a topological space and a Borel space. It is useful in important situations, but much less useful in many other situations, for example because, for an infinite discrete group, it is usually not countably separated as a Borel space.

We discuss other candidates for being a dual space of a group. The two most important ones are the primitive dual, the space of weak equivalence classes of unitary irreducible representations, and the normal quasi-dual, the space of quasi-equivalence classes of traceable factor representations. The last space is parametrized by "characters", which can be finite or infinite. The space of finite characters is an object particularly easy to describe: it is in natural bijection with the space of indecomposable central positive definite functions on the group, that we call the Thoma dual. We discuss other related spaces, such as the space of finite-dimensional irreducible unitary representations.

These spaces are much better behaved than the unitary dual. They deserve to be studied on their own, and related to each other. In contrast to the unitary dual, they can in principle be classified for every specific group. We illustrate the theory by giving an explicit description of these alternative dual spaces for a series of examples: Heisenberg groups over fields and over the integers, affine groups of infinite fields, solvable Baumslag–Solitar groups, lamplighter groups, and general (or special) linear groups over infinite fields.

Operator algebras play an important role in the discussion, in particular von Neumann algebras associated with unitary representations and C*-algebras associated with locally compact groups.

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It is the occasion to celebrate Jacques Dixmier and the writing
of his two books [**Dixm–vN**, **Dixm–C***] *on operator algebras.*
More than 50 years after publication,
they remain unrivalled references on the subject.

Bibliography

[Aaro–97] J. Aaronson, *An introduction to infinite ergodic theory*, Mathematical Surveys and Monographs, vol. 50, American Mathematical Society, Providence, RI, 1997. MR1450400. 368, 390

[AbEL–11] F. Abdelmoula, M. Elloumi, and J. Ludwig, *The C^*-algebra of the motion group* $\mathrm{SO}(n) \ltimes \mathbb{R}^n$, Bull. Sci. Math. **135** (2011), no. 2, 166–177. MR2773395. 246

[AbGV–14] M. Abért, Y. Glasner, and B. Virág, *Kesten's theorem for invariant random subgroups*, Duke Math. J. **163** (2014), no. 3, 465–488. MR3165420. 416, 417

[ABB+–17] M. Abert, N. Bergeron, I. Biringer, T. Gelander, N. Nikolov, J. Raimbault, and I. Samet, *On the growth of L^2-invariants for sequences of lattices in Lie groups*, Ann. of Math. (2) **185** (2017), no. 3, 711–790. MR3664810. 417

[AlBe–95] J. L. Alperin and R. B. Bell, *Groups and representations*, Graduate Texts in Mathematics, vol. 162, Springer-Verlag, New York, 1995. MR1369573. 130, 277

[AlEd–79] S. Alpern and R. D. Edwards, *Lusin's theorem for measure-preserving homeomorphisms*, Mathematika **26** (1979), no. 1, 33–43. MR557124. 412

[Ambr–44] W. Ambrose, *Spectral resolution of groups of unitary operators*, Duke Math. J. **11** (1944), 589–595. MR11172. 82

[AnMa–12] H. Ando and Y. Matsuzawa, *On Polish groups of finite type*, Publ. Res. Inst. Math. Sci. **48** (2012), no. 2, 389–408. MR2928146. 399

[ArKa–97] R. J. Archbold and E. Kaniuth, *On the primitive ideal space of a finitely generated nilpotent group*, Forum Math. **9** (1997), no. 2, 193–210. MR1431120. 331

[Arti–57] E. Artin, *Geometric algebra*, Interscience Publishers, Inc., New York-London, 1957. MR0082463. 144

[Arve–76] W. Arveson, *An invitation to C^*-algebras*, Graduate Texts in Mathematics, No. 39, Springer-Verlag, New York-Heidelberg, 1976. MR0512360. 71

[AuMo–66] L. Auslander and C. C. Moore, *Unitary representations of solvable Lie groups*, Mem. Amer. Math. Soc. **62** (1966), 199. MR207910. 204, 228

[AuKo–71] L. Auslander and B. Kostant, *Polarization and unitary representations of solvable Lie groups*, Invent. Math. **14** (1971), 255–354. MR293012. 209

[Bagg–68] L. Baggett, *A description of the topology on the dual spaces of certain locally compact groups*, Trans. Amer. Math. Soc. **132** (1968), 175–215. MR409720. 38

[Bagg–72] L. Baggett, *A separable group having a discrete dual space is compact*, J. Functional Analysis **10** (1972), 131–148. MR0346090. 35, 36

[BaMe–86] L. Baggett and K. Merrill, *Representations of the Mautner group and cocycles of an irrational rotation*, Michigan Math. J. **33** (1986), no. 2, 221–229. MR837580. 165

[BaSu–81] L. Baggett and T. Sund, *The Hausdorff dual problem for connected groups*, J. Functional Analysis **43** (1981), no. 1, 60–68. MR639797. 38

[BaPa–94] L. Baggett and J. Packer, *The primitive ideal space of two-step nilpotent group C^*-algebras*, J. Funct. Anal. **124** (1994), no. 2, 389–426. MR1289356. 271

[Bana–83] W. Banaszczyk, *On the existence of exotic Banach-Lie groups*, Math. Ann. **264** (1983), no. 4, 485–493. MR716262. 27

[Bana–91] W. Banaszczyk, *Additive subgroups of topological vector spaces*, Lecture Notes in Mathematics, vol. 1466, Springer-Verlag, Berlin, 1991. MR1119302. 27

[Barg–47] V. Bargmann, *Irreducible unitary representations of the Lorentz group*, Ann. of Math. (2) **48** (1947), 568–640. MR21942. 8, 121

[BaLS–64] H. Bass, M. Lazard, and J.-P. Serre, *Sous-groupes d'indice fini dans* $\mathbf{SL}(n, \mathbf{Z})$ (French), Bull. Amer. Math. Soc. **70** (1964), 385–392. MR161913. 143

[Bedo–94] E. Bédos, *On the C^*-algebra generated by the left regular representation of a locally compact group*, Proc. Amer. Math. Soc. **120** (1994), no. 2, 603–608. MR1181157. 260

[Bekk–99] B. Bekka, *On the full C^*-algebras of arithmetic groups and the congruence subgroup problem*, Forum Math. **11** (1999), no. 6, 705–715. MR1725593. 416

[Bekk–07] B. Bekka, *Operator-algebraic superridigity for $\mathrm{SL}_n(\mathbb{Z})$, $n \geq 3$*, Invent. Math. **169** (2007), no. 2, 401–425. MR2318561. 326, 331, 416

[Bekk–19] B. Bekka, *Infinite characters on $GL_n(\mathbf{Q})$, on $\mathrm{SL}_n(\mathbf{Z})$, and on groups acting on trees*, J. Funct. Anal. **277** (2019), no. 7, 2160–2178. MR3989142. 313

[Bekk–20] B. Bekka, *Character rigidity of simple algebraic groups*, Math. Ann. **378** (2020),no. 3–4, 1223–1243. 326, 420

[BeLS–92] B. Bekka, A. T. Lau, and G. Schlichting, *On invariant subalgebras of the Fourier-Stieltjes algebra of a locally compact group*, Math. Ann. **294** (1992), no. 3, 513–522. MR1188135. 251

[BeHa–94] B. Bekka and P. de la Harpe, *Représentations d'un groupe faiblement équivalentes à la représentation régulière* (French, with English and French summaries), Bull. Soc. Math. France **122** (1994), no. 3, 333–342. MR1294459. 225, 261, 290, 432

[BeVa–95] B. Bekka and A. Valette, *Lattices in semi-simple Lie groups, and multipliers of group C^*-algebras*, Recent advances in operator algebras (Orléans, 1992), Astérisque **232** (1995), 67–79. MR1372525. 251

[BeLo–00] B. Bekka and N. Louvet, *Some properties of C^*-algebras associated to discrete linear groups*, C^*-algebras (Münster, 1999), Springer, Berlin, 2000, pp. 1–22. MR1796907. 412

[BeMa–00] B. Bekka and M. Mayer, *Ergodic theory and topological dynamics of group actions on homogeneous spaces*, London Mathematical Society Lecture Note Series, vol. 269, Cambridge University Press, Cambridge, 2000. MR1781937. 108, 112, 117, 118, 168, 275, 386, 434

[BeCu–03] B. Bekka and R. Curtis, *On Mackey's irreducibility criterion for induced representations*, Int. Math. Res. Not. **38** (2003), 2095–2101. MR1994777. 51

[BeHV–08] B. Bekka, P. de la Harpe, and A. Valette, *Kazhdan's Property (T)*, Cambridge Univ. Press, 2008. [An erratum is available on the web, and contains in particular a corrected proof of Proposition F.2.7, quoted on several occasions in the present book. `https://perso.univ-rennes1.fr/bachir.bekka/Erratum-Kazhdan.pdf`] 17, 19, 21, 25, 26, 27, 28, 29, 31, 33, 38, 40, 43, 44, 52, 86, 148, 151, 172, 190, 214, 249, 254, 265, 267, 289, 356, 413, 416, 435, 436, 438

[BeEc–20] B. Bekka and S. Echterhoff, *On unitary representations of algebraic groups over local fields*, `arXiv:2003.02297v1`, 4 Mar 2020. 208

[BeFr–20] B. Bekka and C. Francini, *Characters of algebraic groups over number fields*, `arXiv:2002.07497v1`, 18 Feb 2020. 326

[BYTs–16] I. Ben Yaacov and T. Tsankov, *Weakly almost periodic functions, model-theoretic stability, and minimality of topological groups*, Trans. Amer. Math. Soc. **368** (2016), no. 11, 8267–8294. MR3546800. 27

[BCRZ–16] V. Bergelson, J. C. Christopherson, D. Robertson, and P. Zorin-Kranich, *Finite products sets and minimally almost periodic groups*, J. Funct. Anal. **270** (2016), no. 6, 2126–2167. MR3460236. 145

[Bind–93] M. W. Binder, *Induced factor representations of discrete groups and their types*, J. Funct. Anal. **115** (1993), no. 2, 294–312. MR1234393. 225

[Bern–74] I. N. Bernšteĭn, *All reductive $\mathfrak{p}$-adic groups are of type I* (Russian), Funkcional. Anal. i Priložen. **8** (1974), no. 2, 3–6. MR0348045. 204

[Blac–77] B. E. Blackadar, *The regular representation of restricted direct product groups*, J. Functional Analysis **25** (1977), no. 3, 267–274. MR0439979. 387

[Blat–63] R. J. Blattner, *Positive definite measures*, Proc. Amer. Math. Soc. **14** (1963), 423–428. MR147580. 46

[BLSV–78] J. Boidol, H. Leptin, J. Schürman, and D. Vahle, *Räume primitiver Ideale von Gruppenalgebren* (German), Math. Ann. **236** (1978), no. 1, 1–13. MR498971. 261

[Boga–07] V. I. Bogachev, *Measure theory. Vol. I, II*, Springer-Verlag, Berlin, 2007. MR2267655. 430

[Bore–86] A. Borel, *Hermann Weyl and Lie groups*, Hermann Weyl, 1885–1985, Eidgenössische Tech. Hochschule, Zürich, 1986, pp. 53–82. MR875268. 7

[Boul–16] M. Bouljihad, *Existence of rigid actions for finitely-generated non-amenable linear groups*, `arXiv:1606.05911v1`, 19 Jun 2016. 364

[BA1–3] N. Bourbaki, *Algèbre, chapitres 1 à 3*, Bourbaki, 1970. 124

[BTG1–4] N. Bourbaki, *Topologie générale, chapitres 1 à 4*, Hermann, 1971. 32, 39, 138, 254, 405, 421, 423

[BTG5–10] N. Bourbaki, *Topologie générale, chapitres 5 à 10*, Hermann, 1971. 31, 422, 423

[BEVT1–5] N. Bourbaki, *Espaces vectoriels topologiques, chapitres 1 à 5*, N. Bourbaki et Springer, 2007. 362, 426

[BTS1–2] N. Bourbaki, *Théories spectrales, chapitres 1 et 2*, Hermann, 1967. 25, 100, 115, 116, 385, 437, 438, 439

[BoHo–19] R. Boutonnet and C. Houdayer, *Stationary characters on lattices of semisimple Lie groups*, `arXiv:1908.07812v1`, 21 Aug 2019. 326

[Boye–83] R. P. Boyer, *Infinite traces of AF-algebras and characters of* $\mathrm{U}(\infty)$, J. Operator Theory **9** (1983), no. 2, 205–236. MR703808. 326

[BoMa–77] R. Boyer and R. Martin, *The group C^*-algebra of the de Sitter group*, Proc. Amer. Math. Soc. **65** (1977), no. 1, 177–184. MR473718. 246

[Broo–88] R. Brooks, *Constructing isospectral manifolds*, Amer. Math. Monthly **95** (1988), no. 9, 823–839. MR967343. 51

[Brow–73a] I. D. Brown, *Representation of finitely generated nilpotent groups*, Pacific J. Math. **45** (1973), 13–26. MR352324. 165

[Brow–73b] I. D. Brown, *Dual topology of a nilpotent Lie group*, Ann. Sci. École Norm. Sup. (4) **6** (1973), 407–411. MR352326. 110

[BrOz–08] N. P. Brown and N. Ozawa, *C^*-algebras and finite-dimensional approximations*, Graduate Studies in Mathematics, vol. 88, American Mathematical Society, Providence, RI, 2008. MR2391387. 408

[BrTi–72] F. Bruhat and J. Tits, *Groupes réductifs sur un corps local* (French), Inst. Hautes Études Sci. Publ. Math. **41** (1972), 5–251. MR327923. 190

[Bump–97] D. Bump, *Automorphic forms and representations*, Cambridge Studies in Advanced Mathematics, vol. 55, Cambridge University Press, Cambridge, 1997. MR1431508. 52

[BuHa–97] M. Burger and P. de la Harpe, *Constructing irreducible representations of discrete groups*, Proc. Indian Acad. Sci. Math. Sci. **107** (1997), no. 3, 223–235. MR1467427. 49

[Burn–11] W. Burnside, *The theory of groups of finite order*, 2nd Edition. Cambridge Univ. Press, 1911. [Reprint, Dover, 1955.] 49

[CaRW–17] P.-E. Caprace, C. D. Reid, and G. A. Willis, *Locally normal subgroups of totally disconnected groups. Part II: Compactly generated simple groups*, Forum Math. Sigma **5** (2017), e12, 89. MR3659769. 405

[CaLe–16] A. Carderi and F. Le Maître, *More Polish full groups*, Topology Appl. **202** (2016), 80–105. MR3464151. 327

[CaMo–84] A. L. Carey and W. Moran, *Characters of nilpotent groups*, Math. Proc. Cambridge Philos. Soc. **96** (1984), no. 1, 123–137. MR743708. 320, 326, 329

[Choi–80] M. D. Choi, *The full C^*-algebra of the free group on two generators*, Pacific J. Math. **87** (1980), no. 1, 41–48. MR590864. 410

[Conn–76] A. Connes, *Classification of injective factors. Cases II_1, II_∞, III_λ, $\lambda \neq 1$*, Ann. of Math. (2) **104** (1976), no. 1, 73–115. MR454659. 224, 407

[Conr] K. Conrad, *The character group of* $\mathbf{Q}$, Unpublished note, available on `http://www.math.uconn.edu/~kconrad/blurbs/` 105

[Corn–09] Y. Cornulier, *Infinite conjugacy classes in groups acting on trees*, Groups Geom. Dyn. **3** (2009), no. 2, 267–277. MR2486799. 432

[CoHa–16] Y. Cornulier and P. de la Harpe, *Metric geometry of locally compact groups*, EMS Tracts in Mathematics, vol. 25, European Mathematical Society (EMS), Zürich, 2016. Winner of the 2016 EMS Monograph Award. MR3561300. 114, 254, 404

[Corw–75] L. Corwin, *Induced representations of discrete groups*, Proc. Amer. Math. Soc. **47** (1975), 279–287. MR387487. 51

[CoSt–91] M. Cowling and T. Steger, *The irreducibility of restrictions of unitary representations to lattices*, J. reine angew. Math. **420** (1991), 85–98. MR1124567. 261, 290

[Cuntz–77] J. Cuntz, *Simple C^*-algebras generated by isometries*, Comm. Math. Phys. **57** (1977), no. 2, 173–185. MR467330. 314

[Dixm–vN] J. Dixmier, *Les algèbres d'opérateurs dans l'espace hilbertien (algèbres de von Neumann)*, deuxième édition, revue et augmentée. Gauthier-Villars, 1969. [First Edition 1957, English translation North-Holland 1981.] xi, 12, 53, 54, 56, 59, 60, 64, 80, 184, 195, 211, 218, 219, 220, 222, 230, 259, 298, 304, 306, 307, 308, 309, 311, 312, 315, 321, 353, 361, 362, 402, 429, 440, 443, 444, 445

[Dixm–C*] J. Dixmier, *Les C^*-algèbres et leurs représentations*, deuxième édition. Gauthier–Villars, 1969. [First Edition 1964, English translation North-Holland 1977.] xi, 9, 12, 19, 21, 30, 31, 37, 52, 53, 55, 56, 71, 132, 133, 135, 136, 184, 192, 193, 195, 196, 199, 200, 201, 203, 204, 208, 220, 230, 232, 233, 234, 235, 236, 237, 238, 243, 244, 245, 258, 260, 298, 301, 307, 309, 312, 328, 399, 402, 406, 424, 440, 442, 443

[Dixm–57] J. Dixmier, *Sur les représentations unitaires des groupes de Lie algébriques* (French), Ann. Inst. Fourier (Grenoble) **7** (1957), 315–328. MR99380. 208

[Dixm–59] J. Dixmier, *Sur les représentations unitaires des groupes de Lie nilpotents. V* (French), Bull. Soc. Math. France **87** (1959), 65–79. MR115097. 204, 208

[Dixm–60a] J. Dixmier, *Sur les C^*-algèbres* (French), Bull. Soc. Math. France **88** (1960), 95–112. MR121674. 228, 240, 254, 259

[Dixm–60b] J. Dixmier, *Opérateurs de rang fini dans les représentations unitaires* (French), Inst. Hautes Études Sci. Publ. Math. **6** (1960), 13–25. MR136684. 261

[Dixm–60c] J. Dixmier, *Sur les structures boréliennes du spectre d'une C^*-algèbre* (French), Inst. Hautes Études Sci. Publ. Math. **6** (1960), 5–11. MR124755. 259

[Dixm–61] J. Dixmier, *Sur le revêtement universel d'un groupe de Lie de type I* (French), C. R. Acad. Sci. Paris **252** (1961), 2805–2806. MR133407. 210

[Dixm–63] J. Dixmier, *Traces sur les C^*-algèbres* (French), Ann. Inst. Fourier (Grenoble) **13** (1963), no. fasc., fasc. 1, 219–262. MR149317. 217

[Dixm–64a] J. Dixmier, *Traces sur les C^*-algèbres. II* (French), Bull. Sci. Math. (2) **88** (1964), 39–57. MR181906. 314

[Dixm–64b] J. Dixmier, *Utilisation des facteurs hyperfinis dans la théorie des C^*-algèbres* (French), C. R. Acad. Sci. Paris **258** (1964), 4184–4187. MR161174. 58

[Dixm–66] J. Dixmier, *Existence de traces non normales* (French), C. R. Acad. Sci. Paris Sér. A-B **262** (1966), A1107–A1108. MR196508. 215, 218

[Dixm–69] J. Dixmier, *Sur la représentation régulière d'un groupe localement compact connexe* (French), Ann. Sci. École Norm. Sup. (4) **2** (1969), 423–436. MR260933. 227

[Dudk–11] A. Dudko, *Characters on the full group of an ergodic hyperfinite equivalence relation*, J. Funct. Anal. **261** (2011), no. 6, 1401–1414. MR2813475. 327

[DuMe–14] A. Dudko and K. Medynets, *Finite factor representations of Higman-Thompson groups*, Groups Geom. Dyn. **8** (2014), no. 2, 375–389. MR3231220. 320, 326, 419, 420

[DuGr–17a] A. Dudko and R. Grigorchuk, *On irreducibility and disjointness of Koopman and quasi-regular representations of weakly branch groups*, Modern theory of dynamical systems, Contemp. Math., vol. 692, Amer. Math. Soc., Providence, RI, 2017, pp. 51–66. MR3666066. 290

[DuGr–17b] A. Dudko and R. Grigorchuk, *On spectra of Koopman, groupoid and quasi-regular representations*, J. Mod. Dyn. **11** (2017), 99–123. MR3627119. 290

[DuGr–18] A. Dudko and R. Grigorchuk, *On diagonal actions of branch groups and the corresponding characters*, J. Funct. Anal. **274** (2018), no. 11, 3033–3055. MR3782986. 419

[Dye–59] H. A. Dye, *On groups of measure preserving transformations. I*, Amer. J. Math. **81** (1959), 119–159. MR131516. 165

[EcKl–12] S. Echterhoff and H. Klüver, *A general Kirillov theory for locally compact nilpotent groups*, J. Lie Theory **22** (2012), no. 3, 601–645. MR3012147. 268

[Effr–63] E. G. Effros, *A decomposition theory for representations of C^*-algebras*, Trans. Amer. Math. Soc. **107** (1963), 83–106. MR146682. 9, 242

[Effr–65] E. G. Effros, *Transformation groups and C^*-algebras*, Ann. of Math. (2) **81** (1965), 38–55. MR174987. 259

[Elli–77] G. A. Elliott, *The Mackey Borel structure on the spectrum of an approximately finite-dimensional separable C^*-algebra*, Trans. Amer. Math. Soc. **233** (1977), 59–68. MR454648. 294, 295

[Enge–89] R. Engelking, *General topology*, 2nd ed., Sigma Series in Pure Mathematics, vol. 6, Heldermann Verlag, Berlin, 1989. Translated from the Polish by the author. MR1039321. 423

[EnSc–92] M. Enock and J.-M. Schwartz, *Kac algebras and duality of locally compact groups*, Springer-Verlag, Berlin, 1992. With a preface by Alain Connes; With a postface by Adrian Ocneanu. MR1215933. 10

[Erne–64] J. Ernest, *A new group algebra for locally compact groups*, Amer. J. Math. **86** (1964), 467–492. MR167566. 260

[Erne–65] J. Ernest, *A new group algebra for locally compact groups. II*, Canadian J. Math. **17** (1965), 604–615. MR182676. 260

[Eyma–64] P. Eymard, *L'algèbre de Fourier d'un groupe localement compact* (French), Bull. Soc. Math. France **92** (1964), 181–236. MR228628. 402

[Eyma–72] P. Eymard, *Moyennes invariantes et représentations unitaires* (French), Lecture Notes in Mathematics, Vol. 300, Springer-Verlag, Berlin-New York, 1972. MR0447969. 51

[FeMo–77] J. Feldman and C. C. Moore, *Ergodic equivalence relations, cohomology, and von Neumann algebras. I*, Trans. Amer. Math. Soc. **234** (1977), no. 2, 289–324. MR578656. 292, 365

[Fell–60a] J. M. G. Fell, *The dual spaces of C^*-algebras*, Trans. Amer. Math. Soc. **94** (1960), 365–403. MR146681. 8, 30, 121

[Fell–60b] J. M. G. Fell, *C^*-algebras with smooth dual*, Illinois J. Math. **4** (1960), 221–230. MR124754. 259

[Fell–61] J. M. G. Fell, *The structure of algebras of operator fields*, Acta Math. **106** (1961), 233–280. MR164248. 9, 121, 246

[Fell–62] J. M. G. Fell, *Weak containment and induced representations of groups*, Canadian J. Math. **14** (1962), 237–268. MR150241. 110, 113

[Fell–62b] J. M. G. Fell, *A Hausdorff topology for the closed subsets of a locally compact non-Hausdorff space*, Proc. Amer. Math. Soc. **13** (1962), 472–476. MR139135. 416

[Fell–64] J. M. G. Fell, *Weak containment and induced representations of groups. II*, Trans. Amer. Math. Soc. **110** (1964), 424–447. MR159898. 251

[Foll–16] G. B. Folland, *A course in abstract harmonic analysis*, 2nd ed., Textbooks in Mathematics, CRC Press, Boca Raton, FL, 2016. MR3444405. 21, 35, 40, 42, 46, 54, 56, 92, 106, 110, 113, 121, 128, 152, 165, 203, 243, 438, 443

[FoSW] B. E. Forrest, N. Spronk, and M. Wiersma, *Existence of tracial states on reduced group C*-algebras*, `arXiv:1706.05354v2`, 19 Jul 2017. 215

[Freu–36] H. Freudenthal, *Topologische Gruppen mit genügend vielen fastperiodischen Funktionen* (German), Ann. of Math. (2) **37** (1936), no. 1, 57–77. MR1503269. 136

[Frob–Col] F. G. Frobenius, *Gesammelte Abhandlungen. Bände I, II, III* (German), Herausgegeben von J.-P. Serre, Springer-Verlag, Berlin-New York, 1968. MR0235974. 7, 9

[Fuc73] L. Fuchs, *Infinite abelian groups. Vol. II*, Pure and Applied Mathematics. Vol. 36-II, Academic Press, New York-London, 1973. MR0349869. 106

[Furs–61] H. Furstenberg, *Strict ergodicity and transformation of the torus*, Amer. J. Math. **83** (1961), 573–601. MR133429. 154, 165

[Gela–18] T. Gelander, *A lecture on invariant random subgroups*, New directions in locally compact groups, London Math. Soc. Lecture Note Ser., vol. 447, Cambridge Univ. Press, Cambridge, 2018, pp. 186–204. MR3793288. 416

[Gelf–41] I. M. Gel'fand, *Normierte Ringe*, Mat. Sbornik **51** (1941), 3–24 [**Gelf–Col**, Volume I, 181–201]. 8

[Gelf–50] I. M. Gel'fand, *Spherical functions on symmetric Riemannian spaces*, Doklady Akad. Nauk SSSR (N.S.) **70** (1950), 5–8 [**Gelf–Col**, Volume II, 31–35]. 187

[Gelf–Col] I. M. Gel'fand, *Collected papers. Volumes I, II, III*, Springer, 1987, 1988, 1989. 451, 452

[GeRa–43] I. M. Gel'fand and D. Raikov, *Irreducible unitary representations of locally bicompact groups*, Rec. Math. [Mat. Sbornik] N.S. **13(55)** (1943), 301–316 [**Gelf–Col**, Volume II, 3–17]. 8, 27

[GeNa–43] I. M. Gel'fand and M. A. Naimark, *On the imbedding of normed rings into the ring of operators in Hilbert space*, Mat. Sbornik **54** (1943), 197–213 [**Gelf–Col**, Volume I, 241–257]. 8

[GeNa–47a] I. M. Gel'fand and M. A. Naimark, *Unitary representations of the group of linear transformations of the straight line*, Dokl. Akad. Nauk SSSR (N.S.) **55** (1947), 567–570 [**Gelf–Col**, Volume II, 18–21]. 8

[GeNa–47b] I. M. Gel'fand and M. A. Naimark, *Unitary representations of the Lorentz group*, Izvestiya Akad. Nauk SSSR. Ser. Mat., **11** (1947), 411–504. [**Gelf–Col**, Volume II, 41–123]. 8, 9, 121

[GeNa–48] I. M. Gel′fand and M. A. Naĭmark, *Normed rings with involutions and their representations* (Russian), Izvestiya Akad. Nauk SSSR. Ser. Mat. **12** (1948), 445–480. MR0026763. 21

[GeNa–57] I. M. Gelfand and M. A. Neumark, *Unitäre Darstellungen der klassischen Gruppen* (German), Akademie-Verlag, Berlin, 1957. MR0085262. 9

[GGPS–69] I. M. Gel'fand, M. I. Graev, and I. I. Pyatetskii-Shapiro, *Representation theory and automorphic functions*, translated from the Russian by K. A. Hirsch. W. B. Saunders Company, 1969 [Russian original 1966]. 52, 113, 208

[Glas–98] E. Glasner, *On minimal actions of Polish groups*, 8th Prague Topological Symposium on General Topology and Its Relations to Modern Analysis and Algebra (1996), Topology Appl. **85** (1998), no. 1-3, 119–125. MR1617456. 27

[Glim–61a] J. Glimm, *Type I C^*-algebras*, Ann. of Math. (2) **73** (1961), 572–612. MR124756. 8, 258, 259, 291

[Glim–61b] J. Glimm, *Locally compact transformation groups*, Trans. Amer. Math. Soc. **101** (1961), 124–138. MR136681. 291

[Gode–44] R. Godement, *Sur une généralisation d'un théorème de Stone* (French), C. R. Acad. Sci. Paris **218** (1944), 901–903. MR14589. 82

[Gode–48] R. Godement, *Les fonctions de type positif et la théorie des groupes* (French), Trans. Amer. Math. Soc. **63** (1948), 1–84. MR23243. 30

[Gode–51a] R. Godement, *Sur la théorie des représentations unitaires* (French), Ann. of Math. (2) **53** (1951), 68–124. MR38571. 53, 59, 211

[Gode–51b] R. Godement, *Mémoire sur la théorie des caractères dans les groupes localement compacts unimodulaires* (French), J. Math. Pures Appl. (9) **30** (1951), 1–110. MR41857. 9, 227, 399

[Gode–52] R. Godement, *A theory of spherical functions. I*, Trans. Amer. Math. Soc. **73** (1952), 496–556. MR52444. 204, 208

[Gode–54] R. Godement, *Théorie des caractères. II. Définition et propriétés générales des caractères* (French), Ann. of Math. (2) **59** (1954), 63–85. MR58880. 9, 310, 312, 399

[GoSV–16] M. Göll, K. Schmidt, and E. Verbitskiy, *A Wiener lemma for the discrete Heisenberg group*, Monatsh. Math. **180** (2016), no. 3, 485–525. MR3513217. 165

[GoRo–79] E. C. Gootman and J. Rosenberg, *The structure of crossed product C^*-algebras: a proof of the generalized Effros-Hahn conjecture*, Invent. Math. **52** (1979), no. 3, 283–298. MR537063. 263

[Gorb–80] N. V. Gorbachëv, *C^*-algebra of the Heisenberg group* (Russian), Uspekhi Mat. Nauk **35** (1980), no. 6(216), 157–158. MR601762. 246

[GoKV–14] V. Gorin, S. Kerov, and A. Vershik, *Finite traces and representations of the group of infinite matrices over a finite field*, Adv. Math. **254** (2014), 331–395. MR3161102. 326

[Grig–11] R. I. Grigorchuk, *Some topics of the dynamics of group actions on rooted trees* (Russian, with Russian summary), Tr. Mat. Inst. Steklova **273** (2011), no. Sovremennye Problemy Matematiki, 72–191; English transl., Proc. Steklov Inst. Math. **273** (2011), no. 1, 64–175. MR2893544. 290, 432

[GrKO–15] R. Grigorchuk, R. Kravchenko, and A. Ol'shanskii, *Constructions of torsion-free countable, amenable, weakly mixing groups*, Enseign. Math. **61** (2015), no. 3-4, 321–342. MR3539841. 145

[GrMo–71a] S. Grosser and M. Moskowitz, *Compactness conditions in topological groups*, J. reine angew. Math. **246** (1971), 1–40. MR284541. 136

[GrMo–71b] S. Grosser and M. Moskowitz, *Harmonic analysis on central topological groups*, Trans. Amer. Math. Soc. **156** (1971), 419–454. MR276418. 38

[Guic–63] A. Guichardet, *Caractères des algèbres de Banach involutives* (French), Ann. Inst. Fourier (Grenoble) **13** (1963), 1–81. MR147925. 9, 263, 312, 314, 331, 373, 382, 383

[Guic–65] A. Guichardet, *Utilisation des sous-groupes distingués ouverts dans l'étude des représentations unitaires des groupes localement compacts* (French), Compositio Math. **17** (1965), 1–35 (1965). MR188348. 263, 273

[Haa–33] A. Haar, *Der Massbegriff in der Theorie der kontinuierlichen Gruppen* (German), Ann. of Math. (2) **34** (1933), no. 1, 147–169. MR1503103. 7

[HaKL–90] L. A. Harrington, A. S. Kechris, and A. Louveau, *A Glimm-Effros dichotomy for Borel equivalence relations*, J. Amer. Math. Soc. **3** (1990), no. 4, 903–928. MR1057041. 291

[Halm–51] P. R. Halmos, *Introduction to Hilbert Space and the theory of Spectral Multiplicity*, Chelsea Publishing Company, New York, N. Y., 1951. MR0045309. 71, 80, 441

[Halm–60] P. R. Halmos, *Lectures on ergodic theory*, Chelsea Publishing Co., New York, 1960. MR0111817. 412

[Halp–75] H. Halpern, *Quasi-equivalence classes of normal representations for a separable C^*-algebra*, Trans. Amer. Math. Soc. **203** (1975), 129–140. MR367669. 9, 314

[Hari–47] Harish–Chandra, *Infinite irreducible representations of the Lorentz group*, Proc. Roy. Soc. London, Ser. A **189** (1947), 372–401 [**Hari–Col**, Volume I, 162–191]. 121

[Hari–53] Harish–Chandra, *Representations of a semisimple Lie group on a Banach space. I*, Trans. Amer. Math. Soc. **75** (1953), 185–243 [**Hari–Col**, Volume I, 391–449]. 204

[Hari–54] Harish–Chandra, *Representations of a semisimple Lie group. III*, Trans. Amer. Math. Soc. **76** (1954), 234–253 [**Hari–Col**, Volume I, 490–509]. 6, 317

[Hari–56] Harish–Chandra, *The characters of semisimple Lie groups*, Trans. Amer. Math. Soc. **83** (1956), 98–163 [**Hari–Col**, Volume II, 156–221]. 9

[Hari–Col] Harish–Chandra, *Collected papers. Volumes I, II, III, IV*, Edited by V.S. Varadarajan. Springer, 1984. 8, 453

[Harp–07] P. de la Harpe, *On simplicity of reduced C^*-algebras of groups*, Bull. Lond. Math. Soc. **39** (2007), no. 1, 1–26. MR2303514. 289

[Haue–86] W. Hauenschild, *Zwei Dualräume der unendlichen, symmetrischen Gruppe* (German), Arch. Math. (Basel) **46** (1986), no. 3, 211–215. MR834838. 328

[Haus–14] F. Hausdorff, *Grundzüge der Mengenlehre*, Veit, 1914. 143

[Hawk–71] T. Hawkins, *The origins of the theory of group characters*, Arch. History Exact Sci. **7** (1971), no. 2, 142–170. MR1554141. 8

[Hawk–72] T. Hawkins, *Hypercomplex numbers, lie groups, and the creation of group representation theory*, Arch. History Exact Sci. **8** (1972), no. 4, 243–287. MR1554156. 8

[Hawk–74] T. Hawkins, *New light on Frobenius' creation of the theory of group characters*, Arch. History Exact Sci. **12** (1974), 217–243. MR0446837. 8

[HeRo–63] E. Hewitt and K. A. Ross, *Abstract harmonic analysis. Vol. I: Structure of topological groups. Integration theory, group representations*, Die Grundlehren der mathematischen Wissenschaften, Bd. 115, Academic Press, Inc., Publishers, New York; Springer-Verlag, Berlin-Göttingen-Heidelberg, 1963. MR0156915. 31, 404, 436

[HeRo–70] E. Hewitt and K. A. Ross, *Abstract harmonic analysis. Vol. II: Structure and analysis for compact groups. Analysis on locally compact Abelian groups*, Die Grundlehren der mathematischen Wissenschaften, Band 152, Springer-Verlag, New York-Berlin, 1970. MR0262773. 10, 86

[HiHi–05] T. Hirai and E. Hirai, *Positive definite class functions on a topological group and characters of factor representations*, J. Math. Kyoto Univ. **45** (2005), no. 2, 355–379. MR2161697. 322

[HoRa–19] C. Houdayer and S. Raum, *Locally compact groups acting on trees, the type I conjecture and non-amenable von Neumann algebras*, Comment. Math. Helv. **94** (2019), no. 1, 185–219. MR3941470. 209

[Howe–77a] R. E. Howe, *On representations of discrete, finitely generated, torsion-free, nilpotent groups*, Pacific J. Math. **73** (1977), no. 2, 281–305. MR499004. 271, 326, 329

[Howe–77b] R. E. Howe, *The Fourier transform for nilpotent locally compact groups. I*, Pacific J. Math. **73** (1977), no. 2, 307–327. MR492059. 99, 268

[HoMo–79] R. E. Howe and C. C. Moore, *Asymptotic properties of unitary representations*, J. Functional Analysis **32** (1979), no. 1, 72–96. MR533220. 415

[HoRo–89] R. E. Howe and J. Rosenberg, *The unitary representation theory of* GL(n) *of an infinite discrete field*, Israel J. Math. **67** (1989), no. 1, 67–81. MR1021362. 276

[Hump–80] J. E. Humphreys, *Arithmetic groups*, Lecture Notes in Mathematics, vol. 789, Springer, Berlin, 1980. MR584623. 142

[Isma–69] R. S. Ismagilov, *The irreducible cycles connected with a dynamic system* (Russian), Funkcional. Anal. i Priložen. **3** (1969), no. 3, 92–93. MR0249571. 154, 165

[Iwas–51] K. Iwasawa, *Topological groups with invariant compact neighborhoods of the identity*, Ann. of Math. (2) **54** (1951), 345–348. MR43106. 136, 403

[Jaco–45] N. Jacobson, *A topology for the set of primitive ideals in an arbitrary ring*, Proc. Nat. Acad. Sci. U.S.A. **31** (1945), 333–338. MR13138. 8

[KaSi–52] R. V. Kadison and I. M. Singer, *Some remarks on representations of connected groups*, Proc. Nat. Acad. Sci. U.S.A. **38** (1952), 419–423. MR48456. 227, 404

[Kaim–00] V. A. Kaimanovich, *Ergodic properties of the horocycle flow and classification of Fuchsian groups*, J. Dynam. Control Systems **6** (2000), no. 1, 21–56. MR1738739. 371

[Kaji–83] T. Kajiwara, *Group extension and Plancherel formulas*, J. Math. Soc. Japan **35** (1983), no. 1, 93–115. MR679078. 228

[KaKo–44] S. Kakutani and K. Kodaira, *Über das Haarsche Mass in der lokal bikompakten Gruppe* (German), Proc. Imp. Acad. Tokyo **20** (1944), 444–450. MR14401. 254

[Kall–70] R. R. Kallman, *Certain topological groups are type I*, Bull. Amer. Math. Soc. **76** (1970), 404–406. MR255725. 210

[Kall–73] R. R. Kallman, *Certain topological groups are type I. II*, Advances in Math. **10** (1973), 221–255. MR338256. 209

[Kani–69] E. Kaniuth, *Der Typ der regulären Darstellung diskreter Gruppen* (German), Math. Ann. **182** (1969), 334–339. MR260896. 224

[Kani–79] E. Kaniuth, *Primitive ideal spaces of groups with relatively compact conjugacy classes*, Arch. Math. (Basel) **32** (1979), no. 1, 16–24. MR532844. 40

[Kani–80] E. Kaniuth, *Ideals in group algebras of finitely generated FC-nilpotent discrete groups*, Math. Ann. **248** (1980), no. 2, 97–108. MR573341. 329

[Kani–82] E. Kaniuth, *On primary ideals in group algebras*, Monatsh. Math. **93** (1982), no. 4, 293–302. MR666831. 268

[KaTa–13] E. Kaniuth and K. F. Taylor, *Induced representations of locally compact groups*, Cambridge Tracts in Mathematics, vol. 197, Cambridge University Press, Cambridge, 2013. MR3012851. 38, 40, 92, 121, 152, 271

[KaWe–72] Y. Katznelson and B. Weiss, *The construction of quasi-invariant measures*, Israel J. Math. **12** (1972), 1–4. MR316679. 164

[Kapl–51a] I. Kaplansky, *Group algebras in the large*, Tohoku Math. J. (2) **3** (1951), 249–256. MR48712. 225

[Kapl–51b] I. Kaplansky, *The structure of certain operator algebras*, Trans. Amer. Math. Soc. **70** (1951), 219–255. MR42066. 9, 259

[Kazh–67] D. A. Kazhdan, *Connection of the dual space of a group with the structure of its closed subgroups* (Russian), Funkcional. Anal. i Priložen. **1** (1967), 71–74. MR0209390. 38

[Kean–71] M. Keane, *Sur les mesures quasi-ergodiques des translations irrationnelles* (French), C. R. Acad. Sci. Paris Sér. A-B **272** (1971), A54–A55. MR272982. 164

[Kech–95] A. S. Kechris, *Classical descriptive set theory*, Graduate Texts in Mathematics, vol. 156, Springer-Verlag, New York, 1995. MR1321597. 424, 425, 429

[KeRa–17] M. Kennedy and S. Raum, *Traces on reduced group* C**-algebras*, Bull. Lond. Math. Soc. **49** (2017), no. 6, 988–990. MR3743482. 215

[Khal–74] I. Khalil, *Sur l'analyse harmonique du groupe affine de la droite* (French), Studia Math. **51** (1974), 139–167. MR350330. 203

[Kirc–93] E. Kirchberg, *On nonsemisplit extensions, tensor products and exactness of group* C^**-algebras*, Invent. Math. **112** (1993), no. 3, 449–489. MR1218321. 406

[Kiri–62] A. A. Kirillov, *Unitary representations of nilpotent Lie groups* (Russian), Uspehi Mat. Nauk **17** (1962), no. 4 (106), 57–110. MR0142001. 8, 99, 110, 204, 317

[Kiri–65] A. A. Kirillov, *Positive-definite functions on a group of matrices with elements from a discrete field* (Russian), Dokl. Akad. Nauk SSSR **162** (1965), 503–505. MR0193183. 331, 346

[Kiri–67] A. A. Kirillov, *Dynamical systems, factors and group representations* (Russian), Uspehi Mat. Nauk **22** (1967), no. 5 (137), 67–80. MR0217256. 154

[Kiri–73] A. A. Kirillov, *Representation of the infinite-dimensional unitary group*, Dokl. Akad. Nauk SSSR **212** (1973), 288–290 [English translation: Soviet Math. Dokl. **14** (1973), no. 5, 1355–1358]. 122, 209

[Kiri–76] A. A. Kirillov, *Elements of the theory of representations*, Grundlehren der math. Wiss. **220**, Springer, 1976 [Russian original 1972]. 259

[Kiri–99] A. A. Kirillov, *Merits and demerits of the orbit method*, Bull. Amer. Math. Soc. (N.S.) **36** (1999), no. 4, 433–488. MR1701415. 110

[Klep–61] A. Kleppner, *On the intertwining number theorem*, Proc. Amer. Math. Soc. **12** (1961), 731–733. MR131783. 51

[KlLi–72] A. Kleppner and R. L. Lipsman, *The Plancherel formula for group extensions. I, II*, Ann. Sci. École Norm. Sup. (4) **5** (1972), 459–516; ibid. (4) 6 (1973), 103–132. MR342641. 113

[KlLi–73] A. Kleppner and R. L. Lipsman, *The Plancherel formula for group extensions. I, II*, Ann. Sci. École Norm. Sup. (4) **5** (1972), 459–516; ibid. (4) 6 (1973), 103–132. MR342641. 228

[Knap–02] A. W. Knapp, *Lie groups beyond an introduction*, 2nd ed., Progress in Mathematics, vol. 140, Birkhäuser Boston, Inc., Boston, MA, 2002. MR1920389. 189, 205

[KoKa–43] K. Kodaira and S. Kakutani, *Normed ring of a locally compact Abelian group*, Proc. Imp. Acad. Tokyo **19** (1943), 360–365. MR14094. 260

[Koop–31] B. O. Koopman, *Hamiltonian systems and transformations in Hilbert space*, Proc. Nat. Acad. Sci. U.S.A. **17** (1931), 315–318. 356

[Krie–71] W. Krieger, *On quasi-invariant measures in uniquely ergodic systems*, Invent. Math. **14** (1971), 184–196. MR293060. 164

[Krie–76] W. Krieger, *On Borel automorphisms and their quasi-invariant measures*, Math. Z. **151** (1976), no. 1, 19–24. MR417389. 377

[Kura–66] K. Kuratowski, *Topology. Vol. I*, New edition, revised and augmented, Academic Press, 1966. [First edition in French, 1933.] 425

[Lang–85] S. Lang, $\mathrm{SL}_2(\mathbf{R})$, Graduate Texts in Mathematics, vol. 105, Springer-Verlag, New York, 1985. Reprint of the 1975 edition. MR803508. 205

[Lang–93] S. Lang, *Real and functional analysis*, 3rd ed., Graduate Texts in Mathematics, vol. 142, Springer-Verlag, New York, 1993. MR1216137. 425

[LeLu–94] H. Leptin and J. Ludwig, *Unitary representation theory of exponential Lie groups*, De Gruyter Expositions in Mathematics, vol. 18, Walter de Gruyter & Co., Berlin, 1994. MR1307383. 110

[LeRo–68] H. Leptin and L. Robertson, *Every local compact MAP group is unimodular*, Proc. Amer. Math. Soc. **19** (1968), 1079–1082. MR230839. 136

[Lieb–72] A. Lieberman, *The structure of certain unitary representations of infinite symmetric groups*, Trans. Amer. Math. Soc. **164** (1972), 189–198. MR286940. 209

[Lips–70] R. L. Lipsman, *The dual topology for the principal and discrete series on semisimple groups*, Trans. Amer. Math. Soc. **152** (1970), 399–417. MR269778. 121

[Lips–72] R. L. Lipsman, *Representation theory of almost connected groups*, Pacific J. Math. **42** (1972), 453–467. MR327975. 208

[LiMo–74] J. Liukkonen and R. Mosak, *The primitive dual space of* $[FC]^-$ *groups*, J. Functional Analysis **15** (1974), 279–296. MR0346088. 40

[LuTu–11] J. Ludwig and L. Turowska, *The C^*-algebras of the Heisenberg group and of thread-like Lie groups*, Math. Z. **268** (2011), no. 3-4, 897–930. MR2818736. 246

[LuSh–04] A. Lubotzky and Y. Shalom, *Finite representations in the unitary dual and Ramanujan groups*, Discrete geometric analysis, Contemp. Math., vol. 347, Amer. Math. Soc., Providence, RI, 2004, pp. 173–189. MR2077037. 412, 414

[Mack–51] G. W. Mackey, *On induced representations of groups*, Amer. J. Math. **73** (1951), 576–592. MR42420. 48, 49, 51, 58

[Mack–52] G. W. Mackey, *Induced representations of locally compact groups. I*, Ann. of Math. (2) **55** (1952), 101–139. MR44536. 8, 40, 55, 113

[Mack–53] G. W. Mackey, *Induced representations of locally compact groups. II. The Frobenius reciprocity theorem*, Ann. of Math. (2) **58** (1953), 193–221. MR56611. 211

[Mack–57] G. W. Mackey, *Borel structure in groups and their duals*, Trans. Amer. Math. Soc. **85** (1957), 134–165. MR89999. 2, 258, 424, 425

[Mack–58] G. W. Mackey, *Unitary representations of group extensions. I*, Acta Math. **99** (1958), 265–311. MR98328. 8, 40, 42, 91, 92, 152

[Mack–61] G. W. Mackey, *Induced representations and normal subgroups*, Proc. Internat. Sympos. Linear Spaces (Jerusalem, 1960), Jerusalem Academic Press, Jerusalem; Pergamon, Oxford, 1961, pp. 319–326. MR0139687. 228

[Mack–63] G. W. Mackey, *Infinite-dimensional group representations*, Bull. Amer. Math. Soc. **69** (1963), 628–686. MR153784. 203

[Mack–76] G. W. Mackey, *The theory of unitary group representations*, Chicago Lectures in Mathematics, University of Chicago Press, Chicago, Ill.-London, 1976. Based on notes by James M. G. Fell and David B. Lowdenslager of lectures given at the University of Chicago, Chicago, Ill., 1955. MR0396826. 8, 27, 132, 169, 172, 183, 209, 220, 428

[Mack–78] G. W. Mackey, *Unitary group representations in physics, probability, and number theory*, Mathematics Lecture Note Series, vol. 55, Benjamin/Cummings Publishing Co., Inc., Reading, Mass., 1978. MR515581. 169

[Mal'c–40] A. I. Mal'cev, *On the faithful representation of infinite groups by matrices*, Amer. Math. Soc. Transl. (2) **45** (1965) 1–18 [Russian original: Mat. SS.(N.S.) 8(50) (1940), pp. 405–422]. 136

[Mare–75] O. Maréchal, *Une remarque sur un théorème de Glimm*, Bull. Sci. Math. (2) **99** (1975), no. 1, 41–44. MR430797. 259

[Maut–50a] F. I. Mautner, *Unitary representations of locally compact groups. I*, Ann. of Math. (2) **51** (1950), 1–25. MR32650. 53, 211

[Maut–50b] F. I. Mautner, *Unitary representations of locally compact groups. II*, Ann. of Math. (2) **52** (1950), 528–556. MR36763. 211, 227, 228

[Maut–50c] F. I. Mautner, *The structure of the regular representation of certain discrete groups*, Duke Math. J. **17** (1950), 437–441. MR39727. 221, 223, 224

[Megr–01] M. G. Megrelishvili, *Every semitopological semigroup compactification of the group $H_+[0,1]$ is trivial*, Semigroup Forum **63** (2001), no. 3, 357–370. MR1851816. 27

[Menn–65] J. L. Mennicke, *Finite factor groups of the unimodular group*, Ann. of Math. (2) **81** (1965), 31–37. MR171856. 143

[Mili–71] D. Miličić, *Topological representation of the group C^*-algebra of* SL(2, R) (English, with Serbo-Croatian summary), Glasnik Mat. Ser. III **6(26)** (1971), 231–246. MR308795. 121, 246

[Miln–71] P. Milnes, *Identities of group algebras*, Proc. Amer. Math. Soc. **29** (1971), 421–422. MR274661. 245

[Moor–65] C. C. Moore, *Decomposition of unitary representations defined by discrete subgroups of nilpotent groups*, Ann. of Math. (2) **82** (1965), 146–182. MR181701. 208

[Moor–72] C. C. Moore, *Groups with finite dimensional irreducible representations*, Trans. Amer. Math. Soc. **166** (1972), 401–410. MR302817. 132, 210, 254

[MoRo–76] C. C. Moore and J. Rosenberg, *Groups with T_1 primitive ideal spaces*, J. Functional Analysis **22** (1976), no. 3, 204–224. MR0419675. 39, 40

[Mott–12] L. Motto Ros, *On the complexity of the relations of isomorphism and bi-embeddability*, Proc. Amer. Math. Soc. **140** (2012), no. 1, 309–323. MR2833542. 291

[MuvN–36] F. J. Murray and J. von Neumann, *On rings of operators*, Ann. of Math. (2) **37** (1936), 116–229 [**vNeu–Col**, Volume III, 6–119]. 353, 362

[MuvN–43] F. J. Murray and J. von Neumann, *On rings of operators, IV*, Ann. of Math. (2) **44** (1943), 716–808 [**vNeu–Col**, Volume III, 229–321]. 211

[Naim–43] M. Naimark [also written Neumark], *Positive definite operator functions on a commutative group*, Bull. Acad. Sci. URSS Sér. Math. [Izvestia Akad. Nauk SSSR] **7** (1943), 237–244. 82

[Neeb–14] K.-H. Neeb, *Unitary representations of unitary groups*, Developments and retrospectives in Lie theory, Dev. Math., vol. 37, Springer, Cham, 2014, pp. 197–243. MR3329940. 209

[vNeu–31] J. von Neumann, *Die Eindeutigkeit der Schrödingerschen Operatoren*, Math. Ann. **104** (1931), no. 1, 570–578 [**vNeu–Col**, Volume II, 220–229]. 8, 109

[vNeu–32] J. von Neumann, *Einige Sätze über messbare Abbildungen*, Ann. of Math. (2) **33** (1032), 574–586 [**vNeu–Col**, Volume II, 294-306]. 429

[vNeu–34] J. von Neumann, *Almost periodic functions in a group. I*, Trans. Amer. Math. Soc. **36** (1934), 445–492 [**vNeu–Col**, Volume II, 454–501]. 133, 144

[vNeu–40] J. von Neumann, *On rings of operators. III*, Ann. of Math. (2) **41** (1940), 94–161 [**vNeu–Col**, Volume III, 161–228]. 362

[vNeu–49] J. von Neumann, *On rings of operators. Reduction theory*, Ann. of Math. (2) **50** (1949), 401–485, the article was written in 1937–38 [**vNeu–Col**, Volume III, 400–484]. 53, 59, 428

[vNWi–40] J. von Neumann and E. P. Wigner, *Minimally almost periodic groups*, Ann. of Math. (2) **41** (1940), 746–750 [**vNeu–Col**, Volume IV, 220–224]. 144

[vNeu–Col] J. von Neumann, *Collected works. Volumes I to VI*, General editor: A. H. Taub. Pergamon Press, 1961–1969. 456, 457, 459

[vNeu–Col] J. von Neumann, *Collected works. Vol. III: Rings of operators*, General editor: A. H. Taub, Pergamon Press, New York-Oxford-London-Paris, 1961. MR0157873. 8

[Obat–89] N. Obata, *Some remarks on induced representations of infinite discrete groups*, Math. Ann. **284** (1989), no. 1, 91–102. MR995384. 49

[O'Do–81] D. P. O'Donovan, *Irreducible representations of the irrational rotation covariance C^*-algebra*, Proc. Roy. Irish Acad. Sect. A **81** (1981), no. 2, 231–236. MR654822. 271

[Ol's–77] G. I. Ol'shanskii, *Classification of the irreducible representations of the automorphism groups of Bruhat–Tits trees*, Funct. Anal. Appl. **11** (1977), no. 1, 26–34. 209

[Ol's–78] G. I. Ol'shanskii, *Unitary representations of the infinite-dimensional classical groups $U(p,\infty), \mathrm{SO}(p,\infty), Sp(p,\infty)$ and of the corresponding motion groups*, Funct. Anal. Appl. **12** (1978), no. 1, 185–195. 122, 209

[Ol's–80] G. I. Ol'shanskii, *New "large" groups of type I*, Akad. Nauk. SSSR **16** (1980), 31–52. [English translation: J. Soviet Math. **18** (1992), 22–39.] 209

[OrWe–80] D. S. Ornstein and B. Weiss, *Ergodic theory of amenable group actions. I. The Rohlin lemma*, Bull. Amer. Math. Soc. (N.S.) **2** (1980), no. 1, 161–164. MR551753. 165

[OsAd–76] B. Osofsky and S. Adams, *Problems and Solutions: Advanced Solutions: 6102*, Amer. Math. Monthly **85** (1978), no. 6, 504–505. MR1538750. 143

[Ozaw–13] N. Ozawa, *About the Connes embedding conjecture: algebraic approaches*, Jpn. J. Math. **8** (2013), no. 1, 147–183. MR3067294. 407

[Palm–78] T. W. Palmer, *Classes of nonabelian, noncompact, locally compact groups*, Rocky J. Math. **8** (1978), no. 4, 683–741. Updated and expanded in Sections 12.5 and 12.6 of [**Palm–01**]. 38, 40, 136

[Palm–01] T. W. Palmer, *Banach algebras and the general theory of $*$-algebras. Vol. 2: $*$-algebras*, Encyclopedia of Mathematics and its Applications, vol. 79, Cambridge University Press, Cambridge, 2001. MR1819503. 457

[Pede–79] G. K. Pedersen, *C^*-algebras and their automorphism groups*, London Mathematical Society Monographs, vol. 14, Academic Press, Inc. [Harcourt Brace Jovanovich, Publishers], London-New York, 1979. MR548006. 217

[Pest–07] V. Pestov, *The isometry group of the Urysohn space as a Levy group*, Topology Appl. **154** (2007), no. 10, 2173–2184. MR2324929. 27

[PeWe–27] F. Peter and H. Weyl, *Die Vollständigkeit der primitiven Darstellungen einer geschlossenen kontinuierlichen Gruppe*, Math. Ann. **97** (1927), no. 1, 737–755 [**Weyl–Col**, Volume III, 58–75]. 7

[Pete–15] J. Peterson, *Character rigidity for lattices in higher-rank groups*, Preprint 2015. 326

[PeTh–16] J. Peterson and A. Thom, *Character rigidity for special linear groups*, J. reine angew. Math. **716** (2016), 207–228. MR3518376. 320, 351, 419, 420

[Pfef–95] C. Pfeffer Johnston, *Primitive ideal spaces of discrete rational nilpotent groups*, Amer. J. Math. **117** (1995), no. 2, 323–335. MR1323678. 329

[Pogu–82] D. Poguntke, *Discrete nilpotent groups have a T_1 primitive ideal space*, Studia Math. **71** (1981/82), no. 3, 271–275. MR667315. 39, 329

[Pogu–83] D. Poguntke, *Simple quotients of group C^*-algebras for two-step nilpotent groups and connected Lie groups*, Ann. Sci. École Norm. Sup. (4) **16** (1983), no. 1, 151–172. MR719767. 254, 328

[Pont–39] L. Pontrjagin, *Topological Groups*, Princeton Mathematical Series, v. 2, Princeton University Press, Princeton, 1939. Translated from the Russian by Emma Lehmer. MR0000265. 8

[Popa–07] S. Popa, *Cocycle and orbit equivalence superrigidity for malleable actions of w-rigid groups*, Invent. Math. **170** (2007), no. 2, 243–295. MR2342637. 398

[Powe–75] R. T. Powers, *Simplicity of the C^*-algebra associated with the free group on two generators*, Duke Math. J. **42** (1975), 151–156. MR374334. 313

[Puka–71] L. Pukanszky, *Unitary representations of solvable Lie groups*, Ann. Sci. École Norm. Sup. (4) **4** (1971), 457–608. MR439985. 227

[Puka–74] L. Pukanszky, *Characters of connected Lie groups*, Acta Math. **133** (1974), 81–137. MR409728. 9, 328, 329

[Puka–99] L. Pukánszky, *Characters of connected Lie groups*, Mathematical Surveys and Monographs, vol. 71, American Mathematical Society, Providence, RI, 1999. With a preface by J. Dixmier and M. Duflo. MR1707323. 328, 329

[Raeb–82] I. Raeburn, *On group C^*-algebras of bounded representation dimension*, Trans. Amer. Math. Soc. **272** (1982), no. 2, 629–644. MR662057. 38

[Rams–76] A. Ramsay, *Nontransitive quasi-orbits in Mackey's analysis of group extensions*, Acta Math. **137** (1976), no. 1, 17–48. MR460531. 147, 162

[RiZa–10] L. Ribes and P. Zalesskii, *Profinite groups*, 2nd ed., Ergebnisse der Mathematik und ihrer Grenzgebiete. 3. Folge. A Series of Modern Surveys in Mathematics [Results in Mathematics and Related Areas. 3rd Series. A Series of Modern Surveys in Mathematics], vol. 40, Springer-Verlag, Berlin, 2010. MR2599132. 139

[Rief–74] M. A. Rieffel, *Induced representations of C^*-algebras*, Advances in Math. **13** (1974), 176–257. MR353003. 251

[Rief–81] M. A. Rieffel, *C^*-algebras associated with irrational rotations*, Pacific J. Math. **93** (1981), no. 2, 415–429. MR623572. 271

[Robe–78] A. Robert, *Exemples de groupes de Fell* (French, with English summary), C. R. Acad. Sci. Paris Sér. A-B **287** (1978), no. 8, A603–A606. MR514536. 214

[Robe–83] A. Robert, *Introduction to the representation theory of compact and locally compact groups*, London Mathematical Society Lecture Note Series, vol. 80, Cambridge University Press, Cambridge-New York, 1983. MR690955. 175

[Robe–00] A. M. Robert, *A course in p-adic analysis*, Graduate Texts in Mathematics, vol. 198, Springer-Verlag, New York, 2000. MR1760253. 116

[Robe–69] L. C. Robertson, *A note on the structure of Moore groups*, Bull. Amer. Math. Soc. **75** (1969), 594–599. MR245721. 132

[Robi–96] D. J. S. Robinson, *A course in the theory of groups*, 2nd ed., Graduate Texts in Mathematics, vol. 80, Springer-Verlag, New York, 1996. MR1357169. 137, 141, 431

[Rose–76] J. Rosenberg, *The C^*-algebras of some real and p-adic solvable groups*, Pacific J. Math. **65** (1976), no. 1, 175–192. MR447467. 246

[Rose–04] J. Rosenberg, *A selective history of the Stone-von Neumann theorem*, Operator algebras, quantization, and noncommutative geometry, Contemp. Math., vol. 365, Amer. Math. Soc., Providence, RI, 2004, pp. 331–353. MR2106827. 8

[Rudi–62] W. Rudin, *Fourier analysis on groups*, Interscience Tracts in Pure and Applied Mathematics, No. 12, Interscience Publishers (a division of John Wiley and Sons), New York-London, 1962. MR0152834. 142

[Rudi–66] W. Rudin, *Real and complex analysis*, 3rd ed., McGraw-Hill Book Co., New York, 1987. MR924157. 367, 425, 426, 427, 429, 430, 431

[Rudi–73] W. Rudin, *Functional analysis*, McGraw-Hill Series in Higher Mathematics, McGraw-Hill Book Co., New York-Düsseldorf-Johannesburg, 1973. MR0365062. 32, 441, 442, 444

[Saka–66] S. Sakai, *On a characterization of type I C^*-algebras*, Bull. Amer. Math. Soc. **72** (1966), 508–512. MR192363. 259

[Saka–67] S. Sakai, *On type I C^*-algebras*, Proc. Amer. Math. Soc. **18** (1967), 861–863. MR213887. 259

[Schm–77a] K. Schmidt, *Cocycles on ergodic transformation groups*, Macmillan Lectures in Mathematics, Vol. 1, Macmillan Company of India, Ltd., Delhi, 1977. MR0578731. 168, 365, 366, 367, 368

[Schm–77b] K. Schmidt, *Infinite invariant measures on the circle*, Symposia Mathematica, Vol. XXI (Convegno sulle Misure su Gruppi e su Spazi Vettoriali, Convegno sui Gruppi e Anelli Ordinati, INDAM, Rome, 1975), Academic Press, London, 1977, pp. 37–43. MR0476996. 165, 166

[Schm–84] K. Schmidt, *Asymptotic properties of unitary representations and mixing*, Proc. London Math. Soc. (3) **48** (1984), no. 3, 445–460. MR735224. 144

[Schu–24] I. Schur, *Neue Anwendungen der Integralrechnung auf Probleme der Invariantentheorie, I-III*, Sitzungberichte der Akademie der Wiss. zu Berlin (1924), 189–208; Ges. Abh. **II**, pp. 440–459. 7

[Sega–47] I. E. Segal, *Irreducible representations of operator algebras*, Bull. Amer. Math. Soc. **53** (1947), 73–88. MR20217. 21

[Sega–50a] I. E. Segal, *An extension of Plancherel's formula to separable unimodular groups*, Ann. of Math. (2) **52** (1950), 272–292. MR36765. 227

[Sega–50b] I. E. Segal, *The two-sided regular representation of a unimodular locally compact group*, Ann. of Math. (2) **51** (1950), 293–298. MR36764. 227

[Sega–51] I. E. Segal, *Decompositions of operator algebras. I*, Mem. Amer. Math. Soc. **9** (1951), 67. MR44749. 211

[Sega–57] I. E. Segal, *The structure of a class of representations of the unitary group on a Hilbert space*, Proc. Amer. Math. Soc. **8** (1957), 197–203. MR84122. 122

[SevN–50] I. E. Segal and J. von Neumann, *A theorem on unitary representations of semisimple Lie groups*, Ann. of Math. (2) **52** (1950), 509–517 [**vNeu–Col**, Volume IV, 332–340]. 144, 404

[Serr–77] J.-P. Serre, *Arbres, amalgames,* SL_2 (French), Société Mathématique de France, Paris, 1977. Avec un sommaire anglais. Rédigé avec la collaboration de Hyman Bass. Astérisque, No. 46. MR0476875. 189

[Shod–33] K. Shoda, *Über die monomialen Darstellungen einer endlichen Gruppe*, Proc. Phys. Math. Soc. Japan (3) **15** (1933), 249–257. 51

[Shte–71] A. I. Shtern (= Štern), *The connection between the topologies of a locally bicompact group and its dual space*, Funkcional Anal. i Priložen. **5** (1971), no. 4, 56–63. 35, 36

[Sing–52] I. M. Singer, *Uniformly continuous representations of Lie groups*, Ann. of Math. (2) **56** (1952), 242–247. MR49201. 440

[Skud–76] H.-L. Skudlarek, *Die unzerlegbaren Charaktere einiger diskreter Gruppen* (German), Math. Ann. **223** (1976), no. 3, 213–231. MR463356. 326

[Smir–66] S. V. Smirnov, *Positive definite functions on algebraic nilpotent groups over a discrete field* (Russian), Dokl. Akad. Nauk SSSR **170** (1966), 524–525. MR0201573. 326

[Spat–87] R. J. Spatzier, *An example of an amenable action from geometry*, Ergodic Theory Dynam. Systems **7** (1987), no. 2, 289–293. MR896799. 371

[Stew–76] J. Stewart, *Positive definite functions and generalizations, an historical survey*, Rocky Mountain J. Math. **6** (1976), no. 3, 409–434. MR430674. 19

[Ston–30] M. H. Stone, *Linear transformations in Hilbert space. III. Operational methods and group theory*, Proc. Nat. Acad. Sci. U.S.A. **16** (1930), 172–175. 8, 82, 109

[StZi–94] G. Stuck and R. J. Zimmer, *Stabilizers for ergodic actions of higher rank semisimple groups*, Ann. of Math. (2) **139** (1994), no. 3, 723–747. MR1283875. 416, 417

[StVo–75] S. Stratila and D. Voiculescu, *Representations of AF-algebras and of the group* $U(\infty)$, Lecture Notes in Math. **486**, Springer, 1975. 245, 326

[Suna–86] T. Sunada, *Riemannian coverings and isospectral manifolds*, Ann. of Math. (2) **121** (1985), no. 1, 169–186. MR782558. 51

[Sund–87] V. S. Sunder, *An invitation to von Neumann algebras*, Universitext, Springer-Verlag, New York, 1987. MR866671. 367

[Sund–97] V. S. Sunder, *Functional analysis*: *Spectral theory*, Birkhäuser Advanced Texts: Basler Lehrbücher. [Birkhäuser Advanced Texts: Basel Textbooks], Birkhäuser Verlag, Basel, 1997. MR1646508. 71, 77, 80

[Suth–78] C. E. Sutherland, *Type analysis of the regular representation of a nonunimodular group*, Pacific J. Math. **79** (1978), no. 1, 225–250. MR526681. 387

[Suth–83] C. E. Sutherland, *On a construction of unitary cocycles and the representation theory of amenable groups*, Ergodic Theory Dynam. Systems **3** (1983), no. 1, 129–133. MR743031. 162, 293

[Take–57] O. Takenouchi, *Sur la facteur-représentation d'un groupe de Lie résoluble de type (E)* (French), Math. J. Okayama Univ. **7** (1957), 151–161. MR97464. 208

[Take–79] M. Takesaki, *Theory of operator algebras. I*, Springer-Verlag, New York-Heidelberg, 1979. MR548728. 353

[Tayl–76] K. F. Taylor, *The type structure of the regular representation of a locally compact group*, Math. Ann. **222** (1976), no. 3, 211–224. MR425007. 227

[Thom–64a] E. Thoma, *Über unitäre Darstellungen abzählbarer, diskreter Gruppen*, Math. Ann. **153** (1964), 111–138. 7, 9, 21, 221, 324
Translation by A. Törnquist: *On unitary representations of countable discrete groups*, October 2006, `www.math.toronto.edu/asger/thoma.pdf`

[Thom–64b] E. Thoma, *Über positiv-definite Klassenfunctionen abzählbarer Gruppen* (German), Math. Z. **84** (1964), 389–402. MR170217. 7, 9, 324, 331

[Thom–64c] E. Thoma, *Die unzerlegbaren, positiv-definiten Klassenfunktionen der abzählbar unendlichen, symmetrischen Gruppe* (German), Math. Z. **85** (1964), 40–61. MR173169. 9, 326

[Thom–68] E. Thoma, *Eine Charakterisierung diskreter Gruppen vom Typ I* (German), Invent. Math. **6** (1968), 190–196. MR248288. 221

[Thos–15] S. Thomas, *A descriptive view of unitary group representations*, J. Eur. Math. Soc. (JEMS) **17** (2015), no. 7, 1761–1787. MR3361728. 291, 295, 296

[Tsan–12] T. Tsankov, *Unitary representations of oligomorphic groups*, Geom. Funct. Anal. **22** (2012), no. 2, 528–555. MR2929072. 209

[Vale–84] A. Valette, *Notes on the structure and the K-theory of the C^*-algebras associated to* $\mathrm{SL}_2(\mathbf{R})$, Bull. Soc. Math. Belg. Sér. B **36** (1984), no. 1, 29–56. MR885552. 121, 246

[Vale–86] A. Valette, *Representations of minimally almost periodic groups*, J. Austral. Math. Soc. Ser. A **41** (1986), no. 3, 366–375. MR855403. 144

[Veec–69] W. A. Veech, *Strict ergodicity in zero dimensional dynamical systems and the Kronecker-Weyl theorem* mod 2, Trans. Amer. Math. Soc. **140** (1969), 1–33. MR240056. 165

[VeKe–81] A. M. Vershik and S. V. Kerov, *Asymptotic theory of the characters of a symmetric group* (Russian), Funktsional. Anal. i Prilozhen. **15** (1981), no. 4, 15–27, 96. MR639197. 9, 326

[Vers–11] A. M. Vershik, *Nonfree actions of countable groups and their characters* (Russian, with English and Russian summaries), Zap. Nauchn. Sem. S.-Peterburg. Otdel. Mat. Inst. Steklov. (POMI) **378** (2010), no. Teoriya Predstavleniĭ, Dinamicheskie Sistemy, Kombinatornye Metody. XVIII, 5–16, 228; English transl., J. Math. Sci. (N.Y.) **174** (2011), no. 1, 1–6. MR2749291. 9, 417, 418, 419

[Vers–12] A. M. Vershik, *Totally nonfree actions and the infinite symmetric group* (English, with English and Russian summaries), Mosc. Math. J. **12** (2012), no. 1, 193–212, 216. MR2952431. 417

[Vile–68] N. Ja. Vilenkin, *Special functions and the theory of group representations*, Translated from the Russian by V. N. Singh. Translations of Mathematical Monographs, Vol. 22, American Mathematical Society, Providence, R. I., 1968. MR0229863. 52

[Voic–76] D. Voiculescu, *Représentations factorielles de type II1 de $U(\infty)$* (French), J. Math. Pures Appl. (9) **55** (1976), no. 1, 1–20. MR442153. 326

[Wall–92] N. R. Wallach, *Real reductive groups. II*, Pure and Applied Mathematics, vol. 132, Academic Press, Inc., Boston, MA, 1992. MR1170566. 12

[Walt–82] P. Walters, *An introduction to ergodic theory*, Graduate Texts in Mathematics, vol. 79, Springer-Verlag, New York-Berlin, 1982. MR648108. 342, 393

[Wang–75] P. S. Wang, *On isolated points in the dual spaces of locally compact groups*, Math. Ann. **218** (1975), no. 1, 19–34. MR384993. 35

[Warn–72] G. Warner, *Harmonic analysis on semi-simple Lie groups. I*, Springer-Verlag, New York-Heidelberg, 1972. Die Grundlehren der mathematischen Wissenschaften, Band 188. MR0498999. 203

[Weav–03] N. Weaver, *A prime C^*-algebra that is not primitive*, J. Funct. Anal. **203** (2003), no. 2, 356–361. MR2003352. 240

[Weil–40] A. Weil, *L'intégration dans les groupes topologiques et ses applications* (French), Actual. Sci. Ind., no. 869, Hermann et Cie., Paris, 1940. [This book has been republished by the author at Princeton, N. J., 1941.] MR0005741. 8, 133, 136, 249, 437

[Weyl–24] H. Weyl, *Zur Theorie der Darstellung der einfachen kontinuierlichen Gruppen*, (Aus einem Schreiben an Herrn I. Schur.) Sitzungsber. Akad. Wiss. Berlin, (1924), pp. 338–345, letter dated 28. November 1924 [**Weyl–Col**, Volume II, 453–460]. 7

[Weyl–Col] H. Weyl, *Gesammelte Abhandlungen. Bände I, II, III, IV* (German), Herausgegeben von K. Chandrasekharan, Springer-Verlag, Berlin-New York, 1968. MR0230597. 7, 9, 457, 461

[Wign–39] E. Wigner, *On unitary representations of the inhomogeneous Lorentz group*, Ann. of Math. (2) **40** (1939), no. 1, 149–204. MR1503456. 8

[Will–07] D. P. Williams, *Crossed products of C^*-algebras*, Mathematical Surveys and Monographs, vol. 134, American Mathematical Society, Providence, RI, 2007. MR2288954. 245, 421

[Wils–98] J. S. Wilson, *Profinite groups*, London Mathematical Society Monographs. New Series, vol. 19, The Clarendon Press, Oxford University Press, New York, 1998. MR1691054. 137

[Wolf–07] J. A. Wolf, *Harmonic analysis on commutative spaces*, Mathematical Surveys and Monographs, vol. 142, American Mathematical Society, Providence, RI, 2007. MR2328043. 191

[Yosh–51] H. Yoshizawa, *Some remarks on unitary representations of the free group*, Osaka Math. J. **3** (1951), 55–63. MR41854. 58

[Zimm–84] R. J. Zimmer, *Ergodic theory and semisimple groups*, Monographs in Mathematics, vol. 81, Birkhäuser Verlag, Basel, 1984. MR776417. 27, 108, 150, 365, 371, 413, 417, 424

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Unitary representations of groups play an important role in many subjects, including number theory, geometry, probability theory, partial differential equations, and quantum mechanics. This monograph focuses on dual spaces associated to a group, which are spaces of building blocks of general unitary representations.

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