

Mathematical
Surveys
and
Monographs
Volume 258

Perverse Sheaves and Applications to Representation Theory

Pramod N. Achar



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2020 *Mathematics Subject Classification*. Primary 32S60, 14F08, 20G05, 14M15, 17B08, 17B37.

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Library of Congress Cataloging-in-Publication Data

Names: Achar, Pramod N., 1976- author.
Title: Perverse sheaves and applications to representation theory / Pramod N. Achar.
Description: Providence, Rhode Island : American Mathematical Society, [2021] | Series: Mathematical surveys and monographs, 0076-5376 ; volume 258 | Includes bibliographical references and index.
Identifiers: LCCN 2021015561 | ISBN 9781470455972 (paperback) | 9781470466688 (ebook)
Subjects: LCSH: Sheaf theory. | Representations of groups. | Linear algebraic groups. | AMS: Several complex variables and analytic spaces – Complex singularities – Stratifications; constructible sheaves; intersection cohomology (complex-analytic aspects). | Group theory and generalizations – Linear algebraic groups and related topics – Representation theory for linear algebraic groups. | Algebraic geometry – Special varieties – Grassmannians, Schubert varieties, flag manifolds. | Nonassociative rings and algebras – Lie algebras and Lie superalgebras – Coadjoint orbits; nilpotent varieties. | Nonassociative rings and algebras – Lie algebras and Lie superalgebras – Quantum groups (quantized enveloping algebras) and related deformations.
Classification: LCC QA612.36 .A243 2021 | DDC 514/.224-dc23
LC record available at <https://lcn.loc.gov/2021015561>

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Preface

Perverse sheaves were discovered in the fall of 1980 by Beilinson–Bernstein–Deligne–Gabber [24], sitting at the confluence of two major developments of the 1970s: the intersection homology theory of Goresky–MacPherson [85, 86], and the Riemann–Hilbert correspondence, due to Kashiwara [121] and Mebkhout [175]. Those same two ingredients had already been combined a few months prior for a breakthrough in representation theory: the proof of the Kazhdan–Lusztig conjecture on Lie algebra representations [26, 50, 129, 130]. From today’s perspective, the Kazhdan–Lusztig conjecture may be seen as the spectacular first application of perverse sheaves. Ever since, perverse sheaves have been a powerful tool of fundamental importance in geometric representation theory.

This is partly due to the diversity of perspectives from which one may approach this subject. Perverse sheaves have close connections (especially in their computational aspects) to topics in classical algebraic topology, including fundamental groups, covering spaces, and singular cohomology. On the other hand, perverse sheaves (at least with field coefficients) have algebraic features reminiscent of modules over an artinian ring: every perverse sheaf has a composition series, and one can classify the simple perverse sheaves.

But in my opinion, the most significant reason for the usefulness of perverse sheaves is the following secret known to experts: perverse sheaves are *easy*, in the sense that most arguments come down to a rather short list of tools, such as proper base change, smooth pullback, and open–closed distinguished triangles. In practice, one can reason and compute with perverse sheaves just using a list of these tools, much as calculus students might use a table of integrals. One does not have to dig into the details of flabby resolutions or sheafification any more than a calculus student needs to revisit Riemann sums to integrate a polynomial. In this book, I have tried to emphasize this perspective with computational exercises and with the **Quick Reference** pages near the end of the book.

Organization and prerequisites. This book is divided into two parts: the first six chapters develop the general theory of constructible sheaves on complex algebraic varieties, and the last four chapters give brief introductions to selected applications of perverse sheaves in representation theory. The prerequisites for the first six chapters are: familiarity with the language of derived and triangulated categories; familiarity with introductory algebraic topology; and (starting from Chapter 2) some minimal familiarity with complex algebraic varieties. For the applications in Chapters 7–10, some knowledge of Lie theory is required.

Chapter 1 covers the foundations of sheaf theory on topological spaces, including the definitions of the six basic sheaf operations, and a number of natural compatibilities between them, such as the proper base change theorem and the

projection formula. This chapter also contains material on local systems and fundamental groups. Much of this material can be found in many other textbook-level sources, so a number of proofs in this chapter are merely sketched, or sometimes omitted entirely.

In **Chapter 2** we begin the study of constructible sheaves on complex algebraic varieties. Some highlights of results proved in this chapter include Artin’s vanishing theorem, the Verdier duality theorem, and the “constructibility theorem” (which says that in the algebraic setting, all six sheaf operations preserve constructibility). We also show that in the setting of constructible sheaves, the external tensor product functor and the extension-of-scalars functor commute with all sheaf operations. The chapter ends with a selection of other topics related to constructible sheaves, including hyperbolic localization, Borel–Moore homology, and fundamental classes.

In **Chapter 3**, we begin the study of perverse sheaves, including the important special case of intersection cohomology complexes. Key results in this chapter describe the behavior of perverse sheaves with respect to push-forward along affine morphisms, and pullback along smooth morphisms. The former is very closely related to Artin’s vanishing theorem. In the context of the latter, we prove that perverse sheaves satisfy “smooth descent”—that is, a perverse sheaf can be recovered from its pullback along a smooth surjective morphism. The chapter concludes with a discussion of two of the deepest results about perverse sheaves with coefficients in \mathbb{Q} : the decomposition theorem and the hard Lefschetz theorem.

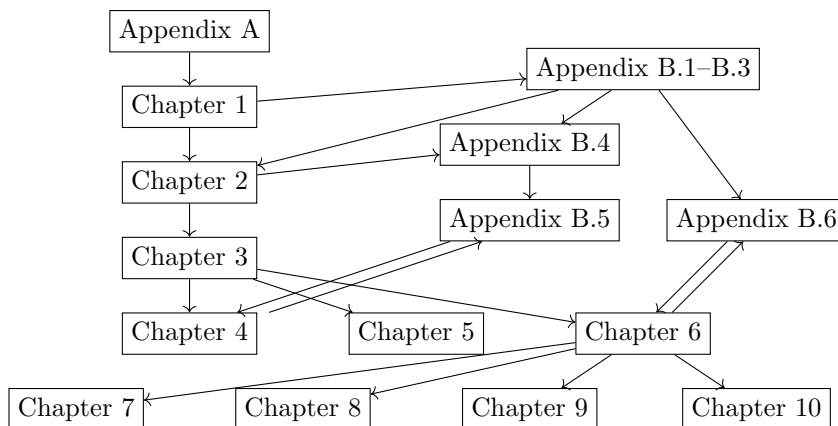
Chapter 4 discusses the nearby cycles functor. The definition of this functor requires leaving the algebraic setting (it involves the exponential map $\exp : \mathbb{C} \rightarrow \mathbb{C}^\times$), so most results from Chapter 2 cannot be applied directly. Nevertheless, we prove that this functor preserves constructibility; that it takes perverse sheaves to perverse sheaves; and that it is compatible with Verdier duality and the extension of scalars. As an application, we prove Beilinson’s theorem, which says that the derived category of perverse sheaves (with coefficients in a field) is equivalent to the constructible derived category.

Chapter 5 gives an overview of two separate (but conceptually related) topics: mixed ℓ -adic sheaves in the étale topology, and mixed Hodge modules. Both of these theories provide a kind of “enrichment” of perverse sheaves: the objects carry additional structure, most notably the weight filtration. This chapter includes discussions of some related side topics, including the sheaf–function correspondence and the Riemann–Hilbert correspondence. Most theorems in this chapter are stated without proof.

The final chapter in the first part of the book, **Chapter 6**, is devoted to the study of equivariant sheaves. It is straightforward to define the abelian category of equivariant sheaves (or equivariant perverse sheaves), but it is rather nontrivial to define the correct triangulated analogue. (The derived category of the abelian category of equivariant sheaves is usually the “wrong” answer.) We present a solution to this problem following Bernstein–Lunts. We also study compatibilities of sheaf functors with various ways of modifying the group action, such as forgetting, inflation, and averaging. Perhaps the two most useful results from this chapter are the quotient equivalence and the induction equivalence.

The remaining chapters deal with applications in representation theory.

Chapter 7 deals with the study of Borel-equivariant perverse sheaves on the flag variety of a reductive group. This chapter contains a proof that these perverse



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sheaves give a categorification of the Hecke algebra. This fact, which essentially goes back to a 1980 paper of Kazhdan–Lusztig, is one of the ingredients in the Kazhdan–Lusztig conjectures for Lie algebra representations. This chapter also discusses some more recent developments around sheaves on flag varieties, including parity sheaves and Soergel bimodules.

Chapter 8 studies perverse sheaves on the nilpotent cone of the Lie algebra of a reductive group. The starting point for this topic was Springer’s discovery in the late 1970s that the stalks of some of these perverse sheaves (with coefficients in \mathbb{Q}) carry a natural action of the Weyl group. By the mid-1980s, Lusztig had extended Springer’s work to cover all perverse sheaves (still with coefficients in \mathbb{Q}); this became the starting point for his theory of character sheaves. This chapter also discusses recent developments on Springer theory for perverse sheaves with coefficients in a field of positive characteristic.

In **Chapter 9**, we study perverse sheaves on the affine Grassmannian of a reductive group G . Results of Lusztig going back to 1983 indicated that these perverse sheaves contained a great deal of information about representations of the Langlands dual group \check{G} . In a landmark 2007 paper, Mirković and Vilonen, following an idea of Drinfeld, proved that this can be upgraded to an equivalence of tensor categories, known as the geometric Satake equivalence. We give proofs of the more sheaf-theoretic steps in this theorem, but we will not prove it in full.

Lastly, in **Chapter 10**, we use perverse sheaves on the space of representations of a quiver to construct the canonical basis for a quantum group. The fact that quantum groups are related to (functions on the space of) quiver representations is due to Ringel. The project of upgrading this by replacing functions by sheaves is due to Lusztig.

The book concludes with two appendices. **Appendix A** contains background (mostly without proofs) on category theory and homological algebra. One fact that is proved is the duality theorem for rings of finite global dimension. This result can be seen as a precursor to Verdier duality. **Appendix B** contains a number of calculations involving sheaves on \mathbb{C}^n . The results in this appendix are enlightening examples in their own right, but they are also needed for the proofs of a number of theorems in the main body of the book.

Acknowledgments. This book grew out of notes for a mini-course I gave at East China Normal University in July 2015 on the topic of “Perverse sheaves in representation theory.” This mini-course was part of a workshop organized by Bin Shu and Weiqiang Wang. I am grateful to them for the opportunity to participate, and especially to Weiqiang Wang for strongly encouraging me to expand my lecture notes into a book.

Over the years, I have had the opportunity to teach a number of graduate courses at Louisiana State University on topics related to this book, including homological algebra, sheaf theory, and Lie theory. In the 2017–2018 academic year, I co-taught a two-semester sequence with my colleague Daniel Sage on sheaves and geometric representation theory. I am grateful to him and to all the students in these courses for the feedback they have given me. These experiences have shaped the presentation of a number of topics in this book.

I learned this subject myself largely from my collaborators. In particular, I have learned a number of explicit examples from Anthony Henderson, Daniel Juteau, Carl Mautner, and Simon Riche. Some of these examples appear as exercises in this book.

I would like to thank Laura Rider for extensive comments on an earlier draft of this book. I am also grateful for suggestions and corrections from Tom Braden, Tamanna Chatterjee, Stefan Dawydiak, Zhijie Dong, Joseph Dorta, Joel Kamnitzer, Maitreyee Kulkarni, Chun-Ju Lai, Ivan Losev, George Lusztig, Jacob Matherne, Carl Mautner, Olaf Schnürer, Vishnu Sivaprasad, Wolfgang Soergel, Kurt Trampel, David Treumann, Kent Vashaw, and Noah Winslow.

This work was supported by the National Science Foundation under Grant Nos. DMS-1500890 and DMS-1802241.

Some notation and conventions. The 1-point topological space is denoted by pt . For any topological space X , the unique continuous map from X to pt is denoted by $a_X : X \rightarrow \text{pt}$.

All rings in this book are unital. Sheaves will almost always have coefficients in a commutative ring, usually denoted by \mathbb{k} . Starting from Chapter 2, the ring \mathbb{k} is almost always assumed to be noetherian and of finite global dimension. The category of \mathbb{k} -modules is denoted by $\mathbb{k}\text{-mod}$, and the category of finitely generated \mathbb{k} -modules by $\mathbb{k}\text{-mod}^{\text{fg}}$. However, if π is a group and $\mathbb{k}[\pi]$ is its group ring, the notation $\mathbb{k}[\pi]\text{-mod}^{\text{fg}}$ means the category of $\mathbb{k}[\pi]$ -modules that are finitely generated over \mathbb{k} (and not merely over $\mathbb{k}[\pi]$).

We write $H^i(A)$ for the the i th cohomology object of a chain complex A . We write $\mathbf{H}^i(X, \mathcal{F})$ for the i th sheaf (hyper)cohomology of a topological space X with coefficients in a sheaf (or chain complex of sheaves) \mathcal{F} .

Sheaf functors such as $f_!$ and f_* are always derived; a separate notation (${}^{\circ}f_!$, ${}^{\circ}f_*$) is used for their non-derived counterparts.

Pramod N. Achar
Baton Rouge
March 2021

Bibliography

- [1] *Revêtements étales et groupe fondamental (SGA 1)* (French), Documents Mathématiques (Paris) [Mathematical Documents (Paris)], vol. 3, Société Mathématique de France, Paris, 2003. Séminaire de géométrie algébrique du Bois Marie 1960–61. [Algebraic Geometry Seminar of Bois Marie 1960–61]; Directed by A. Grothendieck; With two papers by M. Raynaud; Updated and annotated reprint of the 1971 original [Lecture Notes in Math., 224, Springer, Berlin; MR0354651 (50 #7129)]. MR2017446
- [2] *Théorie des topos et cohomologie étale des schémas. Tome 1: Théorie des topos* (French), Lecture Notes in Mathematics, Vol. 269, Springer-Verlag, Berlin-New York, 1972. Séminaire de Géométrie Algébrique du Bois-Marie 1963–1964 (SGA 4); Dirigé par M. Artin, A. Grothendieck, et J. L. Verdier. Avec la collaboration de N. Bourbaki, P. Deligne et B. Saint-Donat. MR0354652
- [3] *Théorie des topos et cohomologie étale des schémas. Tome 3* (French), Lecture Notes in Mathematics, Vol. 305, Springer-Verlag, Berlin-New York, 1973. Séminaire de Géométrie Algébrique du Bois-Marie 1963–1964 (SGA 4); Dirigé par M. Artin, A. Grothendieck et J. L. Verdier. Avec la collaboration de P. Deligne et B. Saint-Donat. MR0354654
- [4] A. Grothendieck et al., *Cohomologie l -adique et fonctions L* , Séminaire de Géométrie Algébrique du Bois-Marie 1965–1966 (SGA 5), Lecture Notes in Mathematics, no. 589, Springer-Verlag, 1977.
- [5] *Groupes de monodromie en géométrie algébrique. II* (French), Lecture Notes in Mathematics, Vol. 340, Springer-Verlag, Berlin-New York, 1973. Séminaire de Géométrie Algébrique du Bois-Marie 1967–1969 (SGA 7 II); Dirigé par P. Deligne et N. Katz. MR0354657
- [6] Pramod N. Achar, Anthony Henderson, Daniel Juteau, and Simon Riche, *Weyl group actions on the Springer sheaf*, Proc. Lond. Math. Soc. (3) **108** (2014), no. 6, 1501–1528, DOI 10.1112/plms/pdt055. MR3218317
- [7] Pramod N. Achar, Anthony Henderson, Daniel Juteau, and Simon Riche, *Modular generalized Springer correspondence I: the general linear group*, J. Eur. Math. Soc. (JEMS) **18** (2016), no. 7, 1405–1436, DOI 10.4171/JEMS/618. MR3506603
- [8] Pramod N. Achar, Anthony Henderson, Daniel Juteau, and Simon Riche, *Constructible sheaves on nilpotent cones in rather good characteristic*, Selecta Math. (N.S.) **23** (2017), no. 1, 203–243, DOI 10.1007/s00029-016-0236-z. MR3595892
- [9] Pramod Achar, Anthony Henderson, Daniel Juteau, and Simon Riche, *Modular generalized Springer correspondence II: classical groups*, J. Eur. Math. Soc. (JEMS) **19** (2017), no. 4, 1013–1070, DOI 10.4171/JEMS/687. MR3626550
- [10] Pramod N. Achar, Anthony Henderson, Daniel Juteau, and Simon Riche, *Modular generalized Springer correspondence III: exceptional groups*, Math. Ann. **369** (2017), no. 1-2, 247–300, DOI 10.1007/s00208-017-1524-4. MR3694647
- [11] Pramod N. Achar, Anthony Henderson, and Simon Riche, *Geometric Satake, Springer correspondence, and small representations II*, Represent. Theory **19** (2015), 94–166, DOI 10.1090/ert/465. MR3347990
- [12] Pramod N. Achar, Shotaro Makisumi, Simon Riche, and Geordie Williamson, *Koszul duality for Kac-Moody groups and characters of tilting modules*, J. Amer. Math. Soc. **32** (2019), no. 1, 261–310, DOI 10.1090/jams/905. MR3868004
- [13] Pramod N. Achar and Carl Mautner, *Sheaves on nilpotent cones, Fourier transform, and a geometric Ringel duality* (English, with English and Russian summaries), Mosc. Math. J. **15** (2015), no. 3, 407–423, 604, DOI 10.17323/1609-4514-2015-15-3-407-423. MR3427432

- [14] Pramod N. Achar and Laura Rider, *Parity sheaves on the affine Grassmannian and the Mirković-Vilonen conjecture*, *Acta Math.* **215** (2015), no. 2, 183–216, DOI 10.1007/s11511-016-0132-6. MR3455233
- [15] Dean Alvis and George Lusztig, *On Springer’s correspondence for simple groups of type E_n ($n = 6, 7, 8$)*, *Math. Proc. Cambridge Philos. Soc.* **92** (1982), no. 1, 65–78, DOI 10.1017/S0305004100059703. With an appendix by N. Spaltenstein. MR662961
- [16] H. H. Andersen, J. C. Jantzen, and W. Soergel, *Representations of quantum groups at a p th root of unity and of semisimple groups in characteristic p : independence of p* (English, with English and French summaries), *Astérisque* **220** (1994), 321. MR1272539
- [17] Aldo Andreotti and Theodore Frankel, *The Lefschetz theorem on hyperplane sections*, *Ann. of Math. (2)* **69** (1959), 713–717, DOI 10.2307/1970034. MR177422
- [18] Donu Arapura, *Algebraic geometry over the complex numbers*, Universitext, Springer, New York, 2012, DOI 10.1007/978-1-4614-1809-2. MR2895485
- [19] M. F. Atiyah and I. G. Macdonald, *Introduction to commutative algebra*, Addison-Wesley Publishing Co., Reading, Mass.-London-Don Mills, Ont., 1969. MR0242802
- [20] G. Barthel, G. Göringer, J. Duskin, R. Fittler, H. Acuña, R. Gergondey, J.-L. Verdier, and M. Zisman, *Séminaire Heidelberg-Strasbourg, 1966/1967: Dualité de Poincaré*, *Publ. I.R.M.A.*, no. 3, Institut de Recherche Mathématique Avancée, Strasbourg, 1969.
- [21] Pierre Baumann and Simon Riche, *Notes on the geometric Satake equivalence*, *Relative aspects in representation theory, Langlands functoriality and automorphic forms*, *Lecture Notes in Math.*, vol. 2221, Springer, Cham, 2018, pp. 1–134. MR3839695
- [22] A. A. Beilinson, *How to glue perverse sheaves*, *K-theory, arithmetic and geometry (Moscow, 1984)*, *Lecture Notes in Math.*, vol. 1289, Springer, Berlin, 1987, pp. 42–51, DOI 10.1007/BFb0078366. MR923134
- [23] A. A. Beilinson, *On the derived category of perverse sheaves*, *K-theory, arithmetic and geometry (Moscow, 1984)*, *Lecture Notes in Math.*, vol. 1289, Springer, Berlin, 1987, pp. 27–41, DOI 10.1007/BFb0078365. MR923133
- [24] A. A. Beilinson, J. Bernstein, and P. Deligne, *Faisceaux pervers (French)*, *Analysis and topology on singular spaces, I (Luminy, 1981)*, *Astérisque*, vol. 100, Soc. Math. France, Paris, 1982, pp. 5–171. MR751966
- [25] A. Beilinson and V. Drinfel’d, *Quantization of Hitchin’s integrable system and Hecke eigen-sheaves*, preprint.
- [26] Alexandre Beilinson and Joseph Bernstein, *Localisation de g -modules (French, with English summary)*, *C. R. Acad. Sci. Paris Sér. I Math.* **292** (1981), no. 1, 15–18. MR610137
- [27] Joseph Bernstein and Valery Lunts, *Equivariant sheaves and functors*, *Lecture Notes in Mathematics*, vol. 1578, Springer-Verlag, Berlin, 1994, DOI 10.1007/BFb0073549. MR1299527
- [28] A. Białynicki-Birula, *Some theorems on actions of algebraic groups*, *Ann. of Math. (2)* **98** (1973), 480–497, DOI 10.2307/1970915. MR366940
- [29] Gebhard Böckle and Chandrashekar Khare, *Mod l representations of arithmetic fundamental groups. I. An analog of Serre’s conjecture for function fields*, *Duke Math. J.* **129** (2005), no. 2, 337–369, DOI 10.1215/S0012-7094-05-12925-8. MR2165545
- [30] Cédric Bonnafé, *Éléments unipotents réguliers des sous-groupes de Levi (French, with English and French summaries)*, *Canad. J. Math.* **56** (2004), no. 2, 246–276, DOI 10.4153/CJM-2004-012-0. MR2040915
- [31] A. Borel and J. C. Moore, *Homology theory for locally compact spaces*, *Michigan Math. J.* **7** (1960), 137–159. MR131271
- [32] A. Borel and N. Spaltenstein, *Sheaf theoretic intersection cohomology*, *Intersection cohomology (Bern, 1983)*, *Progr. Math.*, vol. 50, Birkhäuser Boston, Boston, MA, 1984, pp. 47–182, DOI 10.1007/978-0-8176-4765-0_5. MR788176
- [33] Armand Borel, *Sur la cohomologie des espaces fibrés principaux et des espaces homogènes de groupes de Lie compacts (French)*, *Ann. of Math. (2)* **57** (1953), 115–207, DOI 10.2307/1969728. MR51508
- [34] Armand Borel, *Seminar on transformation groups*, With contributions by G. Bredon, E. E. Floyd, D. Montgomery, R. Palais. *Annals of Mathematics Studies*, No. 46, Princeton University Press, Princeton, N.J., 1960. MR0116341

- [35] Armand Borel, *Sous-groupes commutatifs et torsion des groupes de Lie compacts connexes* (French), *Tohoku Math. J. (2)* **13** (1961), 216–240, DOI 10.2748/tmj/1178244298. MR147579
- [36] Armand Borel, *Linear algebraic groups*, 2nd ed., Graduate Texts in Mathematics, vol. 126, Springer-Verlag, New York, 1991, DOI 10.1007/978-1-4612-0941-6. MR1102012
- [37] A. Borel, P.-P. Grivel, B. Kaup, A. Haefliger, B. Malgrange, and F. Ehlers, *Algebraic D-modules*, Perspectives in Mathematics, vol. 2, Academic Press, Inc., Boston, MA, 1987. MR882000
- [38] A. Borel and et al., *Intersection cohomology*, Modern Birkhäuser Classics, Birkhäuser Boston, Inc., Boston, MA, 2008. Notes on the seminar held at the University of Bern, Bern, 1983; Reprint of the 1984 edition. MR2401086
- [39] Walter Borho and Robert MacPherson, *Représentations des groupes de Weyl et homologie d'intersection pour les variétés nilpotentes* (French, with English summary), *C. R. Acad. Sci. Paris Sér. I Math.* **292** (1981), no. 15, 707–710. MR618892
- [40] Walter Borho and Robert MacPherson, *Partial resolutions of nilpotent varieties*, Analysis and topology on singular spaces, II, III (Luminy, 1981), Astérisque, vol. 101, Soc. Math. France, Paris, 1983, pp. 23–74. MR737927
- [41] Raoul Bott and Loring W. Tu, *Differential forms in algebraic topology*, Graduate Texts in Mathematics, vol. 82, Springer-Verlag, New York-Berlin, 1982. MR658304
- [42] N. Bourbaki, *Éléments de mathématique. Topologie générale. Chapitres 1 à 4*, Hermann, Paris, 1971. MR0358652
- [43] Tom Braden, *Hyperbolic localization of intersection cohomology*, *Transform. Groups* **8** (2003), no. 3, 209–216, DOI 10.1007/s00031-003-0606-4. MR1996415
- [44] Glen E. Bredon, *Sheaf theory*, 2nd ed., Graduate Texts in Mathematics, vol. 170, Springer-Verlag, New York, 1997, DOI 10.1007/978-1-4612-0647-7. MR1481706
- [45] Michel Brion, *Linearization of algebraic group actions*, Handbook of group actions. Vol. IV, Adv. Lect. Math. (ALM), vol. 41, Int. Press, Somerville, MA, 2018, pp. 291–340. MR3888690
- [46] Kenneth S. Brown, *Cohomology of groups*, Graduate Texts in Mathematics, vol. 87, Springer-Verlag, New York-Berlin, 1982. MR672956
- [47] F. Bruhat and J. Tits, *Groupes réductifs sur un corps local* (French), *Inst. Hautes Études Sci. Publ. Math.* **41** (1972), 5–251. MR327923
- [48] Jonathan Brundan, *Quiver Hecke algebras and categorification*, Advances in representation theory of algebras, EMS Ser. Congr. Rep., Eur. Math. Soc., Zürich, 2013, pp. 103–133. MR3220535
- [49] Jean-Luc Brylinski, *Transformations canoniques, dualité projective, théorie de Lefschetz, transformations de Fourier et sommes trigonométriques* (French, with English summary), *Astérisque* **140-141** (1986), 3–134, 251. Géométrie et analyse microlocales. MR864073
- [50] J.-L. Brylinski and M. Kashiwara, *Kazhdan-Lusztig conjecture and holonomic systems*, *Invent. Math.* **64** (1981), no. 3, 387–410, DOI 10.1007/BF01389272. MR632980
- [51] Roger W. Carter, *Finite groups of Lie type: Conjugacy classes and complex characters*, Pure and Applied Mathematics (New York), John Wiley & Sons, Inc., New York, 1985. A Wiley-Interscience Publication. MR794307
- [52] David H. Collingwood and William M. McGovern, *Nilpotent orbits in semisimple Lie algebras*, Van Nostrand Reinhold Mathematics Series, Van Nostrand Reinhold Co., New York, 1993. MR1251060
- [53] Brian Conrad, *Deligne's notes on Nagata compactifications*, *J. Ramanujan Math. Soc.* **22** (2007), no. 3, 205–257. MR2356346
- [54] Mark Andrea A. de Cataldo, *Decomposition theorem for semi-simples*, *J. Singul.* **14** (2016), 194–197, DOI 10.1007/s11856-006-0006-2. MR3595140
- [55] Mark Andrea A. de Cataldo and Luca Migliorini, *The hard Lefschetz theorem and the topology of semismall maps* (English, with English and French summaries), *Ann. Sci. École Norm. Sup. (4)* **35** (2002), no. 5, 759–772, DOI 10.1016/S0012-9593(02)01108-4. MR1951443
- [56] Mark Andrea A. de Cataldo and Luca Migliorini, *The Hodge theory of algebraic maps* (English, with English and French summaries), *Ann. Sci. École Norm. Sup. (4)* **38** (2005), no. 5, 693–750, DOI 10.1016/j.ansens.2005.07.001. MR2195257
- [57] C. De Concini, G. Lusztig, and C. Procesi, *Homology of the zero-set of a nilpotent vector field on a flag manifold*, *J. Amer. Math. Soc.* **1** (1988), no. 1, 15–34, DOI 10.2307/1990965. MR924700

- [58] Pierre Deligne, *Théorie de Hodge. I* (French), Actes du Congrès International des Mathématiciens (Nice, 1970), Gauthier-Villars, Paris, 1971, pp. 425–430. MR0441965
- [59] Pierre Deligne, *Théorie de Hodge. II* (French), Inst. Hautes Études Sci. Publ. Math. **40** (1971), 5–57. MR498551
- [60] Pierre Deligne, *La conjecture de Weil. II* (French), Inst. Hautes Études Sci. Publ. Math. **52** (1980), 137–252. MR601520
- [61] P. Deligne, *Un théorème de finitude pour la monodromie* (French), Discrete groups in geometry and analysis (New Haven, Conn., 1984), Progr. Math., vol. 67, Birkhäuser Boston, Boston, MA, 1987, pp. 1–19, DOI 10.1007/978-1-4899-6664-3_1. MR900821
- [62] P. Deligne and J. S. Milne, *Tannakian categories*, Hodge cycles, motives, and Shimura varieties, Lecture Notes in Math., no. 900, Springer-Verlag, 1982, pp. 101–228.
- [63] Michel Demazure, *Invariants symétriques entiers des groupes de Weyl et torsion* (French), Invent. Math. **21** (1973), 287–301, DOI 10.1007/BF01418790. MR342522
- [64] Michel Demazure, *Désingularisation des variétés de Schubert généralisées* (French), Ann. Sci. École Norm. Sup. (4) **7** (1974), 53–88. MR354697
- [65] Bangming Deng, Jie Du, Brian Parshall, and Jianpan Wang, *Finite dimensional algebras and quantum groups*, Mathematical Surveys and Monographs, vol. 150, American Mathematical Society, Providence, RI, 2008, DOI 10.1090/surv/150. MR2457938
- [66] Harm Derksen and Jerzy Weyman, *An introduction to quiver representations*, Graduate Studies in Mathematics, vol. 184, American Mathematical Society, Providence, RI, 2017, DOI 10.1090/gsm/184. MR3727119
- [67] Alexandru Dimca, *Sheaves in topology*, Universitext, Springer-Verlag, Berlin, 2004, DOI 10.1007/978-3-642-18868-8. MR2050072
- [68] Vladimir Drinfeld, *On a conjecture of Kashiwara*, Math. Res. Lett. **8** (2001), no. 5-6, 713–728, DOI 10.4310/MRL.2001.v8.n6.a3. MR1879815
- [69] V. Drinfeld and D. Gaitsgory, *On a theorem of Braden*, Transform. Groups **19** (2014), no. 2, 313–358, DOI 10.1007/s00031-014-9267-8. MR3200429
- [70] Bjørn Ian Dundas, *A short course in differential topology*, Cambridge Mathematical Textbooks, Cambridge University Press, Cambridge, 2018, DOI 10.1017/9781108349130. MR3793640
- [71] Charles Ehresmann, *Les connexions infinitésimales dans un espace fibré différentiable* (French), Colloque de topologie (espaces fibrés), Bruxelles, 1950, Georges Thone, Liège; Masson et Cie., Paris, 1951, pp. 29–55. MR0042768
- [72] David Eisenbud, *Commutative algebra: With a view toward algebraic geometry*, Graduate Texts in Mathematics, vol. 150, Springer-Verlag, New York, 1995, DOI 10.1007/978-1-4612-5350-1. MR1322960
- [73] Ben Elias and Geordie Williamson, *Soergel calculus*, Represent. Theory **20** (2016), 295–374, DOI 10.1090/ert/481. MR3555156
- [74] Peter Fiebig, *An upper bound on the exceptional characteristics for Lusztig’s character formula*, J. Reine Angew. Math. **673** (2012), 1–31, DOI 10.1515/CRELLE.2011.170. MR2999126
- [75] Eberhard Freitag and Reinhardt Kiehl, *Étale cohomology and the Weil conjecture*, Ergebnisse der Mathematik und ihrer Grenzgebiete (3) [Results in Mathematics and Related Areas (3)], vol. 13, Springer-Verlag, Berlin, 1988. Translated from the German by Betty S. Waterhouse and William C. Waterhouse; With an historical introduction by J. A. Dieudonné, DOI 10.1007/978-3-662-02541-3. MR926276
- [76] Peter Freyd, *Abelian categories. An introduction to the theory of functors*, Harper’s Series in Modern Mathematics, Harper & Row, Publishers, New York, 1964. MR0166240
- [77] Michael D. Fried and Moshe Jarden, *Field arithmetic*, 3rd ed., Ergebnisse der Mathematik und ihrer Grenzgebiete. 3. Folge. A Series of Modern Surveys in Mathematics [Results in Mathematics and Related Areas. 3rd Series. A Series of Modern Surveys in Mathematics], vol. 11, Springer-Verlag, Berlin, 2008. Revised by Jarden. MR2445111
- [78] Peter Gabriel, *Unzerlegbare Darstellungen. I* (German, with English summary), Manuscripta Math. **6** (1972), 71–103; correction, *ibid.* **6** (1972), 309, DOI 10.1007/BF01298413. MR332887
- [79] Pierre Gabriel, *Des catégories abéliennes* (French), Bull. Soc. Math. France **90** (1962), 323–448. MR232821

- [80] D. Gaiitsgory, *On de Jong's conjecture*, Israel J. Math. **157** (2007), 155–191, DOI 10.1007/s11856-006-0006-2. MR2342444
- [81] Sergei I. Gelfand and Yuri I. Manin, *Methods of homological algebra*, Springer-Verlag, Berlin, 1996. Translated from the 1988 Russian original, DOI 10.1007/978-3-662-03220-6. MR1438306
- [82] V. Ginzburg, *Perverse sheaves on a loop group and Langlands' duality*, arXiv:alg-geom/9511007.
- [83] Claude Godbillon, *Éléments de topologie algébrique* (French), Hermann, Paris, 1971. MR0301725
- [84] Roger Godement, *Topologie algébrique et théorie des faisceaux* (French), Actualités Sci. Ind. No. 1252. Publ. Math. Univ. Strasbourg. No. 13, Hermann, Paris, 1958. MR0102797
- [85] Mark Goresky and Robert MacPherson, *Intersection homology theory*, Topology **19** (1980), no. 2, 135–162, DOI 10.1016/0040-9383(80)90003-8. MR572580
- [86] Mark Goresky and Robert MacPherson, *Intersection homology. II*, Invent. Math. **72** (1983), no. 1, 77–129, DOI 10.1007/BF01389130. MR696691
- [87] Mark Goresky and Robert MacPherson, *Morse theory and intersection homology theory*, Analysis and topology on singular spaces, II, III (Luminy, 1981), Astérisque, vol. 101, Soc. Math. France, Paris, 1983, pp. 135–192. MR737930
- [88] Mark Goresky and Robert MacPherson, *Stratified Morse theory*, Ergebnisse der Mathematik und ihrer Grenzgebiete (3) [Results in Mathematics and Related Areas (3)], vol. 14, Springer-Verlag, Berlin, 1988, DOI 10.1007/978-3-642-71714-7. MR932724
- [89] Mark Goresky and Robert MacPherson, *Local contribution to the Lefschetz fixed point formula*, Invent. Math. **111** (1993), no. 1, 1–33, DOI 10.1007/BF01231277. MR1193595
- [90] Phillip A. Griffiths, *On the periods of integrals on algebraic manifolds*, Rice Univ. Stud. **54** (1968), no. 4, 21–38. MR242844
- [91] A. Grothendieck, *Éléments de géométrie algébrique. II. Étude globale élémentaire de quelques classes de morphismes* (French), Inst. Hautes Études Sci. Publ. Math. **8** (1961), 222. MR217084
- [92] A. Grothendieck, *Éléments de géométrie algébrique. IV. Étude locale des schémas et des morphismes de schémas. II* (French), Inst. Hautes Études Sci. Publ. Math. **24** (1965), 231. MR199181
- [93] A. Grothendieck, *Éléments de géométrie algébrique. IV. Étude locale des schémas et des morphismes de schémas IV* (French), Inst. Hautes Études Sci. Publ. Math. **32** (1967), 361. MR238860
- [94] Rajendra V. Gurjar, Kayo Masuda, and Masayoshi Miyanishi, *Deformations of \mathbb{A}^1 -fibrations*, Automorphisms in birational and affine geometry, Springer Proc. Math. Stat., vol. 79, Springer, Cham, 2014, pp. 327–361, DOI 10.1007/978-3-319-05681-4_19. MR3229360
- [95] H. C. Hansen, *On cycles in flag manifolds*, Math. Scand. **33** (1973), 269–274 (1974), DOI 10.7146/math.scand.a-11489. MR376703
- [96] Robin Hartshorne, *Residues and duality*, Lecture notes of a seminar on the work of A. Grothendieck, given at Harvard 1963/64. With an appendix by P. Deligne. Lecture Notes in Mathematics, No. 20, Springer-Verlag, Berlin-New York, 1966. MR0222093
- [97] Robin Hartshorne, *Algebraic geometry*, Graduate Texts in Mathematics, No. 52, Springer-Verlag, New York-Heidelberg, 1977. MR0463157
- [98] Allen Hatcher, *Algebraic topology*, Cambridge University Press, Cambridge, 2002. MR1867354
- [99] Joachim Hilgert and Karl-Hermann Neeb, *Structure and geometry of Lie groups*, Springer Monographs in Mathematics, Springer, New York, 2012, DOI 10.1007/978-0-387-84794-8. MR3025417
- [100] Heisuke Hironaka, *Resolution of singularities of an algebraic variety over a field of characteristic zero. I, II*, Ann. of Math. (2) **79** (1964), 109–203; *ibid.* (2) **79** (1964), 205–326, DOI 10.2307/1970547. MR0199184
- [101] Heisuke Hironaka, *Triangulations of algebraic sets*, Algebraic geometry (Proc. Sympos. Pure Math., Vol. 29, Humboldt State Univ., Arcata, Calif., 1974), Amer. Math. Soc., Providence, R.I., 1975, pp. 165–185. MR0374131
- [102] Chung Wu Ho, *A note on proper maps*, Proc. Amer. Math. Soc. **51** (1975), 237–241, DOI 10.2307/2039880. MR370471

- [103] Jin Hong and Seok-Jin Kang, *Introduction to quantum groups and crystal bases*, Graduate Studies in Mathematics, vol. 42, American Mathematical Society, Providence, RI, 2002, DOI 10.1090/gsm/042. MR1881971
- [104] Ryoshi Hotta, *On Springer's representations*, J. Fac. Sci. Univ. Tokyo Sect. IA Math. **28** (1981), no. 3, 863–876 (1982). MR656061
- [105] R. Hotta and T. A. Springer, *A specialization theorem for certain Weyl group representations and an application to the Green polynomials of unitary groups*, Invent. Math. **41** (1977), no. 2, 113–127, DOI 10.1007/BF01418371. MR486164
- [106] Ryoshi Hotta, Kiyoshi Takeuchi, and Toshiyuki Tanisaki, *D-modules, perverse sheaves, and representation theory*, Progress in Mathematics, vol. 236, Birkhäuser Boston, Inc., Boston, MA, 2008. Translated from the 1995 Japanese edition by Takeuchi, DOI 10.1007/978-0-8176-4523-6. MR2357361
- [107] Christian Houzel, *Histoire de la théorie des faisceaux* (French), Gaz. Math. **84**, suppl. (2000), 35–52. Jean Leray (1906–1998). MR1775588
- [108] James E. Humphreys, *Reflection groups and Coxeter groups*, Cambridge Studies in Advanced Mathematics, vol. 29, Cambridge University Press, Cambridge, 1990, DOI 10.1017/CBO9780511623646. MR1066460
- [109] Luc Illusie, *Autour du théorème de monodromie locale* (French), Astérisque **223** (1994), 9–57. Périodes p -adiques (Bures-sur-Yvette, 1988). MR1293970
- [110] Nagayoshi Iwahori, *On the structure of a Hecke ring of a Chevalley group over a finite field*, J. Fac. Sci. Univ. Tokyo Sect. I **10** (1964), 215–236 (1964). MR165016
- [111] N. Iwahori and H. Matsumoto, *On some Bruhat decomposition and the structure of the Hecke rings of p -adic Chevalley groups*, Inst. Hautes Études Sci. Publ. Math. **25** (1965), 5–48. MR185016
- [112] Nathan Jacobson, *Completely reducible Lie algebras of linear transformations*, Proc. Amer. Math. Soc. **2** (1951), 105–113, DOI 10.2307/2032629. MR49882
- [113] I. M. James, *The topology of Stiefel manifolds*, London Mathematical Society Lecture Note Series, No. 24, Cambridge University Press, Cambridge-New York-Melbourne, 1976. MR0431239
- [114] Jens Carsten Jantzen, *Representations of algebraic groups*, 2nd ed., Mathematical Surveys and Monographs, vol. 107, American Mathematical Society, Providence, RI, 2003. MR2015057
- [115] D. Juteau, *Modular representations of reductive groups and geometry of affine Grassmannians*, arXiv:0804.2041.
- [116] ———, *Modular Springer correspondence, decomposition matrices, and basic sets*, arXiv:1410.1471.
- [117] ———, *Modular Springer correspondence and decomposition matrices*, Ph.D. thesis, Université Paris 7, 2007.
- [118] Daniel Juteau, *Decomposition numbers for perverse sheaves* (English, with English and French summaries), Ann. Inst. Fourier (Grenoble) **59** (2009), no. 3, 1177–1229. MR2543666
- [119] D. Juteau, C. Lecouvey, and K. Sorlin, *Springer basic sets and modular Springer correspondence for classical types*, arXiv:1410.1477.
- [120] Daniel Juteau, Carl Mautner, and Geordie Williamson, *Parity sheaves*, J. Amer. Math. Soc. **27** (2014), no. 4, 1169–1212, DOI 10.1090/S0894-0347-2014-00804-3. MR3230821
- [121] Masaki Kashiwara, *The Riemann-Hilbert problem for holonomic systems*, Publ. Res. Inst. Math. Sci. **20** (1984), no. 2, 319–365, DOI 10.2977/prims/1195181610. MR743382
- [122] M. Kashiwara, *On crystal bases of the Q -analogue of universal enveloping algebras*, Duke Math. J. **63** (1991), no. 2, 465–516, DOI 10.1215/S0012-7094-91-06321-0. MR1115118
- [123] Masaki Kashiwara, *Semisimple holonomic \mathcal{D} -modules*, Topological field theory, primitive forms and related topics (Kyoto, 1996), Progr. Math., vol. 160, Birkhäuser Boston, Boston, MA, 1998, pp. 267–271. MR1653028
- [124] Masaki Kashiwara, *D-modules and microlocal calculus*, Translations of Mathematical Monographs, vol. 217, American Mathematical Society, Providence, RI, 2003. Translated from the 2000 Japanese original by Mutsumi Saito; Iwanami Series in Modern Mathematics, DOI 10.1090/mmono/217. MR1943036

- [125] Masaki Kashiwara and Pierre Schapira, *Sheaves on manifolds*, Grundlehren der Mathematischen Wissenschaften [Fundamental Principles of Mathematical Sciences], vol. 292, Springer-Verlag, Berlin, 1994. With a chapter in French by Christian Houzel; Corrected reprint of the 1990 original. MR1299726
- [126] Masaki Kashiwara and Pierre Schapira, *Categories and sheaves*, Grundlehren der Mathematischen Wissenschaften [Fundamental Principles of Mathematical Sciences], vol. 332, Springer-Verlag, Berlin, 2006, DOI 10.1007/3-540-27950-4. MR2182076
- [127] Masaki Kashiwara and Toshiyuki Tanisaki, *Kazhdan-Lusztig conjecture for affine Lie algebras with negative level*, Duke Math. J. **77** (1995), no. 1, 21–62, DOI 10.1215/S0012-7094-95-07702-3. MR1317626
- [128] Shin-ichi Kato, *Spherical functions and a q -analogue of Kostant's weight multiplicity formula*, Invent. Math. **66** (1982), no. 3, 461–468, DOI 10.1007/BF01389223. MR662602
- [129] David Kazhdan and George Lusztig, *Representations of Coxeter groups and Hecke algebras*, Invent. Math. **53** (1979), no. 2, 165–184, DOI 10.1007/BF01390031. MR560412
- [130] David Kazhdan and George Lusztig, *Schubert varieties and Poincaré duality*, Geometry of the Laplace operator (Proc. Sympos. Pure Math., Univ. Hawaii, Honolulu, Hawaii, 1979), Proc. Sympos. Pure Math., XXXVI, Amer. Math. Soc., Providence, R.I., 1980, pp. 185–203. MR573434
- [131] David Kazhdan and George Lusztig, *Affine Lie algebras and quantum groups*, Internat. Math. Res. Notices **2** (1991), 21–29, DOI 10.1155/S1073792891000041. MR1104840
- [132] G. M. Kelly, *On MacLane's conditions for coherence of natural associativities, commutativities, etc.*, J. Algebra **1** (1964), 397–402, DOI 10.1016/0021-8693(64)90018-3. MR182649
- [133] G. M. Kelly and M. L. Laplaza, *Coherence for compact closed categories*, J. Pure Appl. Algebra **19** (1980), 193–213, DOI 10.1016/0022-4049(80)90101-2. MR593254
- [134] Mikhail Khovanov and Aaron D. Lauda, *A diagrammatic approach to categorification of quantum groups. I*, Represent. Theory **13** (2009), 309–347, DOI 10.1090/S1088-4165-09-00346-X. MR2525917
- [135] Mikhail Khovanov and Aaron D. Lauda, *A diagrammatic approach to categorification of quantum groups II*, Trans. Amer. Math. Soc. **363** (2011), no. 5, 2685–2700, DOI 10.1090/S0002-9947-2010-05210-9. MR2763732
- [136] Reinhardt Kiehl and Rainer Weissauer, *Weil conjectures, perverse sheaves and l -adic Fourier transform*, Ergebnisse der Mathematik und ihrer Grenzgebiete. 3. Folge. A Series of Modern Surveys in Mathematics [Results in Mathematics and Related Areas. 3rd Series. A Series of Modern Surveys in Mathematics], vol. 42, Springer-Verlag, Berlin, 2001, DOI 10.1007/978-3-662-04576-3. MR1855066
- [137] Frances Kirwan, *Intersection homology and torus actions*, J. Amer. Math. Soc. **1** (1988), no. 2, 385–400, DOI 10.2307/1990921. MR928263
- [138] Steven L. Kleiman, *The development of intersection homology theory*, Pure Appl. Math. Q. **3** (2007), no. 1, Special Issue: In honor of Robert D. MacPherson., 225–282, DOI 10.4310/PAMQ.2007.v3.n1.a8. MR2330160
- [139] Bertram Kostant, *The principal three-dimensional subgroup and the Betti numbers of a complex simple Lie group*, Amer. J. Math. **81** (1959), 973–1032, DOI 10.2307/2372999. MR114875
- [140] Bertram Kostant, *Lie group representations on polynomial rings*, Amer. J. Math. **85** (1963), 327–404, DOI 10.2307/2373130. MR158024
- [141] Emmanuel Kowalski, *An introduction to the representation theory of groups*, Graduate Studies in Mathematics, vol. 155, American Mathematical Society, Providence, RI, 2014, DOI 10.1090/gsm/155. MR3236265
- [142] Henning Krause, *Krull-Schmidt categories and projective covers*, Expo. Math. **33** (2015), no. 4, 535–549, DOI 10.1016/j.exmath.2015.10.001. MR3431480
- [143] Shrawan Kumar, *Kac-Moody groups, their flag varieties and representation theory*, Progress in Mathematics, vol. 204, Birkhäuser Boston, Inc., Boston, MA, 2002, DOI 10.1007/978-1-4612-0105-2. MR1923198
- [144] T. Y. Lam, *A first course in noncommutative rings*, Graduate Texts in Mathematics, vol. 131, Springer-Verlag, New York, 1991, DOI 10.1007/978-1-4684-0406-7. MR1125071
- [145] T. Y. Lam, *Lectures on modules and rings*, Graduate Texts in Mathematics, vol. 189, Springer-Verlag, New York, 1999, DOI 10.1007/978-1-4612-0525-8. MR1653294

- [146] Yves Laszlo and Martin Olsson, *The six operations for sheaves on Artin stacks. I. Finite coefficients*, Publ. Math. Inst. Hautes Études Sci. **107** (2008), 109–168, DOI 10.1007/s10240-008-0011-6. MR2434692
- [147] Yves Laszlo and Martin Olsson, *The six operations for sheaves on Artin stacks. II. Adic coefficients*, Publ. Math. Inst. Hautes Études Sci. **107** (2008), 169–210, DOI 10.1007/s10240-008-0012-5. MR2434693
- [148] Yves Laszlo and Martin Olsson, *Perverse t -structure on Artin stacks*, Math. Z. **261** (2009), no. 4, 737–748, DOI 10.1007/s00209-008-0348-z. MR2480756
- [149] G. Laumon, *Transformation de Fourier, constantes d'équations fonctionnelles et conjecture de Weil* (French), Inst. Hautes Études Sci. Publ. Math. **65** (1987), 131–210. MR908218
- [150] Gérard Laumon, *Transformation de Fourier homogène* (French, with English and French summaries), Bull. Soc. Math. France **131** (2003), no. 4, 527–551, DOI 10.24033/bsmf.2454. MR2044494
- [151] Robert Lazarsfeld, *Positivity in algebraic geometry. I: Classical setting: line bundles and linear series*, Ergebnisse der Mathematik und ihrer Grenzgebiete. 3. Folge. A Series of Modern Surveys in Mathematics [Results in Mathematics and Related Areas. 3rd Series. A Series of Modern Surveys in Mathematics], vol. 48, Springer-Verlag, Berlin, 2004, DOI 10.1007/978-3-642-18808-4. MR2095471
- [152] Lê Dũng Tráng, *Some remarks on relative monodromy*, Real and complex singularities (Proc. Ninth Nordic Summer School/NAVF Sympos. Math., Oslo, 1976), Sijthoff and Noordhoff, Alphen aan den Rijn, 1977, pp. 397–403. MR0476739
- [153] Jue Le and Xiao-Wu Chen, *Karoubianness of a triangulated category*, J. Algebra **310** (2007), no. 1, 452–457, DOI 10.1016/j.jalgebra.2006.11.027. MR2307804
- [154] Qing Liu, *Algebraic geometry and arithmetic curves*, Oxford Graduate Texts in Mathematics, vol. 6, Oxford University Press, Oxford, 2002. Translated from the French by Reinie Ern e; Oxford Science Publications. MR1917232
- [155] G. Lusztig, *Character sheaves. I–V*, Adv. in Math. **56** (1985), 193–237; **57** (1985), 226–265, 266–315; **59** (1986), 1–63; **61** (1986), 103–155. MR0792706, MR0806210, MR0825086, MR0849848.
- [156] George Lusztig, *Some problems in the representation theory of finite Chevalley groups*, The Santa Cruz Conference on Finite Groups (Univ. California, Santa Cruz, Calif., 1979), Proc. Sympos. Pure Math., vol. 37, Amer. Math. Soc., Providence, R.I., 1980, pp. 313–317. MR604598
- [157] G. Lusztig, *Green polynomials and singularities of unipotent classes*, Adv. in Math. **42** (1981), no. 2, 169–178, DOI 10.1016/0001-8708(81)90038-4. MR641425
- [158] George Lusztig, *Singularities, character formulas, and a q -analog of weight multiplicities*, Analysis and topology on singular spaces, II, III (Luminy, 1981), Ast risque, vol. 101, Soc. Math. France, Paris, 1983, pp. 208–229. MR737932
- [159] G. Lusztig, *Intersection cohomology complexes on a reductive group*, Invent. Math. **75** (1984), no. 2, 205–272, DOI 10.1007/BF01388564. MR732546
- [160] George Lusztig, *Fourier transforms on a semisimple Lie algebra over \mathbf{F}_q* , Algebraic groups Utrecht 1986, Lecture Notes in Math., vol. 1271, Springer, Berlin, 1987, pp. 177–188, DOI 10.1007/BFb0079237. MR911139
- [161] G. Lusztig, *Canonical bases arising from quantized enveloping algebras*, J. Amer. Math. Soc. **3** (1990), no. 2, 447–498, DOI 10.2307/1990961. MR1035415
- [162] George Lusztig, *Quantum groups at roots of 1*, Geom. Dedicata **35** (1990), no. 1-3, 89–113, DOI 10.1007/BF00147341. MR1066560
- [163] W. L utkebohmert, *On compactification of schemes*, Manuscripta Math. **80** (1993), no. 1, 95–111, DOI 10.1007/BF03026540. MR1226600
- [164] Saunders Mac Lane, *Categories for the working mathematician*, 2nd ed., Graduate Texts in Mathematics, vol. 5, Springer-Verlag, New York, 1998. MR1712872
- [165] I. G. Macdonald, *Spherical functions on a group of p -adic type*, Publications of the Ramanujan Institute, No. 2, Ramanujan Institute, Centre for Advanced Study in Mathematics, University of Madras, Madras, 1971. MR0435301
- [166] Robert MacPherson and Kari Vilonen, *Elementary construction of perverse sheaves*, Invent. Math. **84** (1986), no. 2, 403–435, DOI 10.1007/BF01388812. MR833195
- [167] A. Malcev, *On semi-simple subgroups of Lie groups* (Russian, with English summary), Bull. Acad. Sci. URSS. S er. Math. [Izvestia Akad. Nauk SSSR] **8** (1944), 143–174. MR0011303

- [168] J. G. M. Mars and T. A. Springer, *Hecke algebra representations related to spherical varieties*, Represent. Theory **2** (1998), 33–69, DOI 10.1090/S1088-4165-98-00027-2. MR1600804
- [169] David Massey, *Natural commuting of vanishing cycles and the Verdier dual*, Pacific J. Math. **284** (2016), no. 2, 431–437, DOI 10.2140/pjm.2016.284.431. MR3544308
- [170] John Mather, *Notes on topological stability*, Bull. Amer. Math. Soc. (N.S.) **49** (2012), no. 4, 475–506, DOI 10.1090/S0273-0979-2012-01383-6. MR2958928
- [171] Hideya Matsumoto, *Analyse harmonique dans les systèmes de Tits bornologiques de type affine* (French), Lecture Notes in Mathematics, Vol. 590, Springer-Verlag, Berlin-New York, 1977. MR0579177
- [172] Hideyuki Matsumura, *Commutative ring theory*, 2nd ed., Cambridge Studies in Advanced Mathematics, vol. 8, Cambridge University Press, Cambridge, 1989. Translated from the Japanese by M. Reid. MR1011461
- [173] Carl Mautner and Simon Riche, *Exotic tilting sheaves, parity sheaves on affine Grassmannians, and the Mirković-Vilonen conjecture*, J. Eur. Math. Soc. (JEMS) **20** (2018), no. 9, 2259–2332, DOI 10.4171/JEMS/812. MR3836847
- [174] J. P. May, *The additivity of traces in triangulated categories*, Adv. Math. **163** (2001), no. 1, 34–73, DOI 10.1006/aima.2001.1995. MR1867203
- [175] Z. Mebkhout, *Une autre équivalence de catégories* (French), Compositio Math. **51** (1984), no. 1, 63–88. MR734785
- [176] John Milnor, *Singular points of complex hypersurfaces*, Annals of Mathematics Studies, No. 61, Princeton University Press, Princeton, N.J.; University of Tokyo Press, Tokyo, 1968. MR0239612
- [177] I. Mirković, *Character sheaves on reductive Lie algebras* (English, with English and Russian summaries), Mosc. Math. J. **4** (2004), no. 4, 897–910, 981, DOI 10.17323/1609-4514-2004-4-4-897-910. MR2124171
- [178] Ivan Mirković and Kari Vilonen, *Perverse sheaves on affine Grassmannians and Langlands duality*, Math. Res. Lett. **7** (2000), no. 1, 13–24, DOI 10.4310/MRL.2000.v7.n1.a2. MR1748284
- [179] I. Mirković and K. Vilonen, *Geometric Langlands duality and representations of algebraic groups over commutative rings*, Ann. of Math. (2) **166** (2007), no. 1, 95–143, DOI 10.4007/annals.2007.166.95. MR2342692
- [180] Takuro Mochizuki, *Asymptotic behaviour of tame harmonic bundles and an application to pure twistor D -modules. I*, Mem. Amer. Math. Soc. **185** (2007), no. 869, xii+324, DOI 10.1090/memo/0869. MR2281877
- [181] Takuro Mochizuki, *Asymptotic behaviour of tame harmonic bundles and an application to pure twistor D -modules. II*, Mem. Amer. Math. Soc. **185** (2007), no. 870, xii+565, DOI 10.1090/memo/0870. MR2283665
- [182] V. V. Morozov, *On a nilpotent element in a semi-simple Lie algebra*, C. R. (Doklady) Acad. Sci. URSS (N.S.) **36** (1942), 83–86. MR0007750
- [183] David Mumford, *The red book of varieties and schemes*, Second, expanded edition, Lecture Notes in Mathematics, vol. 1358, Springer-Verlag, Berlin, 1999. Includes the Michigan lectures (1974) on curves and their Jacobians; With contributions by Enrico Arbarello, DOI 10.1007/b62130. MR1748380
- [184] D. Mumford, J. Fogarty, and F. Kirwan, *Geometric invariant theory*, 3rd ed., Ergebnisse der Mathematik und ihrer Grenzgebiete (2) [Results in Mathematics and Related Areas (2)], vol. 34, Springer-Verlag, Berlin, 1994, DOI 10.1007/978-3-642-57916-5. MR1304906
- [185] James R. Munkres, *Elements of algebraic topology*, Addison-Wesley Publishing Company, Menlo Park, CA, 1984. MR755006
- [186] Amnon Neeman, *Triangulated categories*, Annals of Mathematics Studies, vol. 148, Princeton University Press, Princeton, NJ, 2001, DOI 10.1515/9781400837212. MR1812507
- [187] Madhav V. Nori, *Constructible sheaves*, Algebra, arithmetic and geometry, Part I, II (Mumbai, 2000), Tata Inst. Fund. Res. Stud. Math., vol. 16, Tata Inst. Fund. Res., Bombay, 2002, pp. 471–491. MR1940678
- [188] Chris A. M. Peters and Joseph H. M. Steenbrink, *Mixed Hodge structures*, Ergebnisse der Mathematik und ihrer Grenzgebiete. 3. Folge. A Series of Modern Surveys in Mathematics [Results in Mathematics and Related Areas. 3rd Series. A Series of Modern Surveys in Mathematics], vol. 52, Springer-Verlag, Berlin, 2008. MR2393625

- [189] S. Ramanan, *Global calculus*, Graduate Studies in Mathematics, vol. 65, American Mathematical Society, Providence, RI, 2005, DOI 10.1090/gsm/065. MR2104612
- [190] Michel Raynaud, *Faisceaux amples sur les schémas en groupes et les espaces homogènes* (French), Lecture Notes in Mathematics, Vol. 119, Springer-Verlag, Berlin-New York, 1970. MR0260758
- [191] Ryan Reich, *Notes on Beilinson's "How to glue perverse sheaves"* [MR0923134], *J. Singul.* **1** (2010), 94–115. MR2671769
- [192] Ryan Cohen Reich, *Obvious natural morphisms of sheaves are unique*, *Theory Appl. Categ.* **29** (2014), No. 4, 48–99. MR3192770
- [193] S. Riche and G. Williamson, *Smith–Treumann theory and the linkage principle*, arXiv: 2003.08522.
- [194] Simon Riche and Geordie Williamson, *Tilting modules and the p -canonical basis* (English, with English and French summaries), *Astérisque* **397** (2018), ix+184. MR3805034
- [195] Claus Michael Ringel, *Hall algebras*, Topics in algebra, Part 1 (Warsaw, 1988), Banach Center Publ., vol. 26, PWN, Warsaw, 1990, pp. 433–447. MR1171248
- [196] Claus Michael Ringel, *Hall algebras and quantum groups*, *Invent. Math.* **101** (1990), no. 3, 583–591, DOI 10.1007/BF01231516. MR1062796
- [197] Raphaël Rouquier, *Quiver Hecke algebras and 2-Lie algebras*, *Algebra Colloq.* **19** (2012), no. 2, 359–410, DOI 10.1142/S1005386712000247. MR2908731
- [198] Claude Sabbah, *Polarizable twistor \mathcal{D} -modules* (English, with English and French summaries), *Astérisque* **300** (2005), vi+208. MR2156523
- [199] Morihiko Saito, *Modules de Hodge polarisables* (French), *Publ. Res. Inst. Math. Sci.* **24** (1988), no. 6, 849–995 (1989), DOI 10.2977/prims/1195173930. MR1000123
- [200] Morihiko Saito, *Extension of mixed Hodge modules*, *Compositio Math.* **74** (1990), no. 2, 209–234. MR1047741
- [201] Morihiko Saito, *Mixed Hodge modules*, *Publ. Res. Inst. Math. Sci.* **26** (1990), no. 2, 221–333, DOI 10.2977/prims/1195171082. MR1047415
- [202] Ichirō Satake, *Theory of spherical functions on reductive algebraic groups over p -adic fields*, *Inst. Hautes Études Sci. Publ. Math.* **18** (1963), 5–69. MR195863
- [203] Ralf Schiffler, *Quiver representations*, CMS Books in Mathematics/Ouvrages de Mathématiques de la SMC, Springer, Cham, 2014, DOI 10.1007/978-3-319-09204-1. MR3308668
- [204] Wilfried Schmid, *Variation of Hodge structure: the singularities of the period mapping*, *Invent. Math.* **22** (1973), 211–319, DOI 10.1007/BF01389674. MR382272
- [205] J.-P. Schneiders, *Introduction to characteristic classes and index theory*, *Textos de Matemática*, no. 13, Departamento de Matemática, Faculdade de Ciências da Universidade de Lisboa, 2000.
- [206] Jörg Schürmann, *Topology of singular spaces and constructible sheaves*, Instytut Matematyczny Polskiej Akademii Nauk. Monografie Matematyczne (New Series) [Mathematics Institute of the Polish Academy of Sciences. Mathematical Monographs (New Series)], vol. 63, Birkhäuser Verlag, Basel, 2003, DOI 10.1007/978-3-0348-8061-9. MR2031639
- [207] Y. Sella, *Comparison of sheaf cohomology and singular cohomology*, arXiv:1602.06674.
- [208] J.-P. Serre, *Espaces fibrés algébriques*, Séminaire C. Chevalley; 2e année: 1958. Anneaux de Chow et applications, Secrétariat mathématique, Paris, 1958, pp. 1–37.
- [209] Jean-Pierre Serre, *Local fields*, Graduate Texts in Mathematics, vol. 67, Springer-Verlag, New York-Berlin, 1979. Translated from the French by Marvin Jay Greenberg. MR554237
- [210] Igor R. Shafarevich, *Basic algebraic geometry. 1*, Translated from the 2007 third Russian edition, Springer, Heidelberg, 2013. Varieties in projective space. MR3100243
- [211] Igor R. Shafarevich, *Basic algebraic geometry. 2*, Translated from the 2007 third Russian edition, Springer, Heidelberg, 2013. Schemes and complex manifolds. MR3100288
- [212] Toshiaki Shoji, *On the Springer representations of the Weyl groups of classical algebraic groups*, *Comm. Algebra* **7** (1979), no. 16, 1713–1745, DOI 10.1080/00927877908822425. MR546195
- [213] Toshiaki Shoji, *On the Springer representations of Chevalley groups of type F_4* , *Comm. Algebra* **8** (1980), no. 5, 409–440, DOI 10.1080/00927878008822466. MR561538
- [214] Toshiaki Shoji, *Geometry of orbits and Springer correspondence: Orbites unipotentes et représentations, I*, *Astérisque* **168** (1988), 9, 61–140. MR1021493

- [215] Peter Slodowy, *Four lectures on simple groups and singularities*, Communications of the Mathematical Institute, Rijksuniversiteit Utrecht, vol. 11, Rijksuniversiteit Utrecht, Mathematical Institute, Utrecht, 1980. MR563725
- [216] Peter Slodowy, *Simple singularities and simple algebraic groups*, Lecture Notes in Mathematics, vol. 815, Springer, Berlin, 1980. MR584445
- [217] Karen E. Smith, Lauri Kahanpää, Pekka Kekäläinen, and William Traves, *An invitation to algebraic geometry*, Universitext, Springer-Verlag, New York, 2000, DOI 10.1007/978-1-4757-4497-2. MR1788561
- [218] Wolfgang Soergel, *Kategorie \mathcal{O} , perverse Garben und Moduln über den Koinvarianten zur Weylgruppe* (German, with English summary), J. Amer. Math. Soc. **3** (1990), no. 2, 421–445, DOI 10.2307/1990960. MR1029692
- [219] Wolfgang Soergel, *The combinatorics of Harish-Chandra bimodules*, J. Reine Angew. Math. **429** (1992), 49–74, DOI 10.1515/crll.1992.429.49. MR1173115
- [220] Wolfgang Soergel, *Kazhdan-Lusztig polynomials and a combinatoric[s] for tilting modules*, Represent. Theory **1** (1997), 83–114, DOI 10.1090/S1088-4165-97-00021-6. MR1444322
- [221] Wolfgang Soergel, *On the relation between intersection cohomology and representation theory in positive characteristic*, J. Pure Appl. Algebra **152** (2000), no. 1-3, 311–335, DOI 10.1016/S0022-4049(99)00138-3. Commutative algebra, homological algebra and representation theory (Catania/Genoa/Rome, 1998). MR1784005
- [222] Wolfgang Soergel, *Kazhdan-Lusztig-Polynome und unzerlegbare Bimoduln über Polynomringen* (German, with English and German summaries), J. Inst. Math. Jussieu **6** (2007), no. 3, 501–525, DOI 10.1017/S1474748007000023. MR2329762
- [223] N. Spaltenstein, *Resolutions of unbounded complexes*, Compositio Math. **65** (1988), no. 2, 121–154. MR932640
- [224] Nicolas Spaltenstein, *Classes unipotentes et sous-groupes de Borel* (French), Lecture Notes in Mathematics, vol. 946, Springer-Verlag, Berlin-New York, 1982. MR672610
- [225] Edwin H. Spanier, *Algebraic topology*, Springer-Verlag, New York, 1990. Corrected reprint of the 1966 original. MR1325242
- [226] T. A. Springer, *Trigonometric sums, Green functions of finite groups and representations of Weyl groups*, Invent. Math. **36** (1976), 173–207, DOI 10.1007/BF01390009. MR442103
- [227] T. A. Springer, *A construction of representations of Weyl groups*, Invent. Math. **44** (1978), no. 3, 279–293, DOI 10.1007/BF01403165. MR491988
- [228] T. A. Springer, *Quelques applications de la cohomologie d'intersection* (French), Bourbaki Seminar, Vol. 1981/1982, Astérisque, vol. 92, Soc. Math. France, Paris, 1982, pp. 249–273. MR689533
- [229] T. A. Springer, *A purity result for fixed point varieties in flag manifolds*, J. Fac. Sci. Univ. Tokyo Sect. IA Math. **31** (1984), no. 2, 271–282. MR763421
- [230] T. A. Springer, *Linear algebraic groups*, 2nd ed., Progress in Mathematics, vol. 9, Birkhäuser Boston, Inc., Boston, MA, 1998, DOI 10.1007/978-0-8176-4840-4. MR1642713
- [231] T. A. Springer and R. Steinberg, *Conjugacy classes*, Seminar on Algebraic Groups and Related Finite Groups (The Institute for Advanced Study, Princeton, N.J., 1968/69), Lecture Notes in Mathematics, Vol. 131, Springer, Berlin, 1970, pp. 167–266. MR0268192
- [232] N. E. Steenrod, *Homology with local coefficients*, Ann. of Math. (2) **44** (1943), 610–627, DOI 10.2307/1969099. MR9114
- [233] Robert Steinberg, *On the desingularization of the unipotent variety*, Invent. Math. **36** (1976), 209–224, DOI 10.1007/BF01390010. MR430094
- [234] Hideyasu Sumihiro, *Equivariant completion*, J. Math. Kyoto Univ. **14** (1974), 1–28, DOI 10.1215/kjm/1250523277. MR337963
- [235] Shenghao Sun, *Generic base change, Artin's comparison theorem, and the decomposition theorem for complex Artin stacks*, J. Algebraic Geom. **26** (2017), no. 3, 513–555, DOI 10.1090/jag/683. MR3647792
- [236] R. Thom, *Ensembles et morphismes stratifiés* (French), Bull. Amer. Math. Soc. **75** (1969), 240–284, DOI 10.1090/S0002-9904-1969-12138-5. MR239613
- [237] M. Varagnolo and E. Vasserot, *Canonical bases and KLR-algebras*, J. Reine Angew. Math. **659** (2011), 67–100, DOI 10.1515/CRELLE.2011.068. MR2837011
- [238] Jean-Louis Verdier, *Dualité dans la cohomologie des espaces localement compacts* (French), Séminaire Bourbaki, Vol. 9, Soc. Math. France, Paris, 1995, pp. Exp. No. 300, 337–349. MR1610971

- [239] Jean-Louis Verdier, *Classe d'homologie associée à un cycle* (French), Séminaire de géométrie analytique (École Norm. Sup., Paris, 1974-75), Soc. Math. France, Paris, 1976, pp. 101–151. Astérisque, No. 36–37. MR0447623
- [240] Jean-Louis Verdier, *Extension of a perverse sheaf over a closed subspace*, Astérisque **130** (1985), 210–217. Differential systems and singularities (Luminy, 1983). MR804054
- [241] Jean-Louis Verdier, *Des catégories dérivées des catégories abéliennes* (French, with French summary), Astérisque **239** (1996), xii+253 pp. (1997). With a preface by Luc Illusie; Edited and with a note by Georges Maltsiniotis. MR1453167
- [242] Daya-Nand Verma, *The rôle of affine Weyl groups in the representation theory of algebraic Chevalley groups and their Lie algebras*, Lie groups and their representations (Proc. Summer School, Bolyai János Math. Soc., Budapest, 1971), Halsted, New York, 1975, pp. 653–705. MR0409673
- [243] Hassler Whitney, *Local properties of analytic varieties*, Differential and Combinatorial Topology (A Symposium in Honor of Marston Morse), Princeton Univ. Press, Princeton, N.J., 1965, pp. 205–244. MR0188486
- [244] Geordie Williamson, *On torsion in the intersection cohomology of Schubert varieties*, J. Algebra **475** (2017), 207–228, DOI 10.1016/j.jalgebra.2016.06.006. MR3612469
- [245] Geordie Williamson, *Schubert calculus and torsion explosion*, J. Amer. Math. Soc. **30** (2017), no. 4, 1023–1046, DOI 10.1090/jams/868. With a joint appendix with Alex Kontorovich and Peter J. McNamara. MR3671935
- [246] Xinwen Zhu, *An introduction to affine Grassmannians and the geometric Satake equivalence*, Geometry of moduli spaces and representation theory, IAS/Park City Math. Ser., vol. 24, Amer. Math. Soc., Providence, RI, 2017, pp. 59–154. MR3752460

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