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This monograph is dedicated to Mary Ann Heinzer, to Maria Rotthaus, to Roger Wiegand, and to the past, present and future students of the authors.

Contents

Preface

The authors have had as a long-term project the creation of examples using power series to analyze and distinguish several properties of commutative rings and their spectra. This monograph is our attempt to expose the results that have been obtained in this endeavor, to put these results in better perspective and to clarify their proofs. We hope in this way to assist current and future researchers in commutative algebra in utilizing the techniques described here.

William Heinzer, Christel Rotthaus, Sylvia Wiegand

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