

Mathematical
Surveys
and
Monographs
Volume 260

Hopf Algebras and Galois Module Theory

Lindsay N. Childs
Cornelius Greither
Kevin P. Keating
Alan Koch
Timothy Kohl
Paul J. Truman
Robert G. Underwood



Hopf Algebras and Galois Module Theory

Mathematical
Surveys
and
Monographs

Volume 260

Hopf Algebras and Galois Module Theory

Lindsay N. Childs
Cornelius Greither
Kevin P. Keating
Alan Koch
Timothy Kohl
Paul J. Truman
Robert G. Underwood



Providence, Rhode Island

EDITORIAL COMMITTEE

Ana Caraiani
Robert Guralnick, Chair
Bryna Kra
Natasa Sesum
Constantin Teleman
Anna-Karin Tornberg

2020 *Mathematics Subject Classification*. Primary 16T05, 11R33;
Secondary 11S15, 12F10, 20B35, 16T25.

For additional information and updates on this book, visit
www.ams.org/bookpages/surv-260

Library of Congress Cataloging-in-Publication Data

Names: Childs, Lindsay, author.

Title: Hopf algebras and Galois module theory / Lindsay N. Childs, Cornelius Greither, Kevin P. Keating, Alan Koch, Timothy Kohl, Paul J. Truman, Robert G. Underwood.

Description: Providence, Rhode Island: American Mathematical Society, [2021] | Series: Mathematical surveys and monographs, 0076-5376; Volume 260 | Includes bibliographical references and index.

Identifiers: LCCN 2021021477 | ISBN 9781470465162 (paperback) | 9781470467371 (ebook)

Subjects: LCSH: Hopf algebras. | Galois modules (Algebra) | AMS: Associative rings and algebras – Hopf algebras, quantum groups and related topics – Hopf algebras and their applications. | Number theory – Algebraic number theory: global fields – Integral representations related to algebraic numbers; Galois module structure of rings of integers. | Field theory and polynomials – Field extensions – Separable extensions, Galois theory. | Group theory and generalizations – Permutation groups – Subgroups of symmetric groups. | Associative rings and algebras – Hopf algebras, quantum groups and related topics – Yang-Baxter equations.

Classification: LCC QA613.8 .C47 2021 | DDC 512/.55–dc23

LC record available at <https://lcn.loc.gov/2021021477>

Copying and reprinting. Individual readers of this publication, and nonprofit libraries acting for them, are permitted to make fair use of the material, such as to copy select pages for use in teaching or research. Permission is granted to quote brief passages from this publication in reviews, provided the customary acknowledgment of the source is given.

Republication, systematic copying, or multiple reproduction of any material in this publication is permitted only under license from the American Mathematical Society. Requests for permission to reuse portions of AMS publication content are handled by the Copyright Clearance Center. For more information, please visit www.ams.org/publications/pubpermissions.

Send requests for translation rights and licensed reprints to reprint-permission@ams.org.

© 2021 by the American Mathematical Society. All rights reserved.

The American Mathematical Society retains all rights
except those granted to the United States Government.
Printed in the United States of America.

∞ The paper used in this book is acid-free and falls within the guidelines
established to ensure permanence and durability.

Visit the AMS home page at <https://www.ams.org/>

10 9 8 7 6 5 4 3 2 1 26 25 24 23 22 21

Contents

Chapter 1. Introduction: What is this book about?	1
1.1. Background: Hopf-Galois theory and Galois module theory	1
1.2. Hopf-Galois structures since 2000	4
1.3. Galois module theory since 2000	6
1.4. What's not in this book	9
Acknowledgments	10
Part I: Hopf-Galois Extensions	11
Chapter 2. Hopf-Galois structures on Galois extensions of fields, regular subgroups, and skew braces	13
2.1. Introduction	13
2.2. Greither-Pareigis theory	14
2.3. Byott translation theory	16
2.4. Actions by the left regular representations	18
2.5. Counting	19
2.6. Working with regular subgroups of $\text{Hol}(N)$	19
2.7. Radical algebras	22
2.8. Skew (left) braces	23
2.9. Connecting skew braces with Hopf-Galois structures	24
2.10. Isomorphic skew braces	25
Chapter 3. (Non)-existence results on Hopf-Galois structures	27
3.1. Introduction	27
3.2. p -groups, p an odd prime, cyclic case	28
3.3. 2-groups	30
3.4. Groups of composite order n that decompose	31
3.5. Cases where G must be isomorphic to N	33
3.6. Realizability when $G = S_n$ or A_n	35
3.7. Cases where given G , N can be any group with $ G = N $	35
3.8. If G is abelian or nilpotent, then N is ...?	36
3.9. Kohl's non-existence theorem	37
3.10. The case G metabelian and radical algebras	38
3.11. Other realizability results	39
Chapter 4. Hopf-Galois structures arising from fixed point free pairs of homomorphisms	41
4.1. Introduction	41
4.2. Fixed point free pairs of homomorphisms	42
4.3. The case $N = G$	43

4.4.	The action of $L[N]^G$ on L	46
4.5.	Fixed point free endomorphisms	48
4.6.	Examples with $N \neq G$	50
4.7.	Bi-skew braces and semidirect products	54
4.8.	Bi-skew braces and nilpotent rings	56
Chapter 5.	Quantitative results	59
5.1.	Introduction	59
5.2.	Regular subgroups and nilpotent algebras	60
5.3.	Elementary abelian p -groups	61
5.4.	Almost trivial algebras	62
5.5.	Asymptotic results on $e(C_p^n, C_p^n)$ for large n	65
5.6.	Other counting results	66
Chapter 6.	Enumeration of Hopf-Galois structures on Galois extensions of degree mp	69
6.1.	Introduction	69
6.2.	Preliminaries	70
6.3.	Twisted wreath products	71
6.4.	Enumeration within S_{mp}	73
6.5.	Block systems and Hopf-Galois structures	78
Chapter 7.	On the Galois correspondence for Hopf-Galois structures	83
7.1.	Introduction	83
7.2.	K -Hopf subalgebras	85
7.3.	On the Galois correspondence for Hopf-Galois structures	87
7.4.	Kohl's application of Corollary 7.6	89
7.5.	A skew brace setting	90
7.6.	Fixed point free pairs	92
7.7.	Radical algebras and the Galois correspondence	95
7.8.	Commutative examples	97
7.9.	Elementary abelian p -groups	98
7.10.	Non-normal Hopf-Galois structures	99
Chapter 8.	Normality in Hopf-Galois extensions	103
8.1.	Normality for Hopf-Galois structures on Galois extensions	104
8.2.	Skew braces and normality in Hopf-Galois extensions	108
8.3.	Induced Hopf-Galois structures	111
Chapter 9.	Descent theory, and the structure of Hopf algebras acting on separable field extensions	117
9.1.	General descent theory	118
9.2.	Galois descent for Hopf algebras	126
9.3.	Absolutely semisimple forms	142
Chapter 10.	Hopf-Galois actions on purely inseparable extensions	145
10.1.	A little bit of algebraic geometry	145
10.2.	A short history	152
10.3.	Hopf-Galois structures on modular extensions	159

Part II: Hopf-Galois Module Theory	163
Chapter 11. Hopf-Galois module theory	165
11.1. The Normal Basis Theorem for Hopf-Galois structures	166
11.2. Hopf orders and Childs' theorem	169
11.3. Associated orders for opposite Hopf-Galois structures	174
11.4. Subextension techniques	175
11.5. Tamely ramified extensions	179
11.6. Extensions of number fields	183
Chapter 12. Hopf orders in group rings	189
12.1. Hopf orders and Galois module theory	189
12.2. Dual Hopf orders	191
12.3. Byott's theorem on realizability	195
12.4. Group valuations and Larson orders	197
12.5. Hopf orders in $K[G]$, $G = C_p$	204
12.6. Hopf orders in $K[G]$, $G = C_p \times C_p$, $G = C_{p^2}$	210
12.7. Hopf orders in $K[G]$, $G = C_{p^3}$	213
12.8. General constructions in the cases $G = C_{p^n}$, $G = C_p^n$	215
12.9. Truncated exponential Hopf orders	216
12.10. Models of $\mu_{p^n, K}$ for $n = 1, 2, 3$	218
12.11. Hopf orders and realizability	219
12.12. Realizable Hopf orders in $K[C_{p^n}]$, $K[C_p^n]$	224
12.13. When K has characteristic p	225
Chapter 13. Ramification theory for separable extensions of local fields	237
13.1. Basic theory	237
13.2. Power series and Herbrand's theorem	240
13.3. Properties of lower ramification breaks	247
Chapter 14. Stable and semistable Hopf-Galois extensions	255
14.1. Bondarko's map φ for Hopf-Galois extensions	255
14.2. Some technical lemmas	261
14.3. Diagrams of elements of $L \otimes_K L$	266
14.4. H -stable and H -semistable extensions	271
14.5. Hopf-Galois module structure	273
14.6. A non-classical example	280
Chapter 15. Hopf-Galois scaffolds	285
15.1. H -scaffolds	285
15.2. Examples of H -scaffolds	286
15.3. Some basic properties of H -scaffolds	289
15.4. H -semistable extensions and H -scaffolds	291
15.5. Ramified extensions of degree p	293
Bibliography	297
Index	309

Bibliography

- [AB18] Ali A. Alabdali and Nigel P. Byott, *Counting Hopf-Galois structures on cyclic field extensions of squarefree degree*, J. Algebra **493** (2018), 1–19, DOI 10.1016/j.jalgebra.2017.09.009. MR3715201 ↑35, 53, 66, 92
- [AB20a] E. Acri and M. Bonatto, *Skew braces of size pq* , Comm. Algebra **48** (2020), no. 5, 1872–1881, DOI 10.1080/00927872.2019.1709480. MR4085764 ↑110
- [AB20b] Ali A. Alabdali and Nigel P. Byott, *Hopf-Galois structures of squarefree degree*, J. Algebra **559** (2020), 58–86, DOI 10.1016/j.jalgebra.2020.04.019. MR4093704 ↑35, 53, 66
- [AB20c] A. A. Alabdali and N. P. Byott, *Skew braces of squarefree order*, J. Algebra Appl. (2020), 2150128. ↑53
- [Abh64] Shreeram Shankar Abhyankar, *Local analytic geometry*, Pure and Applied Mathematics, Vol. XIV, Academic Press, New York-London, 1964. MR0175897 ↑240
- [Art42] E. Artin, *Galois Theory - Notre Dame mathematical lectures no. 2*, Field theory: Lectures delivered at the university of Notre Dame, 1942. ↑1
- [AW73] J. C. Ault and J. F. Watters, *Circle groups of nilpotent rings*, Amer. Math. Monthly **80** (1973), 48–52, DOI 10.2307/2319260. MR316493 ↑38
- [Bac15] David Bachiller, *Classification of braces of order p^3* , J. Pure Appl. Algebra **219** (2015), no. 8, 3568–3603, DOI 10.1016/j.jpaa.2014.12.013. MR3320237 ↑29, 30, 67
- [Bac16] David Bachiller, *Counterexample to a conjecture about braces*, J. Algebra **453** (2016), 160–176, DOI 10.1016/j.jalgebra.2016.01.011. MR3465351 ↑14, 28, 29, 30, 38
- [Bat18] Giulia Battiston, *A theory of Galois descent for finite inseparable extensions*, Proc. Amer. Math. Soc. **146** (2018), no. 1, 69–83, DOI 10.1090/proc/13713. MR3723121 ↑157
- [BB99] Werner Bley and Robert Boltje, *Lubin-Tate formal groups and module structure over Hopf orders* (English, with English and French summaries), J. Théor. Nombres Bordeaux **11** (1999), no. 2, 269–305. MR1745880 ↑181
- [BC12] Nigel P. Byott and Lindsay N. Childs, *Fixed-point free pairs of homomorphisms and nonabelian Hopf-Galois structures*, New York J. Math. **18** (2012), 707–731. MR2991421 ↑34, 42, 50, 52, 53, 113
- [BCE18] Nigel P. Byott, Lindsay N. Childs, and G. Griffith Elder, *Scaffolds and generalized integral Galois module structure* (English, with English and French summaries), Ann. Inst. Fourier (Grenoble) **68** (2018), no. 3, 965–1010. MR3805766 ↑8, 9, 217, 277, 285, 286, 287, 288, 293
- [BE05] Nigel P. Byott and G. Griffith Elder, *New ramification breaks and additive Galois structure* (English, with English and French summaries), J. Théor. Nombres Bordeaux **17** (2005), no. 1, 87–107. MR2152213 ↑233
- [BE07] Nigel P. Byott and G. Griffith Elder, *A valuation criterion for normal bases in elementary abelian extensions*, Bull. Lond. Math. Soc. **39** (2007), no. 5, 705–708, DOI 10.1112/blms/bdm036. MR2365217 ↑168
- [BE13] Nigel P. Byott and G. Griffith Elder, *Galois scaffolds and Galois module structure in extensions of characteristic p local fields of degree p^2* , J. Number Theory **133** (2013), no. 11, 3598–3610, DOI 10.1016/j.jnt.2013.04.021. MR3084290 ↑174
- [BE14] Nigel P. Byott and G. Griffith Elder, *Integral Galois module structure for elementary abelian extensions with a Galois scaffold*, Proc. Amer. Math. Soc. **142** (2014), no. 11, 3705–3712, DOI 10.1090/S0002-9939-2014-12126-5. MR3251712 ↑174

- [BE18] Nigel P. Byott and G. Griffith Elder, *Sufficient conditions for large Galois scaffolds*, *J. Number Theory* **182** (2018), 95–130, DOI 10.1016/j.jnt.2017.06.004. MR3703934 ↑8, 9, 216, 217, 234
- [Ber79] Françoise Bertrandias, *Sur les extensions cycliques de degré p^n d'un corps local* (French), *Acta Arith.* **34** (1979), no. 4, 361–377, DOI 10.4064/aa-34-4-361-377. MR543208 ↑173
- [BF72] Françoise Bertrandias and Marie-Josée Ferton, *Sur l'anneau des entiers d'une extension cyclique de degré premier d'un corps local* (French), *C. R. Acad. Sci. Paris Sér. A-B* **274** (1972), A1330–A1333. MR296047 ↑288
- [BH36] Garrett Birkhoff and Philip Hall, *On the order of groups of automorphisms*, *Trans. Amer. Math. Soc.* **39** (1936), no. 3, 496–499, DOI 10.2307/1989764. MR1501860 ↑78
- [BJ08] Werner Bley and Henri Johnston, *Computing generators of free modules over orders in group algebras*, *J. Algebra* **320** (2008), no. 2, 836–852, DOI 10.1016/j.jalgebra.2008.01.042. MR2422318 ↑186
- [BL96] Nigel P. Byott and Günter Lettl, *Relative Galois module structure of integers of abelian fields* (English, with English and French summaries), *J. Théor. Nombres Bordeaux* **8** (1996), no. 1, 125–141. MR1399950 ↑176, 178
- [Bon00] M. V. Bondarko, *Local Leopoldt's problem for rings of integers in abelian p -extensions of complete discrete valuation fields*, *Doc. Math.* **5** (2000), 657–693. MR1808921 ↑7, 9, 217, 255, 256, 294
- [Bon02] M. V. Bondarko, *Local Leopoldt's problem for ideals in totally ramified p -extensions of complete discrete valuation fields*, *Algebraic number theory and algebraic geometry*, *Contemp. Math.*, vol. 300, Amer. Math. Soc., Providence, RI, 2002, pp. 27–57, DOI 10.1090/conm/300/05142. MR1936366 ↑8, 9, 255, 256, 266, 269, 271, 294
- [Bon06] M. V. Bondarko, *The Leopoldt problem for totally ramified abelian extensions of complete discrete valuation fields* (Russian, with Russian summary), *Algebra i Analiz* **18** (2006), no. 5, 99–129, DOI 10.1090/S1061-0022-07-00972-7; English transl., *St. Petersburg Math. J.* **18** (2007), no. 5, 757–778. MR2301042 ↑8, 9, 255
- [Bur11] W. Burnside, *Theory of groups of finite order*, Cambridge University Press, 1911. ↑80
- [Byo00] N. P. Byott, *Galois module theory and Kummer theory for Lubin-Tate formal groups*, *Algebraic number theory and diophantine analysis (proceedings of conference in graz, 1998)*, 2000, pp. 55–67. ↑4, 165, 171, 173
- [Byo02] Nigel P. Byott, *Integral Hopf-Galois structures on degree p^2 extensions of p -adic fields*, *J. Algebra* **248** (2002), no. 1, 334–365, DOI 10.1006/jabr.2001.9053. MR1879021 ↑4, 6, 7, 66, 108, 172, 173, 184
- [Byo04a] Nigel P. Byott, *Hopf-Galois structures on field extensions with simple Galois groups*, *Bull. London Math. Soc.* **36** (2004), no. 1, 23–29, DOI 10.1112/S0024609303002595. MR2011974 ↑16, 33, 45
- [Byo04b] Nigel P. Byott, *Hopf-Galois structures on Galois field extensions of degree pq* , *J. Pure Appl. Algebra* **188** (2004), no. 1-3, 45–57, DOI 10.1016/j.jpaa.2003.10.010. MR2030805 ↑35, 36, 59, 101, 107, 113, 140
- [Byo04c] Nigel P. Byott, *Monogenic Hopf orders and associated orders of valuation rings*, *J. Algebra* **275** (2004), no. 2, 575–599, DOI 10.1016/j.jalgebra.2003.07.003. MR2052627 ↑76, 195, 196
- [Byo07] Nigel P. Byott, *Hopf-Galois structures on almost cyclic field extensions of 2-power degree*, *J. Algebra* **318** (2007), no. 1, 351–371, DOI 10.1016/j.jalgebra.2007.04.010. MR2363137 ↑28, 30, 32, 59
- [Byo08] Nigel P. Byott, *On the integral Galois module structure of cyclic extensions of p -adic fields*, *Q. J. Math.* **59** (2008), no. 2, 149–162, DOI 10.1093/qmath/ham037. MR2428073 ↑173
- [Byo11] Nigel P. Byott, *A valuation criterion for normal basis generators of Hopf-Galois extensions in characteristic p* (English, with English and French summaries), *J. Théor. Nombres Bordeaux* **23** (2011), no. 1, 59–70. MR2780619 ↑169, 256
- [Byo13] Nigel P. Byott, *Nilpotent and abelian Hopf-Galois structures on field extensions*, *J. Algebra* **381** (2013), 131–139, DOI 10.1016/j.jalgebra.2013.02.008. MR3030514 ↑14, 31, 34, 52

- [Byo15] Nigel P. Byott, *Solubility criteria for Hopf-Galois structures*, New York J. Math. **21** (2015), 883–903. MR3425626 ↑31, 36, 37, 53, 55, 94
- [Byo93] N. P. Byott, *Cleft extensions of Hopf algebras. II*, Proc. London Math. Soc. (3) **67** (1993), no. 2, 277–304, DOI 10.1112/plms/s3-67.2.277. MR1226603 ↑172, 212
- [Byo96] N. P. Byott, *Corrigendum: “Uniqueness of Hopf Galois structure for separable field extensions”*, Comm. Algebra **24** (1996), no. 11, 3705, DOI 10.1080/00927879608825779. MR1405283 ↑4, 13, 16, 27, 33, 52, 66, 69, 113
- [Byo97] Nigel P. Byott, *Galois structure of ideals in wildly ramified abelian p -extensions of a p -adic field, and some applications* (English, with English and French summaries), J. Théor. Nombres Bordeaux **9** (1997), no. 1, 201–219. MR1469668 ↑7
- [Byo99] Nigel P. Byott, *Integral Galois module structure of some Lubin-Tate extensions*, J. Number Theory **77** (1999), no. 2, 252–273, DOI 10.1006/jnth.1999.2385. MR1702149 ↑6, 7
- [Car13] A. Caranti, *Quasi-inverse endomorphisms*, J. Group Theory **16** (2013), no. 5, 779–792, DOI 10.1515/jgt-2013-0012. MR3101012 ↑47, 48, 49, 50
- [Car20] A. Caranti, *Bi-skew braces and regular subgroups of the holomorph*, 2020. ↑56
- [CC07] Lindsay N. Childs and Jesse Corradino, *Cayley’s theorem and Hopf Galois structures for semidirect products of cyclic groups*, J. Algebra **308** (2007), no. 1, 236–251, DOI 10.1016/j.jalgebra.2006.09.016. MR2290920 ↑44, 49
- [CC99] Scott Carnahan and Lindsay Childs, *Counting Hopf Galois structures on non-abelian Galois field extensions*, J. Algebra **218** (1999), no. 1, 81–92, DOI 10.1006/jabr.1999.7861. MR1704676 ↑34, 35, 45, 48
- [CCDC20] E. Campedel, A. Caranti, and I. Del Corso, *Hopf-Galois structures on extensions of degree p^2q and skew braces of order p^2q : the cyclic Sylow p -subgroup case*, J. Algebra **556** (2020), 1165–1210, DOI 10.1016/j.jalgebra.2020.04.009. MR4089566 ↑110
- [CDVS06] A. Caranti, F. Dalla Volta, and M. Sala, *Abelian regular subgroups of the affine group and radical rings*, Publ. Math. Debrecen **69** (2006), no. 3, 297–308. MR2273982 ↑4, 13, 22, 25, 28, 61, 64, 97
- [CF67] J. W. S. Cassels and A. Fröhlich (eds.), *Algebraic number theory*, Academic Press, London, 1967. ↑196
- [CG18] Lindsay N. Childs and Cornelius Greither, *Bounds on the number of ideals in finite commutative nilpotent \mathbb{F}_p -algebras*, Publ. Math. Debrecen **92** (2018), no. 3-4, 495–516, DOI 10.5486/pmd.2018.8081. MR3789700 ↑92, 98, 99
- [CH86] Lindsay N. Childs and Susan Hurley, *Tameness and local normal bases for objects of finite Hopf algebras*, Trans. Amer. Math. Soc. **298** (1986), no. 2, 763–778, DOI 10.2307/2000648. MR860392 ↑3, 6, 170
- [Cha72] Stephen U. Chase, *On the automorphism scheme of a purely inseparable field extension*, Ring theory (Proc. Conf., Park City, Utah, 1971), Academic Press, New York, 1972, pp. 75–106. MR0354629 ↑157
- [Cha76] Stephen U. Chase, *Infinitesimal group scheme actions on finite field extensions*, Amer. J. Math. **98** (1976), no. 2, 441–480, DOI 10.2307/2373897. MR424773 ↑157, 158, 159, 160
- [Chi00] Lindsay N. Childs, *Taming wild extensions: Hopf algebras and local Galois module theory*, Mathematical Surveys and Monographs, vol. 80, American Mathematical Society, Providence, RI, 2000, DOI 10.1090/surv/080. MR1767499 ↑4, 6, 7, 9, 14, 16, 28, 45, 48, 59, 84, 86, 87, 99, 105, 145, 165, 166, 168, 169, 170, 171, 183, 189, 190, 194, 197, 199, 202, 203, 205, 206, 207, 232, 257, 258
- [Chi03] Lindsay N. Childs, *On Hopf Galois structures and complete groups*, New York J. Math. **9** (2003), 99–115. MR2016184 ↑35, 50, 53, 77
- [Chi05] Lindsay N. Childs, *Elementary abelian Hopf Galois structures and polynomial formal groups*, J. Algebra **283** (2005), no. 1, 292–316, DOI 10.1016/j.jalgebra.2004.07.009. MR2102084 ↑20, 21, 22, 61, 62
- [Chi07] Lindsay N. Childs, *Some Hopf Galois structures arising from elementary abelian p -groups*, Proc. Amer. Math. Soc. **135** (2007), no. 11, 3453–3460, DOI 10.1090/S0002-9939-07-08888-0. MR2336557 ↑29, 62, 65, 98
- [Chi11] Lindsay N. Childs, *Hopf Galois structures on Kummer extensions of prime power degree*, New York J. Math. **17** (2011), 51–74. MR2781908 ↑49

- [Chi13] Lindsay N. Childs, *Fixed-point free endomorphisms and Hopf Galois structures*, Proc. Amer. Math. Soc. **141** (2013), no. 4, 1255–1265, DOI 10.1090/S0002-9939-2012-11418-2. MR3008873 ↑49, 160
- [Chi15] Lindsay N. Childs, *On abelian Hopf Galois structures and finite commutative nilpotent rings*, New York J. Math. **21** (2015), 205–229. MR3336553 ↑65, 68, 98
- [Chi17] Lindsay N. Childs, *On the Galois correspondence for Hopf Galois structures*, New York J. Math. **23** (2017), 1–10. MR3611070 ↑90, 92, 97, 98
- [Chi18] Lindsay N. Childs, *Skew braces and the Galois correspondence for Hopf Galois structures*, J. Algebra **511** (2018), 270–291, DOI 10.1016/j.jalgebra.2018.06.023. MR3834774 ↑91, 92, 93, 94, 96
- [Chi19a] Lindsay N. Childs, *Abelian Hopf Galois structures from almost trivial commutative nilpotent algebras*, New York J. Math. **25** (2019), 1421–1437. MR4044375 ↑62, 65
- [Chi19b] Lindsay N. Childs, *Bi-skew braces and Hopf Galois structures*, New York J. Math. **25** (2019), 574–588. MR3982254 ↑54, 55, 56
- [Chi21] Lindsay N. Childs, *On the Galois correspondence for Hopf Galois structures arising from finite radical algebras and Zappa-Szép products*, Publ. Mat. **65** (2021), no. 1, 141–163, DOI 10.5565/PUBLMAT6512105. MR4185830 ↑92, 94, 95, 96
- [Chi87] Lindsay N. Childs, *Taming wild extensions with Hopf algebras*, Trans. Amer. Math. Soc. **304** (1987), no. 1, 111–140, DOI 10.2307/2000707. MR906809 ↑6, 171, 220
- [Chi89] Lindsay N. Childs, *On the Hopf Galois theory for separable field extensions*, Comm. Algebra **17** (1989), no. 4, 809–825, DOI 10.1080/00927878908823760. MR990979 ↑13, 15, 100, 108, 113, 186, 293
- [Chi96] Lindsay N. Childs, *Hopf Galois structures on degree p^2 cyclic extensions of local fields*, New York J. Math. **2** (1996), 86–102. MR1420597 ↑6, 172
- [CHR65] S. U. Chase, D. K. Harrison, and Alex Rosenberg, *Galois theory and Galois cohomology of commutative rings*, Mem. Amer. Math. Soc. **52** (1965), 15–33. MR195922 ↑122, 171, 180
- [Cla58] L. E. Clarke, *On Cayley’s formula for counting trees*, J. London Math. Soc. **33** (1958), 471–474, DOI 10.1112/jlms/s1-33.4.471. MR100854 ↑45
- [CM94] L. N. Childs and D. J. Moss, *Hopf algebras and local Galois module theory*, Advances in Hopf algebras, 1994, pp. 1–24. ↑3
- [CR90] Charles W. Curtis and Irving Reiner, *Methods of representation theory. Vol. I*, Wiley Classics Library, John Wiley & Sons, Inc., New York, 1990. With applications to finite groups and orders; Reprint of the 1981 original; A Wiley-Interscience Publication. MR1038525 ↑181
- [CRV15a] T. Crespo, A. Rio, and M. Vela, *From Galois to Hopf Galois: theory and practice*, Trends in number theory, Contemp. Math., vol. 649, Amer. Math. Soc., Providence, RI, 2015, pp. 29–46, DOI 10.1090/conm/649/13018. MR3415265 ↑97, 101
- [CRV15b] Teresa Crespo, Anna Rio, and Montserrat Vela, *Non-isomorphic Hopf Galois structures with isomorphic underlying Hopf algebras*, J. Algebra **422** (2015), 270–276, DOI 10.1016/j.jalgebra.2014.07.038. MR3272077 ↑160
- [CRV16a] T. Crespo, A. Rio, and M. Vela, *Induced Hopf Galois structures*, J. Algebra **457** (2016), 312–322. ↑55
- [CRV16b] Teresa Crespo, Anna Rio, and Montserrat Vela, *On the Galois correspondence theorem in separable Hopf Galois theory*, Publ. Mat. **60** (2016), no. 1, 221–234. MR3447739 ↑85, 100, 101, 111, 113
- [CRV18] T. Crespo, A. Rio, and M. Vela, *Hopf Galois structures on symmetric and alternating extensions*, New York J. Math. **24** (2018), 451–457. ↑35, 108, 113
- [CS05] Lindsay N. Childs and Harold H. Smith III, *Dual Hopf orders in group rings of elementary abelian p -groups*, J. Algebra **294** (2005), no. 2, 489–518, DOI 10.1016/j.jalgebra.2005.06.015. MR2183362 ↑216, 224
- [CS19] Teresa Crespo and Marta Salguero, *Hopf Galois structures on separable field extensions of odd prime power degree*, J. Algebra **519** (2019), 424–439, DOI 10.1016/j.jalgebra.2018.11.004. MR3880638 ↑28, 97
- [CS69] Stephen U. Chase and Moss E. Sweedler, *Hopf algebras and Galois theory*, Lecture Notes in Mathematics, Vol. 97, Springer-Verlag, Berlin-New York, 1969. MR0260724 ↑1, 2, 5, 9, 83, 84, 87, 103

- [CU03] Lindsay N. Childs and Robert G. Underwood, *Cyclic Hopf orders defined by isogenies of formal groups*, Amer. J. Math. **125** (2003), no. 6, 1295–1334. MR2018662 ↑212, 214, 215
- [CU04] Lindsay N. Childs and Robert G. Underwood, *Duals of formal group Hopf orders in cyclic groups*, Illinois J. Math. **48** (2004), no. 3, 923–940. MR2114259 ↑216
- [DG18] Willem A. De Graaf, *Classification of nilpotent associative algebras of small dimension*, Internat. J. Algebra Comput. **28** (2018), no. 1, 133–161, DOI 10.1142/S0218196718500078. MR3768261 ↑67, 96
- [Del84] P. Deligne, *Les corps locaux de caractéristique p , limites de corps locaux de caractéristique 0* (French), Representations of reductive groups over a local field, Travaux en Cours, Hermann, Paris, 1984, pp. 119–157. MR771673 ↑237
- [Dic05] Leonard Eugene Dickson, *Definitions of a group and a field by independent postulates*, Trans. Amer. Math. Soc. **6** (1905), no. 2, 198–204, DOI 10.2307/1986298. MR1500706 ↑36, 52
- [dJ93] A. J. de Jong, *Finite locally free group schemes in characteristic p and Dieudonné modules*, Invent. Math. **114** (1993), no. 1, 89–137, DOI 10.1007/BF01232664. MR1235021 ↑226
- [DM96] J. Dixon and B. Mortimer, *Permutation groups*, GTM, vol. 163, Springer, New York, 1996. ↑79
- [DCR13] Ilaria Del Corso and Lorenzo Paolo Rossi, *Normal integral bases and tameness conditions for Kummer extensions*, Acta Arith. **160** (2013), no. 1, 1–23, DOI 10.4064/aa160-1-1. MR3085149 ↑183
- [dSFT12] B. de Smit, M. Florence, and L. Thomas, *The valuation criterion for normal basis generators*, Bull. Lond. Math. Soc. **44** (2012), no. 4, 729–737, DOI 10.1112/blms/bds005. MR2967240 ↑168, 169
- [Eld09] G. Griffith Elder, *Galois scaffolding in one-dimensional elementary abelian extensions*, Proc. Amer. Math. Soc. **137** (2009), no. 4, 1193–1203, DOI 10.1090/S0002-9939-08-09710-4. MR2465640 ↑174
- [Eld10] G. Griffith Elder, *A valuation criterion for normal basis generators in local fields of characteristic p* , Arch. Math. (Basel) **94** (2010), no. 1, 43–47, DOI 10.1007/s00013-009-0076-6. MR2581332 ↑168, 169
- [Eld18] G. Griffith Elder, *Ramified extensions of degree p and their Hopf-Galois module structure* (English, with English and French summaries), J. Théor. Nombres Bordeaux **30** (2018), no. 1, 19–40. MR3809707 ↑293
- [ESS99] Pavel Etingof, Travis Schedler, and Alexandre Soloviev, *Set-theoretical solutions to the quantum Yang-Baxter equation*, Duke Math. J. **100** (1999), no. 2, 169–209, DOI 10.1215/S0012-7094-99-10007-X. MR1722951 ↑31
- [EU17] G. Griffith Elder and Robert G. Underwood, *Finite group scheme extensions, and Hopf orders in KC_p^2 over a characteristic p discrete valuation ring*, New York J. Math. **23** (2017), 11–39. MR3611071 ↑9, 218, 225, 227, 228, 229, 230, 231, 232, 233
- [FCC12] S. C. Featherstonhaugh, A. Caranti, and L. N. Childs, *Abelian Hopf Galois structures on prime-power Galois field extensions*, Trans. Amer. Math. Soc. **364** (2012), no. 7, 3675–3684, DOI 10.1090/S0002-9947-2012-05503-6. MR2901229 ↑4, 13, 14, 15, 28, 29
- [Fea03] S. C. Featherstonhaugh, *Hopf algebra structures on abelian Galois extensions of fields*, Ph.D. Thesis, 2003. ↑15, 28
- [Fer74] Marie-José Ferton, *Sur l’anneau des entiers d’extensions cycliques d’un corps local* (French), Journées Arithmétiques (Grenoble, 1973), Soc. Math. France, Paris, 1974, pp. 69–74. Bull. Soc. Math. France Mém., No. 37, DOI 10.24033/msmf.130. MR0374104 ↑171
- [Frö83] A. Fröhlich, *Galois module structure of algebraic integers*, Springer Verlag, 1983. ↑165, 169, 183
- [FT93] A. Fröhlich and M. J. Taylor, *Algebraic number theory*, Cambridge Studies in Advanced Mathematics, vol. 27, Cambridge University Press, Cambridge, 1993. MR1215934 ↑178
- [Fuj90] Genjiro Fujisaki, *An elementary construction of Galois quaternion extension*, Proc. Japan Acad. Ser. A Math. Sci. **66** (1990), no. 3, 80–83. MR1051598 ↑185

- [FV02] I. B. Fesenko and S. V. Vostokov, *Local fields and their extensions*, 2nd ed., Translations of Mathematical Monographs, vol. 121, American Mathematical Society, Providence, RI, 2002. With a foreword by I. R. Shafarevich, DOI 10.1090/mmono/121. MR1915966 ↑238
- [GA94] E. J. Gómez Ayala, *Bases normales d'entiers dans les extensions de Kummer de degré premier* (French, with English and French summaries), J. Théor. Nombres Bordeaux **6** (1994), no. 1, 95–116. MR1305289 ↑183, 186, 187
- [GC98] C. Greither and L. N. Childs, *p-elementary group schemes—constructions, and Raynaud's theory*, Hopf algebras, polynomial formal groups, and raynaud orders, 1998. ↑216
- [Ger68] Murray Gerstenhaber, *On modular field extensions*, J. Algebra **10** (1968), 478–484, DOI 10.1016/0021-8693(68)90073-2. MR231814 ↑154
- [GP87] Cornelius Greither and Bodo Pareigis, *Hopf Galois theory for separable field extensions*, J. Algebra **106** (1987), no. 1, 239–258, DOI 10.1016/0021-8693(87)90029-9. MR878476 ↑4, 13, 14, 26, 83, 86, 89, 100, 109, 114, 117, 126, 140, 166
- [GR21] D. Gil Muñoz and A. Rio, *On induced Hopf Galois structures and its local Hopf Galois modules*, Publ. Mat. (Barcelona) (2021), (to appear). ↑178
- [Gre92] C. Greither, *Extensions of finite group schemes, and Hopf Galois theory over a complete discrete valuation ring*, Math. Z. **210** (1992), no. 1, 37–67, DOI 10.1007/BF02571782. MR1161169 ↑6, 172, 195, 210, 221, 228
- [Gro79] Benedict H. Gross, *Ramification in p-adic Lie extensions*, Journées de Géométrie Algébrique de Rennes (Rennes, 1978), Astérisque, vol. 65, Soc. Math. France, Paris, 1979, pp. 81–102. MR563473 ↑242
- [GRRS99] Cornelius Greither, Daniel R. Replogle, Karl Rubin, and Anupam Srivastav, *Swan modules and Hilbert-Speiser number fields*, J. Number Theory **79** (1999), no. 1, 164–173, DOI 10.1006/jnth.1999.2425. MR1718724 ↑183
- [GV17] L. Guarnieri and L. Vendramin, *Skew braces and the Yang-Baxter equation*, Math. Comp. **86** (2017), no. 307, 2519–2534, DOI 10.1090/mcom/3161. MR3647970 ↑4, 9, 14, 23, 25
- [Hal59] Marshall Hall Jr., *The theory of groups*, The Macmillan Co., New York, N.Y., 1959. MR0103215 ↑38, 86
- [Hee71] Nickolas Heerema, *A Galois theory for inseparable field extensions*, Trans. Amer. Math. Soc. **154** (1971), 193–200, DOI 10.2307/1995437. MR269632 ↑156
- [Hei96] Volker Heiermann, *De nouveaux invariants numériques pour les extensions totalement ramifiées de corps locaux* (French, with French summary), J. Number Theory **59** (1996), no. 1, 159–202, DOI 10.1006/jnth.1996.0092. MR1399703 ↑241, 248
- [Hel90] Charles Helou, *Non-Galois ramification theory of local fields*, Algebra Berichte [Algebra Reports], vol. 64, Verlag Reinhard Fischer, Munich, 1990. MR1076620 ↑237
- [Hel91] Charles Helou, *On the ramification breaks*, Comm. Algebra **19** (1991), no. 8, 2267–2279, DOI 10.1080/00927879108824258. MR1123123 ↑237
- [Her61] I. N. Herstein, *Theory of rings*, Math Lecture Notes, Univ. of Chicago, 1961. ↑96
- [Hil65] David Hilbert, *Gesammelte Abhandlungen. Dritter Band. Analysis, Grundlagen der Mathematik, Physik, Verschiedenes, nebst einer Lebensgeschichte* (German), Chelsea Publishing Co., New York, 1965. MR0188048 ↑183
- [HP86] Rudolf Hagenmüller and Bodo Pareigis, *Hopf algebra forms of the multiplicative group and other groups*, Manuscripta Math. **55** (1986), no. 2, 121–136, DOI 10.1007/BF01168681. MR833240 ↑124
- [HS71] Peter John Hilton and Urs Stambach, *A course in homological algebra*, Springer-Verlag, New York-Berlin, 1971. Graduate Texts in Mathematics, Vol. 4. MR0346025 ↑211
- [Hyo87] Osamu Hyodo, *Wild ramification in the imperfect residue field case*, Galois representations and arithmetic algebraic geometry (Kyoto, 1985/Tokyo, 1986), Adv. Stud. Pure Math., vol. 12, North-Holland, Amsterdam, 1987, pp. 287–314, DOI 10.2969/aspm/01210287. MR948250 ↑241
- [Itô55] Noboru Itô, *Über das Produkt von zwei abelschen Gruppen* (German), Math. Z. **62** (1955), 400–401, DOI 10.1007/BF01180647. MR71426 ↑36
- [Jac37] Nathan Jacobson, *Abstract derivation and Lie algebras*, Trans. Amer. Math. Soc. **42** (1937), no. 2, 206–224, DOI 10.2307/1989656. MR1501922 ↑152

- [Jac44] N. Jacobson, *Galois theory of purely inseparable fields of exponent one*, Amer. J. Math. **66** (1944), 645–648, DOI 10.2307/2371772. MR11079 ↑152, 153
- [Jac62] Nathan Jacobson, *Lie algebras*, Interscience Tracts in Pure and Applied Mathematics, No. 10, Interscience Publishers (a division of John Wiley & Sons), New York-London, 1962. MR0143793 ↑152
- [Jac75] Nathan Jacobson, *Lectures in abstract algebra. III*, Springer-Verlag, New York-Heidelberg, 1975. Theory of fields and Galois theory; Second corrected printing; Graduate Texts in Mathematics, No. 32. MR0392906 ↑160
- [Joh15] Henri Johnston, *Explicit integral Galois module structure of weakly ramified extensions of local fields*, Proc. Amer. Math. Soc. **143** (2015), no. 12, 5059–5071, DOI 10.1090/proc/12634. MR3411126 ↑175
- [JY88] C. U. Jensen and N. Yui, *Quaternion extensions*, Algebraic geometry and commutative algebra, 1988, pp. 155–182. ↑185
- [Kea20] Kevin Keating, *Galois scaffolds and semistable extensions*, J. Number Theory **207** (2020), 110–121, DOI 10.1016/j.jnt.2019.07.002. MR4017940 ↑9, 291
- [Keg61] Otto H. Kegel, *Produkte nilpotenter Gruppen* (German), Arch. Math. (Basel) **12** (1961), 90–93, DOI 10.1007/BF01650529. MR133365 ↑36
- [Ker90] Ina Kersten, *Brauergruppen von Körpern* (German), Aspects of Mathematics, D6, Friedr. Vieweg & Sohn, Braunschweig, 1990. MR1137014 ↑132
- [KH75] H. F. Kreimer and N. Heerema, *Modularity vs. separability for field extensions*, Canadian J. Math. **27** (1975), no. 5, 1176–1182. ↑154
- [KKTU19a] Alan Koch, Timothy Kohl, Paul J. Truman, and Robert Underwood, *Normality and short exact sequences of Hopf-Galois structures*, Comm. Algebra **47** (2019), no. 5, 2086–2101, DOI 10.1080/00927872.2018.1529237. MR3977722 ↑103, 104, 111
- [KKTU19b] Alan Koch, Timothy Kohl, Paul J. Truman, and Robert Underwood, *Isomorphism problems for Hopf-Galois structures on separable field extensions*, J. Pure Appl. Algebra **223** (2019), no. 5, 2230–2245, DOI 10.1016/j.jpaa.2018.07.014. MR3906546 ↑118, 127, 138, 139, 140, 160
- [KKTU19c] A. Koch, T. Kohl, P. J. Truman, and R. Underwood, *The structure of Hopf algebras acting on dihedral extensions*, Advances in algebra, 2019, pp. 201–218. ↑118, 140, 141
- [KO74] Max-Albert Knus and Manuel Ojanguren, *Théorie de la descente et algèbres d’Azumaya* (French), Lecture Notes in Mathematics, Vol. 389, Springer-Verlag, Berlin-New York, 1974. MR0417149 ↑117, 119, 120, 122, 123
- [Koc14] Alan Koch, *Hopf Galois structures on primitive purely inseparable extensions*, New York J. Math. **20** (2014), 779–797. MR3262032 ↑160, 161
- [Koc17] Alan Koch, *Primitively generated Hopf orders in characteristic p* , Comm. Algebra **45** (2017), no. 6, 2673–2689, DOI 10.1080/00927872.2016.1233235. MR3594547 ↑225, 226, 227
- [Koc21a] A. Koch, *Abelian maps, bi-skew braces, and opposite pairs of Hopf-Galois structures*, Proc. Amer. Math. Soc. Series B (2021), (to appear). arXiv:2007.08967. ↑56, 57
- [Koc21b] A. Koch, *Abelian maps, brace blocks, and solutions to the Yang-Baxter equation*, 2021. arXiv:2102.06104. ↑57
- [Koh07] Timothy Kohl, *Groups of order $4p$, twisted wreath products and Hopf-Galois theory*, J. Algebra **314** (2007), no. 1, 42–74, DOI 10.1016/j.jalgebra.2007.04.001. MR2331752 ↑70, 72, 74, 75, 109, 113
- [Koh13] Timothy Kohl, *Regular permutation groups of order mp and Hopf Galois structures*, Algebra Number Theory **7** (2013), no. 9, 2203–2240, DOI 10.2140/ant.2013.7.2203. MR3152012 ↑35, 39, 53, 70, 72, 75, 76
- [Koh15] Timothy Kohl, *Multiple holomorphs of dihedral and quaternionic groups*, Comm. Algebra **43** (2015), no. 10, 4290–4304, DOI 10.1080/00927872.2014.943842. MR3366576 ↑82
- [Koh16] Timothy Kohl, *Hopf-Galois structures arising from groups with unique subgroup of order p* , Algebra Number Theory **10** (2016), no. 1, 37–59, DOI 10.2140/ant.2016.10.37. MR3463035 ↑70, 71, 72, 75, 77, 78
- [Koh19] Timothy Kohl, *Characteristic subgroup lattices and Hopf-Galois structures*, Internat. J. Algebra Comput. **29** (2019), no. 2, 391–405, DOI 10.1142/S0218196719500073. MR3934792 ↑35, 36, 84, 89, 90

- [Koh20] Timothy Kohl, *Enumerating dihedral Hopf-Galois structures acting on dihedral extensions*, J. Algebra **542** (2020), 93–115, DOI 10.1016/j.jalgebra.2019.08.040. MR4018326 ↑82
- [Koh98] Timothy Kohl, *Classification of the Hopf Galois structures on prime power radical extensions*, J. Algebra **207** (1998), no. 2, 525–546, DOI 10.1006/jabr.1998.7479. MR1644203 ↑15, 27, 28, 33, 59, 139
- [KP69] Robert L. Kruse and David T. Price, *Nilpotent rings*, Gordon and Breach Science Publishers, New York-London-Paris, 1969. MR0266956 ↑66
- [Kra37] M. Krasner, *Sur la primitivité des corps \mathfrak{F} -adiques*, Mathematica (Cluj) **13** (1937), 72–191. ↑237
- [Kru70] R. L. Kruse, *On the circle group of a nilpotent ring*, Amer. Math. Monthly **77** (1970), 168–170, DOI 10.2307/2317333. MR257130 ↑38, 39
- [KT20] Alan Koch and Paul J. Truman, *Opposite skew left braces and applications*, J. Algebra **546** (2020), 218–235, DOI 10.1016/j.jalgebra.2019.10.033. MR4033084 ↑91, 109, 110
- [Lam05] T. Y. Lam, *Introduction to quadratic forms over fields*, Springer Graduate Studies in Mathematics, vol. 67, Springer, 2005. ↑130
- [Lan65] Serge Lang, *Algebra*, Addison-Wesley Publishing Co., Inc., Reading, Mass., 1965. MR0197234 ↑191, 219
- [Lar67] Richard G. Larson, *Group rings over Dedekind domains*, J. Algebra **5** (1967), 358–361, DOI 10.1016/0021-8693(67)90045-2. MR209368 ↑193
- [Lar76] Richard Gustavus Larson, *Hopf algebra orders determined by group valuations*, J. Algebra **38** (1976), no. 2, 414–452, DOI 10.1016/0021-8693(76)90232-5. MR404413 ↑197, 199, 202
- [Leo59] Heinrich-Wolfgang Leopoldt, *Über die Hauptordnung der ganzen Elemente eines abelschen Zahlkörpers* (German), J. Reine Angew. Math. **201** (1959), 119–149, DOI 10.1515/crll.1959.201.119. MR108479 ↑3, 165
- [Li03] Cai Heng Li, *The finite primitive permutation groups containing an abelian regular subgroup*, Proc. London Math. Soc. (3) **87** (2003), no. 3, 725–747, DOI 10.1112/S0024611503014266. MR2005881 ↑37
- [Lub13] Jonathan Lubin, *Elementary analytic methods in higher ramification theory*, J. Number Theory **133** (2013), no. 3, 983–999, DOI 10.1016/j.jnt.2012.02.017. MR2997782 ↑237, 244
- [Mar71] Jacques Martinet, *Modules sur l’algèbre du groupe quaternionien* (French), Ann. Sci. École Norm. Sup. (4) **4** (1971), 399–408. MR291208 ↑183, 185
- [MBD61] G. A. Miller, H. F. Blichfeldt, and L. E. Dickson, *Theory and applications of finite groups*, Dover Publications, Inc., New York, 1961. MR0123600 ↑73
- [Miy07] Yoshimasa Miyata, *On Galois structure of the integers in elementary abelian extensions of local number fields*, J. Number Theory **125** (2007), no. 2, 442–458, DOI 10.1016/j.jnt.2006.12.005. MR2332598 ↑174
- [Miy98] Y. Miyata, *On the module structure of rings of integers in \mathfrak{p} -adic number fields over associated orders*, Math. Proc. Cambridge Philos. Soc. **123** (1998), no. 2, 199–212, DOI 10.1017/S0305004197002016. MR1490195 ↑173
- [MM84] M. Ram Murty and V. Kumar Murty, *On groups of squarefree order*, Math. Ann. **267** (1984), no. 3, 299–309, DOI 10.1007/BF01456092. MR738255 ↑35, 52
- [Mon93] Susan Montgomery, *Hopf algebras and their actions on rings*, CBMS Regional Conference Series in Mathematics, vol. 82, Published for the Conference Board of the Mathematical Sciences, Washington, DC; by the American Mathematical Society, Providence, RI, 1993, DOI 10.1090/cbms/082. MR1243637 ↑153
- [MRT13] A. Mézard, M. Romagny, and D. Tossici, *Models of group schemes of roots of unity* (English, with English and French summaries), Ann. Inst. Fourier (Grenoble) **63** (2013), no. 3, 1055–1135, DOI 10.5802/aif.2784. MR3137480 ↑218
- [Nas19] Timur Nasybullov, *Connections between properties of the additive and the multiplicative groups of a two-sided skew brace*, J. Algebra **540** (2019), 156–167, DOI 10.1016/j.jalgebra.2019.05.005. MR4003478 ↑37
- [Nej18] K. Nejabati Zenouz, *On Hopf-Galois structures and skew braces of order p^3* , Ph.D. Thesis, 2018. ↑60, 67, 111

- [NZ19] Kayvan Nejabati Zenouz, *Skew braces and Hopf-Galois structures of Heisenberg type*, *J. Algebra* **524** (2019), 187–225, DOI 10.1016/j.jalgebra.2019.01.012. MR3905210 ↑25, 55, 67, 136, 138
- [Noe32] Emmy Noether, *Normalbasis bei Körpern ohne höhere Verzweigung* (German), *J. Reine Angew. Math.* **167** (1932), 147–152, DOI 10.1515/crll.1932.167.147. MR1581331 ↑3, 165
- [NSW08] Jürgen Neukirch, Alexander Schmidt, and Kay Wingberg, *Cohomology of number fields*, 2nd ed., Grundlehren der Mathematischen Wissenschaften [Fundamental Principles of Mathematical Sciences], vol. 323, Springer-Verlag, Berlin, 2008, DOI 10.1007/978-3-540-37889-1. MR2392026 ↑132
- [Osi15] D. V. Osipov, *The discrete Heisenberg group and its automorphism group* (Russian), *Mat. Zametki* **98** (2015), no. 1, 152–155, DOI 10.4213/mzm10694; English transl., *Math. Notes* **98** (2015), no. 1-2, 185–188. MR3399166 ↑136
- [Par90] Bodo Pareigis, *Forms of Hopf algebras and Galois theory*, Topics in algebra, Part 1 (Warsaw, 1988), Banach Center Publ., vol. 26, PWN, Warsaw, 1990, pp. 75–93. MR1171227 ↑142
- [Poo08a] Bjorn Poonen, *Isomorphism types of commutative algebras of finite rank over an algebraically closed field*, Computational arithmetic geometry, *Contemp. Math.*, vol. 463, Amer. Math. Soc., Providence, RI, 2008, pp. 111–120, DOI 10.1090/conm/463/09050. MR2459993 ↑68
- [Poo08b] Bjorn Poonen, *The moduli space of commutative algebras of finite rank*, *J. Eur. Math. Soc. (JEMS)* **10** (2008), no. 3, 817–836, DOI 10.4171/JEMS/131. MR2421162 ↑66, 98
- [Ras71] Richard Rasala, *Inseparable splitting theory*, *Trans. Amer. Math. Soc.* **162** (1971), 411–448, DOI 10.2307/1995762. MR284421 ↑154
- [Ray74] Michel Raynaud, *Schémas en groupes de type (p, \dots, p)* (French), *Bull. Soc. Math. France* **102** (1974), 241–280. MR419467 ↑216
- [Ree55] D. Rees, *Valuations associated with a local ring. I*, *Proc. London Math. Soc. (3)* **5** (1955), 107–128, DOI 10.1112/plms/s3-5.1.107. MR67095 ↑270
- [Rei75] I. Reiner, *Maximal orders*, Academic Press [A subsidiary of Harcourt Brace Jovanovich, Publishers], London-New York, 1975. London Mathematical Society Monographs, No. 5. MR0393100 ↑181, 182
- [Rem32] Robert Remak, *Über Unterguppen direkter Produkte von drei Faktoren* (German), *J. Reine Angew. Math.* **166** (1932), 65–100, DOI 10.1515/crll.1932.166.65. MR1581299 ↑92
- [Rob93] Derek J. S. Robinson, *A course in the theory of groups*, Graduate Texts in Mathematics, vol. 80, Springer-Verlag, New York, 1993. MR1261639 ↑31
- [Rot17] Joseph J. Rotman, *Advanced modern algebra. Part 2*, 3rd ed., Graduate Studies in Mathematics, vol. 180, American Mathematical Society, Providence, RI, 2017. With a foreword by Bruce Reznick, DOI 10.1090/gsm/180. MR3677125 ↑211
- [Rum07a] Wolfgang Rump, *Braces, radical rings, and the quantum Yang-Baxter equation*, *J. Algebra* **307** (2007), no. 1, 153–170, DOI 10.1016/j.jalgebra.2006.03.040. MR2278047 ↑4, 9, 14, 26
- [Rum07b] Wolfgang Rump, *Classification of cyclic braces*, *J. Pure Appl. Algebra* **209** (2007), no. 3, 671–685, DOI 10.1016/j.jpaa.2006.07.001. MR2298848 ↑28, 30, 32, 33
- [Rum19] Wolfgang Rump, *Classification of cyclic braces, II*, *Trans. Amer. Math. Soc.* **372** (2019), no. 1, 305–328, DOI 10.1090/tran/7569. MR3968770 ↑32
- [Sch77] Hans-Jürgen Schneider, *Cartan matrix of liftable finite group schemes*, *Comm. Algebra* **5** (1977), no. 8, 795–819, DOI 10.1080/00927877708822195. MR439857 ↑170, 181
- [Ser62] Jean-Pierre Serre, *Corps locaux* (French), Publications de l’Institut de Mathématique de l’Université de Nancago, VIII, Actualités Sci. Indust., No. 1296. Hermann, Paris, 1962. MR0150130 ↑168, 237, 241
- [ST68] D. A. Suprunenko and R. I. Tyskevič, *Commutative matrices*, Academic Press, New York, 1968. ↑68
- [SV18] Agata Smoktunowicz and Leandro Vendramin, *On skew braces (with an appendix by N. Byott and L. Vendramin)*, *J. Comb. Algebra* **2** (2018), no. 1, 47–86, DOI 10.4171/JCA/2-1-3. MR3763907 ↑4, 9, 14, 25, 31, 51, 66, 108, 185

- [Swa60] Richard G. Swan, *Induced representations and projective modules*, Ann. of Math. (2) **71** (1960), 552–578, DOI 10.2307/1969944. MR138688 ↑170
- [Swe68] Moss Eisenberg Sweedler, *Structure of inseparable extensions*, Ann. of Math. (2) **87** (1968), 401–410, DOI 10.2307/1970711. MR223343 ↑154, 155, 156
- [Tay20] S. Taylor, *Hopf-Galois module structure of a class of tame quaternionic fields*, Ph.D. Thesis, 2020. ↑108, 170, 176, 185
- [Tay81] M. J. Taylor, *On Fröhlich’s conjecture for rings of integers of tame extensions*, Invent. Math. **63** (1981), no. 1, 41–79, DOI 10.1007/BF01389193. MR608528 ↑183
- [Tho08] Lara Thomas, *A valuation criterion for normal basis generators in equal positive characteristic*, J. Algebra **320** (2008), no. 10, 3811–3820, DOI 10.1016/j.jalgebra.2008.05.024. MR2457723 ↑168
- [TO70] John Tate and Frans Oort, *Group schemes of prime order*, Ann. Sci. École Norm. Sup. (4) **3** (1970), 1–21. MR265368 ↑6, 171, 205, 225, 227
- [Tos10] Dajano Tossici, *Models of $\mu_{p^2, K}$ over a discrete valuation ring*, J. Algebra **323** (2010), no. 7, 1908–1957, DOI 10.1016/j.jalgebra.2010.01.012. With an appendix by Xavier Caruso. MR2594655 ↑218
- [TQ20] Cindy Tsang and Chao Qin, *On the solvability of regular subgroups in the holomorph of a finite solvable group*, Internat. J. Algebra Comput. **30** (2020), no. 2, 253–265, DOI 10.1142/S0218196719500735. MR4077413 ↑36, 37
- [Tru11] Paul J. Truman, *Towards a generalisation of Noether’s theorem to nonclassical Hopf-Galois structures*, New York J. Math. **17** (2011), 799–810. MR2862153 ↑181, 183, 184
- [Tru12] Paul J. Truman, *Hopf-Galois module structure of tame biquadratic extensions* (English, with English and French summaries), J. Théor. Nombres Bordeaux **24** (2012), no. 1, 173–199. MR2914905 ↑170, 184
- [Tru13] Paul J. Truman, *Integral Hopf-Galois structures for tame extensions*, New York J. Math. **19** (2013), 647–655. MR3119101 ↑180
- [Tru16a] Paul J. Truman, *Canonical nonclassical Hopf-Galois module structure of non-abelian Galois extensions*, Comm. Algebra **44** (2016), no. 3, 1119–1130, DOI 10.1080/00927872.2014.999930. MR3463133 ↑109, 166, 174, 175
- [Tru16b] Paul J. Truman, *Hopf-Galois module structure of tame $C_p \times C_p$ extensions* (English, with English and French summaries), J. Théor. Nombres Bordeaux **28** (2016), no. 2, 557–582. MR3509724 ↑170, 184
- [Tru18a] Paul J. Truman, *Commutative Hopf-Galois module structure of tame extensions*, J. Algebra **503** (2018), 389–408, DOI 10.1016/j.jalgebra.2018.01.047. MR3780002 ↑115, 179, 180, 183
- [Tru18b] Paul J. Truman, *Commuting Hopf-Galois structures on a separable extension*, Comm. Algebra **46** (2018), no. 4, 1420–1427, DOI 10.1080/00927872.2017.1346107. MR3780516 ↑109, 166, 174, 181
- [Tru20] Paul J. Truman, *Hopf-Galois module structure of tamely ramified radical extensions of prime degree*, J. Pure Appl. Algebra **224** (2020), no. 5, 106231, 13, DOI 10.1016/j.jpaa.2019.106231. MR4046237 ↑186, 187
- [Tsa19a] Cindy Tsang, *Hopf-Galois structures of isomorphic-type on a non-abelian characteristically simple extension*, Proc. Amer. Math. Soc. **147** (2019), no. 12, 5093–5103, DOI 10.1090/proc/14627. MR4021072 ↑34, 45
- [Tsa19b] Cindy Tsang, *Hopf-Galois structures on a Galois S_n -extension*, J. Algebra **531** (2019), 349–360, DOI 10.1016/j.jalgebra.2019.05.006. MR3953015 ↑35
- [Tsa19c] Cindy Tsang, *Non-existence of Hopf-Galois structures and bijective crossed homomorphisms*, J. Pure Appl. Algebra **223** (2019), no. 7, 2804–2821, DOI 10.1016/j.jpaa.2018.09.016. MR3912948 ↑34, 45
- [Tsa20] Cindy Tsang, *Hopf-Galois structures on finite extensions with almost simple Galois group*, J. Number Theory **214** (2020), 286–311, DOI 10.1016/j.jnt.2020.04.003. MR4105712 ↑45
- [Tsa21] Cindy Tsang, *Hopf-Galois structures on finite extensions with quasisimple Galois group*, Bull. Lond. Math. Soc. **53** (2021), no. 1, 148–160, DOI 10.1112/blms.12407. MR4224519 ↑45

- [TT19] Stuart Taylor and Paul J. Truman, *The structure of Hopf algebras giving Hopf-Galois structures on quaternionic extensions*, New York J. Math. **25** (2019), 219–237. MR3933762 ↑118, 130, 131, 133, 185
- [TT20] S. Taylor and P. J. Truman, *On associated orders in separable Hopf-Galois extensions*, 2020. ↑176, 178
- [UC06] Robert G. Underwood and Lindsay N. Childs, *Duality for Hopf orders*, Trans. Amer. Math. Soc. **358** (2006), no. 3, 1117–1163, DOI 10.1090/S0002-9947-05-03728-1. MR2187648 ↑212, 214, 215, 221, 222, 223, 224
- [Und08] Robert G. Underwood, *Realizable Hopf orders in KC_{p^3}* , J. Algebra **319** (2008), no. 11, 4426–4455, DOI 10.1016/j.jalgebra.2008.02.029. MR2416729 ↑7, 224
- [Und11] R. Underwood, *An introduction to Hopf algebras*, Springer, New York, 2011. ↑190, 191, 192, 193, 195, 197, 204, 209, 210, 212, 219, 220
- [Und15] Robert G. Underwood, *Fundamentals of Hopf algebras*, Universitext, Springer, Cham, 2015, DOI 10.1007/978-3-319-18991-8. MR3379140 ↑86, 189, 191, 192, 193, 223, 232
- [Und16] R. Underwood, *The structure of Hopf algebras acting on Galois extensions*, 2016. talk at the Hopf Galois Theory conference Omaha. ↑127
- [Und94] Robert G. Underwood, *R-Hopf algebra orders in KC_{p^2}* , J. Algebra **169** (1994), no. 2, 418–440, DOI 10.1006/jabr.1994.1293. MR1297158 ↑6, 172, 202, 212
- [Und96] Robert Underwood, *The valuative condition and R-Hopf algebra orders in KC_{p^3}* , Amer. J. Math. **118** (1996), no. 4, 701–743. MR1400057 ↑202, 214, 215
- [Und97] Robert Underwood, *The group of Galois extensions over orders in KC_{p^2}* , Trans. Amer. Math. Soc. **349** (1997), no. 4, 1503–1514, DOI 10.1090/S0002-9947-97-01914-4. MR1407713 ↑218
- [Ven19] Leandro Vendramin, *Problems on skew left braces*, Adv. Group Theory Appl. **7** (2019), 15–37, DOI 10.32037/agta-2019-003. MR3974481 ↑9, 30, 31, 36
- [VL15] Michael Vaughan-Lee, *Groups of order p^8 and exponent p* , Int. J. Group Theory **4** (2015), no. 4, 25–42. MR3416635 ↑30
- [Wat79] William C. Waterhouse, *Introduction to affine group schemes*, Graduate Texts in Mathematics, vol. 66, Springer-Verlag, New York-Berlin, 1979. MR547117 ↑145, 146, 151, 197, 207
- [Wil09] Robert A. Wilson, *The finite simple groups*, Graduate Texts in Mathematics, vol. 251, Springer-Verlag London, Ltd., London, 2009, DOI 10.1007/978-1-84800-988-2. MR2562037 ↑65
- [Win72] David L. Winter, *The automorphism group of an extraspecial p -group*, Rocky Mountain J. Math. **2** (1972), no. 2, 159–168, DOI 10.1216/RMJ-1972-2-2-159. MR297859 ↑136
- [Wit37] Ernst Witt, *Zyklische Körper und Algebren der Charakteristik p vom Grad p^n . Struktur diskret bewerteter perfekter Körper mit vollkommenem Restklassenkörper der Charakteristik p* (German), J. Reine Angew. Math. **176** (1937), 126–140, DOI 10.1515/crll.1937.176.126. MR1581526 ↑280

Index

- H -comodule algebra, 149
- H -module algebra, 2, 84
- H_λ , 86, 89
- \mathfrak{A}_H -tame extension, 170, 189
- \mathfrak{o} -stable subgroup, 90, 93, 94
- $e(G, N)$, 59
- $e_b(G, N)$, 59
- p -adic
 - integers, 3
 - numbers, 3

- Absolutely semisimple, 142
- Adjoint group, 14, 28
 - solvable, 31
- Affine group, 4, 13, 20, 61
- Amitsur cohomology, 121
- Antipode = coinverse, 2
- Associated order, 3, 165, 166

- Bachiller's Theorem, 29
- Baer product, 211
- Base change, 88
- Bi-skew brace, 5, 39, 94
- Bialgebra, 2
- Block system, 79
- Bondarko diagram, 266–270, 274, 290
- Brace, 4
 - additive group, 23
 - of order 4, 32
 - adjoint group, 23
 - bi-skew, *see* Bi-skew brace
 - bicyclic, 28, 32
 - circle group, 23
 - cyclic, 28, 32
 - of order p^3 , 29
 - of order p^n , 67
 - skew, *see* Skew brace
- Byott's
 - conjecture, 35, 37
 - counting formula, 19
 - simple groups theorem, 16, 33, 35
 - translation theorem, 4
 - uniqueness theorem, 27, 33, 52, 113

- Canonical non-classical Hopf algebra, 47
- Canonical non-classical Hopf-Galois structure, 44
- Caranti's Lemma, 65, 98
- Characteristic subgroup, 38, 89
- Childs' Theorem, 169
- Circle group, *see* Adjoint group
- Circle operation, 61
- Classical Hopf-Galois structure, 27, 44
- Coassociative, 2
- Cohomological Hopf order, 214
- Coinverse = antipode, 2
- Commutative local algebras, 68
- Comultiplication, 2
- Counit, 2
- Crossed product, 130

- Derivation, 152
 - field of constants, 152
 - higher derivation, 154
- Descent datum, 119
- Dieudonné modules, 226
- Discriminant
 - Hopf algebra, 190
- Duality Hopf order, 215

- Faithfully flat, 119
- Featherstonhaugh's Theorem, 15
- Field extension
 - almost classical, 83, 100
 - almost classically Galois, 114, 180
 - exponent, 154
 - modular, 154
 - primitive, 154
 - separable of degrees 2, 3, 4, 5, 100
- Fixed field of a Hopf subalgebra, 2, 103
- Fixed point free pair of homomorphisms, 35, 37, 42, 92
- Formal group Hopf order, 216
- Fundamental Theorem of Galois Theory,
 - see also* Galois correspondence, 103
 - for Hopf-Galois extensions, 5
 - strong form, 83
 - weak form, 83

- Galois cohomology, 125
- Galois correspondence, 1, 87
 - for Hopf-Galois extensions, 2
 - not surjective, 97
 - image of, 101
 - ratio, 83, 91, 97
 - strong form, 97
 - surjective, 97, 101
- Galois descent, 5
- Galois extension, 1
- Galois module theory
 - global, 9, 183
- Galois object, 145, 194
- Generalized binomial coefficient, 217
- Generalized Greither order, 215
- Generalized quaternion algebras, 130
- Global Field, 3
- Greither order, 212
 - generalized, 215
- Greither-Pareigis theory, 34
 - non-normal setting, 99
- Group
 - 2-nilpotent, 32
 - almost Sylow-cyclic, 32
 - order 16, 32
 - alternating, 35, 89, 90
 - alternating group, 35, 94
 - double cover, 34
 - central series, 38
 - characteristically simple, 33
 - classification of finite simple groups, 34
 - complementary subgroups, 93
 - cyclic of odd order, 59
 - cyclic of order 2^n , 30, 59
 - cyclic of order p^n , 28
 - dihedral, 30, 32, 59, 101
 - elementary abelian, 59
 - elementary abelian p -group, 61
 - exponent p , 30
 - Frobenius, 101
 - Hamiltonian, 86
 - Heisenberg, 29, 67
 - Mathieu group of degree 10, 35
 - metabelian, 36
 - and radical algebras, 38
 - metacyclic, 60, 107, 110
 - nilpotent, 31, 32, 36
 - class of, 38
 - non-abelian simple, 33
 - number of subgroups, 92
 - of order 2^n with a cyclic subgroup of index 2, 30
 - of order $p(p-1)$, p a safeprime, 35, 39
 - of order p^3q , 52
 - of order pq , 35, 59, 107, 110
 - of order 24, 90
 - of order 80, 32
 - of principal units, 62
 - orthogonal, 65
 - quaternion, 30, 32, 59, 108, 111
 - simple non-abelian, 89
 - solvable, 31, 36
 - squarefree order, 35, 39, 53, 55, 66, 95
 - Sylow-cyclic, 32
 - symmetric, 89
 - symmetric group, 35, 94
- Group ring
 - as a Hopf algebra, 2
- Group scheme
 - action, 148
 - affine, 147
- Group valuation, 197
 - order bounded, 199
 - p -adic, 199
- Grouplike elements, 14, 86

- Hall's Theorem, 31
- Hasse-Herbrand functions, 237, 239, 240, 242, 244
- Heisenberg group, 135
- Herbrand's Theorem, 246
- Higher derivation, 154
- Holomorph, 4, 13, 15
 - stabilizer of a regular subgroup in, 20
- Hopf algebra, 2
- Hopf comodule, 194
- Hopf order, 169, 189
 - cohomological, 214
 - duality, 215
 - formal group, 216
 - ILD, 214
 - largest Larson order, 202
 - Larson order, 200
 - realizable, 6, 9, 170, 190
 - triangular, 216
 - truncated exponential, 217, 225
- Hopf-Galois extension, 2, 85
 - almost classical, 100
 - not almost classical, 100
 - separable, non-normal, 99
- Hopf-Galois module theory, 6, 165
- Hopf-Galois structure, 2
 - canonical non-classical, 34, 44
 - classical, 44, 59
 - cyclic type, 34
 - induced, 111
 - non-realizability, 89
 - normality in, 5, 104
 - opposite, 109, 174
 - quotient, 103
 - type, 4, 13, 15

- ILD Hopf orders, 214
- Inner cocycle, 128
- Inner form, 128
- Integral, 190

- Jordan matrix, 62
- Kohl's Theorem, 15
- Larson order
 - one-parameter, 201
- Left regular representation, 13, 14, 23
- Leopoldt's Theorem, 3
- Liftable cocycle, 128
- Local field, 3
 - wildly ramified extensions of degree p^2 , 6, 171
- Model, 218
- Morita theory, 86
- Nilpotent
 - \mathbb{F}_p -algebra
 - almost trivial, 63
 - commutative, 97
 - commutative of dimension 5, 68
 - commutative of dimension 6, 68
 - isomorphic, 61
 - number of ideals, 99
 - primitive, 98
 - ring, 96
 - commutative, 61
- Nilradical, 195
- Noether's Theorem, 3, 165, 169
- Normal basis generator, 166
 - valuation criteria, 168
- Normal Basis Theorem, 1, 3
 - Hopf-Galois, 166
- One-parameter Larson order, 201
- Order, 165
- Outer form, 128
- Precision, 291
- Principal homogeneous space, 149
- Radical algebra, 13, 28, 38
- Radical algebra structures on an elementary abelian p -group, 4
- Radical ring, 96
 - commutative, 28
 - left ideal, 96
- Ramification
 - breaks, 7, 237–240, 246, 247, 251–253, 263, 271, 272, 279, 280, 286, 288, 292–295
 - ramified prime, 3
 - tame, 3, 179
 - unramified, 3
 - wild, 3
- Realizable pair of groups, 27, 37, 59
- Regular embedding, 20
 - equivalence, 20, 41
- Regular subgroup, 13, 14
 - equivalence, 61
- Scaffold, 277, 285–295
 - Galois, 8
- Scheme
 - affine, 145
 - affine group scheme, 147
 - geometric points, 146
 - morphism (natural transformation), 146
 - truncated, 157
 - truncated automorphism, 157
- Semidirect product, 30, 39, 93, 94
- Semiregular subgroup, 70
- Semistable
 - with precision, 291
- Semistable extension, 255, 271–274, 281, 282, 285, 291–293, 295
- Skew brace, 4, 14, 23, 90
 - \circ -quasi ideal, 109
 - \star -quasi ideal, 109
 - homomorphism, 23
 - ideal, 109
 - isomorphism of, 25
 - of order p^3 , 60
 - opposite, 91, 109
 - quasi-ideal, 91, 109
- Smash product, 85, 260
- Stabilizer (of a regular subgroup of the holomorph), 20
- Stable extension, 255, 271, 273–275, 277, 279, 281, 282, 285, 291–293, 295
- Sweedler notation, 84
- Tamely ramified, *see* Ramification, tame
- Teichmüller character, 205
- Teichmüller
 - map, 238
 - representatives, 238, 239, 248, 262, 266
- Torsor, 149
- Triangular Hopf order, 216
- Truncated
 - automorphism scheme, 157
 - polynomial algebra, 157
- Truncated exponential, 216, 232
- Truncated exponential Hopf order, 217, 225
- Twisted form, 120
- Wildly ramified, *see* Ramification, wild
- Wreath product, 71
- Zappa-Szép group, 92

SELECTED PUBLISHED TITLES IN THIS SERIES

- 260 **Lindsay N. Childs, Cornelius Greither, Kevin P. Keating, Alan Koch, Timothy Kohl, Paul J. Truman, and Robert G. Underwood**, Hopf Algebras and Galois Module Theory, 2021
- 258 **Pramod N. Achar**, Perverse Sheaves and Applications to Representation Theory, 2021
- 257 **Juha Kinnunen, Juha Lehrbäck, and Antti Vähäkangas**, Maximal Function Methods for Sobolev Spaces, 2021
- 256 **Michio Jimbo, Tetsuji Miwa, and Fedor Smirnov**, Local Operators in Integrable Models I, 2021
- 255 **Alexandre Boritchev and Sergei Kuksin**, One-Dimensional Turbulence and the Stochastic Burgers Equation, 2021
- 254 **Karim Belabas and Henri Cohen**, Numerical Algorithms for Number Theory, 2021
- 253 **Robert R. Bruner and John Rognes**, The Adams Spectral Sequence for Topological Modular Forms, 2021
- 252 **Julie Déserti**, The Cremona Group and Its Subgroups, 2021
- 251 **David Hoff**, Linear and Quasilinear Parabolic Systems, 2020
- 250 **Bachir Bekka and Pierre de la Harpe**, Unitary Representations of Groups, Duals, and Characters, 2020
- 249 **Nikolai M. Adrianov, Fedor Pakovich, and Alexander K. Zvonkin**, Davenport–Zannier Polynomials and Dessins d’Enfants, 2020
- 248 **Paul B. Larson and Jindřich Zapletal**, Geometric Set Theory, 2020
- 247 **István Heckenberger and Hans-Jürgen Schneider**, Hopf Algebras and Root Systems, 2020
- 246 **Matheus C. Bortolan, Alexandre N. Carvalho, and José A. Langa**, Attractors Under Autonomous and Non-autonomous Perturbations, 2020
- 245 **Aiping Wang and Anton Zettl**, Ordinary Differential Operators, 2019
- 244 **Nabile Boussaïd and Andrew Comech**, Nonlinear Dirac Equation, 2019
- 243 **José M. Isidro**, Jordan Triple Systems in Complex and Functional Analysis, 2019
- 242 **Bhargav Bhatt, Ana Caraiani, Kiran S. Kedlaya, Peter Scholze, and Jared Weinstein**, Perfectoid Spaces, 2019
- 241 **Dana P. Williams**, A Tool Kit for Groupoid C^* -Algebras, 2019
- 240 **Antonio Fernández López**, Jordan Structures in Lie Algebras, 2019
- 239 **Nicola Arcozzi, Richard Rochberg, Eric T. Sawyer, and Brett D. Wick**, The Dirichlet Space and Related Function Spaces, 2019
- 238 **Michael Tsfasman, Serge Vlăduț, and Dmitry Nogin**, Algebraic Geometry Codes: Advanced Chapters, 2019
- 237 **Dusa McDuff, Mohammad Tehrani, Kenji Fukaya, and Dominic Joyce**, Virtual Fundamental Cycles in Symplectic Topology, 2019
- 236 **Bernard Host and Bryna Kra**, Nilpotent Structures in Ergodic Theory, 2018
- 235 **Habib Ammari, Brian Fitzpatrick, Hyeonbae Kang, Matias Ruiz, Sanghyeon Yu, and Hai Zhang**, Mathematical and Computational Methods in Photonics and Phononics, 2018
- 234 **Vladimir I. Bogachev**, Weak Convergence of Measures, 2018
- 233 **N. V. Krylov**, Sobolev and Viscosity Solutions for Fully Nonlinear Elliptic and Parabolic Equations, 2018
- 232 **Dmitry Khavinson and Erik Lundberg**, Linear Holomorphic Partial Differential Equations and Classical Potential Theory, 2018
- 231 **Eberhard Kaniuth and Anthony To-Ming Lau**, Fourier and Fourier-Stieltjes Algebras on Locally Compact Groups, 2018

For a complete list of titles in this series, visit the
AMS Bookstore at www.ams.org/bookstore/survseries/.

Hopf algebras have been shown to play a natural role in studying questions of integral module structure in extensions of local or global fields. This book surveys the state of the art in Hopf-Galois theory and Hopf-Galois module theory and can be viewed as a sequel to the first author's book, *Taming Wild Extensions: Hopf Algebras and Local Galois Module Theory*, which was published in 2000.

The book is divided into two parts. Part I is more algebraic and focuses on Hopf-Galois structures on Galois field extensions, as well as the connection between this topic and the theory of skew braces. Part II is more number theoretical and studies the application of Hopf algebras to questions of integral module structure in extensions of local or global fields.

Graduate students and researchers with a general background in graduate-level algebra, algebraic number theory, and some familiarity with Hopf algebras will appreciate the overview of the current state of this exciting area and the suggestions for numerous avenues for further research and investigation.

ISBN 978-1-4704-6516-2



9 781470 465162

SURV/260



For additional information
and updates on this book, visit

www.ams.org/bookpages/surv-260

