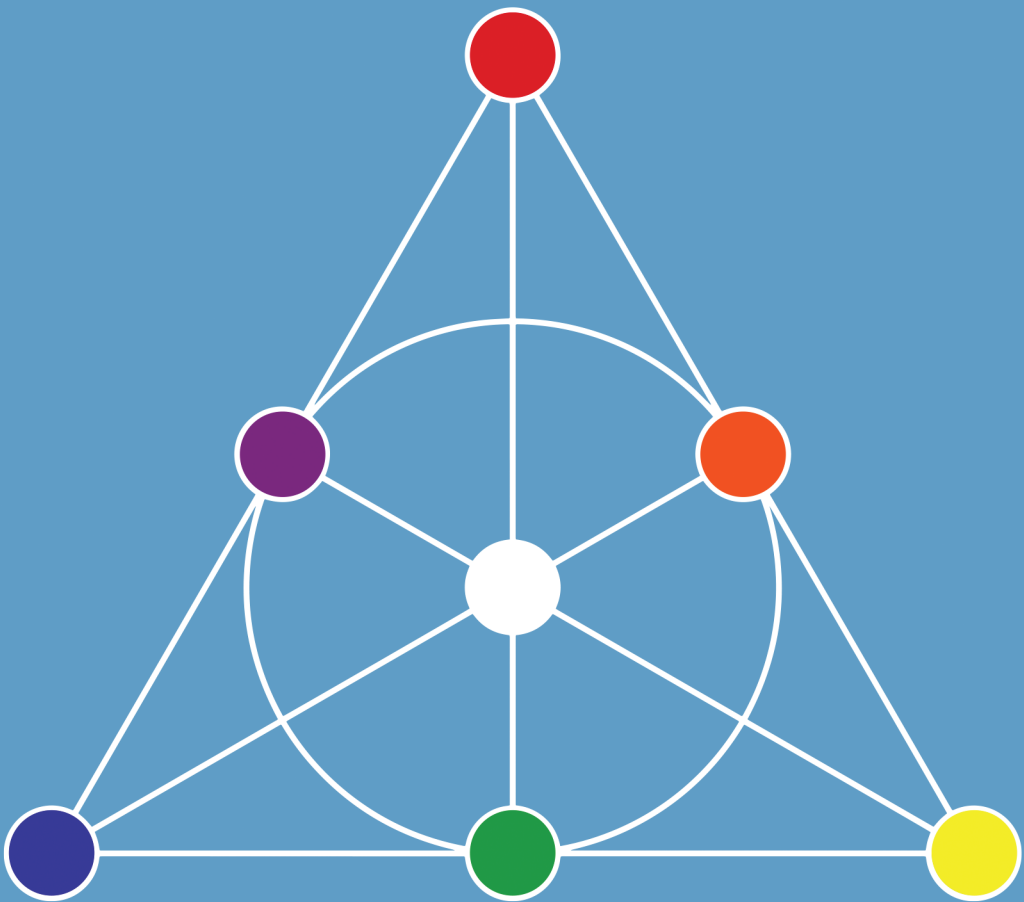


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VOL 48

A TeXas-Style Introduction to Proof

Ron Taylor and Patrick X. Rault



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A T_EXas Style Introduction to Proof

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Dedicated to our families,
especially our wives Kirsten and Masha.

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Of course, we were once students ourselves, and we still think of ourselves as students, particularly when it comes to learning how to be more effective teachers. We have both been fortunate to have good teaching mentors, too numerous to mention here, but there are a few people that have directly impacted our teaching, particularly with regard to this textbook. These are wonderful people that, in addition to being great colleagues, we are lucky enough to call friends. They include Jacqueline Jensen-Vallin, Ted Mahavier, Eric McDowell, Olympia Nicodemi, Ed Parker, Carol Schumacher, Todd Timberlake, Robert Vallin, Walker White and Stan Yoshinobu. These folks helped us along the way by doing things like reading early drafts, offering helpful feedback from the perspective of experienced practitioners of student centered pedagogy, or just being supportive when we got bogged down. We are both better teachers because of your collective influence. Additionally, we would like to thank Harry Lucas for his generous support of the IBL mathematics initiative. Our experiences as part of this community ignited our interest in using the teaching style that we advocate for here, and almost certainly gave rise to the existence of this book.

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R.T. & P.X.R.

APPENDIX A

Mathematical Writing [by Neal Carothers]

Mathematics is suffering from a bad public image. This is actually a recent phenomenon: Fifty years ago mathematics had no public image at all, let alone a poor one. But a lot has changed in those fifty years. Calculus, for example, was at one time taught only to highly specialized scientists and engineers—now it's taught in most high schools. Mathematics affects a larger portion of our society than ever before and yet, somehow, elicits more disdain than excitement. Our society is becoming more mathematically literate, but evidently no more sympathetic to mathematics. Why?

Critics point to a growing inability (or unwillingness) of mathematics teachers to communicate. Those most able to express their interest in mathematics are apparently failing to do so. And if one generation of teachers does a poor job in communicating mathematical ideas, the next generation of teachers suffers. In other words, love and enthusiasm for mathematics is contracted from our teachers, much like a virus. Only those infected can pass it on.

At the heart of society's misconceptions about mathematics is the failure to recognize mathematics as a *human* endeavor. Human beings study mathematics because it pleases them to do so. Not because it builds better mouse traps. We study mathematics for the same reasons that we study art, or music, or literature. That mathematics is frequently useful to engineers and businessmen is typically of more interest to engineers and businessmen than mathematicians.

Our challenge as mathematicians is to communicate the elegance and beauty of mathematics to the unenlightened without relying on its utility as a crutch. Your goal as a student of mathematics is to learn its language and its culture well enough to meet this challenge. The alternative is an unfulfilled life in which your work is misunderstood and unappreciated.

The following suggestions may prove useful in improving your communication skills.

1. Mathematics is written in *complete sentences*. Any mathematically literate reader with the ability to translate the symbols should be able to understand each statement. A student taking the same course as you are, but in a different state, should be able to read and understand your writing. You are *not* writing simply for the benefit of the instructor! It might help if you imagined that you were writing for the benefit of some mythical person (who may not have access to a particular reference or textbook).
2. It's polite, both to your instructor and to your mythical friend, to include the *statement* of the problem (or the theorem you're about to prove) along with your solution (or proof). This not only makes the solution self-contained, and so easier to read, it also acts as a reminder of just what it is that you need to do.
3. Proofread, edit, rewrite, proofread, edit, rewrite, Try reading your solution aloud. Does it make sense? Is it clumsy or confusing? If not, then delete a few offensive lines, or add a few extra lines of clarification. A proof is judged first and foremost by its clarity. Elegance and simplicity are icing on the cake; they can only be introduced after a proof is "fully baked."
4. Be direct. Although proofs by contradiction are often short and "slick," you should avoid them, when possible, in favor of direct, more easily understood proofs. When contradiction seems the only logical course of action, then say so: A proof by contradiction should begin by announcing itself to the reader.
5. Write. You will notice that most "professional" proofs are absolutely thick with prose. Very few intelligible proofs are written using only mathematical symbols.¹

Biographical note: Neal Carothers is Professor Emeritus of mathematics at Bowling Green State University. This appendix being used with his permission and has been adapted from handouts given to students in his courses at BGSU. Professor Carothers is the author of *Real Analysis* published by Cambridge University Press and *A Short Course on Banach Space Theory* published by the London Mathematical Society.

¹For more on this see Appendix B *Comments on Style*.

APPENDIX B

Comments on Style [by James R. Munkres]

The proofs are to be written out carefully and correctly, in good mathematical style. This means:¹

1. Write in complete sentences.
2. Punctuate! (Correctly, if possible.²)
3. Avoid such abbreviations as \exists , \forall , \wedge , \vee , s.t., \ni , w.r.t and similar vulgarisms.³ All are acceptable in informal mathematical conversations, or in a research paper in Logic. In mathematics research journals or texts they are not allowed by editors. There are a few horrendous exceptions. Here is an example, quoted from a textbook on topology:

“Let $f : [0, \Omega) \rightarrow [0, \Omega)$ be s.t. $f(\alpha) < \alpha$ for all $\alpha \geq$ some α_0 . Then $\exists \beta_0 \forall \beta \exists \alpha \geq \beta : f(\alpha) \leq \beta_0$.”

Most mathematicians find this sentence unreadable “as is”; mentally they translate it into the English language. It is an example of bad mathematical style.

4. About the symbols \Rightarrow (implies) and \Leftarrow (is implied by), there is some disagreement among mathematics editors as to their acceptability. They are coming into more widespread use, in any case. In this course, they will be acceptable.⁴

¹The footnotes are the authors', not Munkres'.

²Mathematicians don't like to be picky about this, we're not the grammar police. However, the quality of your writing does reflect on you and as such you should try to adhere to the established rules you learned in your writing classes.

³These abbreviations will likely be acceptable if you are doing presentations at the board. However, in your written solutions you should generally avoid them.

⁴Moreover, if you are doing an *if and only if* proof, you should use these markers to indicate to the reader which direction of the proof you are about to do.

5. Try to steer a middle course between too much detail and not enough. Give reasons for your answers sufficient to convince the reader that your argument is correct and that you understand why it is correct. But don't bore the reader (and get writer's cramp⁵) by checking each tiny detail laboriously in writing. At one extreme of style (bad) are those texts written so concisely that the reader must fill in most of the details himself. At the other extreme (also bad) are the problem set solutions written by your most conscientious fellow student, from which it is almost impossible to extract the basic idea because of the wealth of detail included! Try to hit somewhere in the middle.
6. Incidentally, an illegible proof is incorrect by definition! ⁶
7. A common error is to write in what I call "stream of consciousness" style, à la William Faulkner. When you finish a thought, stop, put down a period, and take a good breath before you begin the next sentence (with a capital letter, please).⁷

Biographical note: James Munkres is Professor Emeritus of mathematics at MIT. This appendix has been adapted from handouts given to students in his courses at MIT. Professor Munkres is the author of *Topology: A first course* published by Pearson and *Analysis on Manifolds* published by Westview Press.

⁵or carpal tunnel syndrome

⁶This one won't come up very often, if your homework is done in L^AT_EX.

⁷It may be a good idea to really embrace the spirit of this if you are doing presentations at the board. As you are writing down your proof, you should explain it line by line as you are transcribing it on the board. If you write it all down in advance and then explain it, your classmates may not be able to keep up with what you are doing if they are trying to follow a lengthy explanation with no pauses while they listen to your explanation.

APPENDIX C

The Structure of a \LaTeX Document

This appendix is a short guide to writing documents in \LaTeX . It supplements the material in Chapter 0, which was essentially an overview. While it should suffice for this course, we will not discuss things like including graphics. If you intend to use mathematics in your future career, then an understanding of \LaTeX will be helpful as you embark on your chosen path. If you are planning on being a teacher, then you can use \LaTeX to write exams so that the mathematics is correctly presented. If you are bound for graduate school, then you will use \LaTeX to write research papers. If you are planning to work in industry, then you can use it to write professional looking reports for your employer. Eventually it is likely that you will find the need to create lists or graphs or use graphics in your documents. At this point, you would need a more comprehensive resource like [15], [8] or [19]. The page at [23] is a nice list of hints that is organized by topic.

In general, a \LaTeX input file will look something like:

```
\documentclass[options]{classname}
:
\begin{document}
:
\end{document}
```

The first set of vertical dots is a placeholder for the rest of the *preamble* and the second set of vertical dots is a placeholder for the body of your file. The *options* box is where you would specify the font size and any deviation from printing on letter sized paper, such as the European A4 size or legal-size paper. There are several built-in choices for *classname*, as well as many custom classes provided by journals. The

`\begin{document}` and `\end{document}` commands tell T_EX where the content of your document begins and ends. (Coincidence? We think not.) Anything in your file after `\end{document}` will be ignored by L^AT_EX. Section C.2 contains more detailed information about how to write the preamble along with a sample that will suffice for your homework solutions, but first let's write a complete, albeit short, L^AT_EX document.

C.1 A sample L^AT_EX document

Before we get to the inner workings of writing a full fledged L^AT_EX document we will start with a simple example. The following program will output some text and some mathematics, both inline and displayed.

```
\documentclass[12pt]{article}
\begin{document}
The distance between points $(a,b)$
and $(x,y)$ in the plane is:
\[
d=\sqrt{(a-x)^2+(b-y)^2}.
\]
\end{document}
```

Type this and save it as `sample.tex`. After running it through your compiler you should get:

The distance between points (a, b) and (x, y) in the plane is:

$$d = \sqrt{(a - x)^2 + (b - y)^2}.$$

although your margins will look a bit different. Can you figure out what the commands mean? What are the dollar signs (\$) for? Let's move on and examine the structure of a L^AT_EX document piece by piece.

C.2 The Preamble

As you saw in Section 0.4, L^AT_EX needs to know in advance what kind of document you want to produce, as well as information about margins and other global properties of the document. The preamble is the part of the document where this information resides. In the previous section, the preamble contained only the line containing the `\documentclass`

command. This provided $\text{T}_{\text{E}}\text{X}$ with the information that you wanted to typeset an article in a 12 point font. In general, however, the preamble for your homework assignments needs to contain a bit more information. The following preamble should suffice for the documents you will write for this course. It will be the default preamble for the remainder of this Appendix.

```
\documentclass[12pt]{article}
\usepackage{amssymb, amsthm, amsmath, fullpage}
\renewcommand{\baselinestretch}{1.25}
\newtheorem*{st}{Statement}
\newtheorem*{ex}{Exercise}
```

There are three choices for the font size, 10pt, 11pt and 12pt. It is possible to alter the size of the font as you go along, but these are the three basic choices. We will see more about changing the font later on. The five built-in choices for class are: `article`, `book`, `letter`, `report` and `slides`. For now the `article` class lends itself well to being used for homework. The three packages `amssymb`, `amsthm` and `amsmath` provide $\text{T}_{\text{E}}\text{X}$ with a larger set of math symbols and a general theorem-proof setup. The `fullpage` package defines the page so that the print will be bordered by one-inch margins, rather than the larger margins that are the $\text{L}_{\text{A}}\text{T}_{\text{E}}\text{X}$ default. There are many, many more packages that can be used with $\text{L}_{\text{A}}\text{T}_{\text{E}}\text{X}$ to control how your fonts look and to place pictures in your documents, but we will stick with these three for now. The `\baselinestretch` line is for changing the spacing of your document and the 1.25 option at the end of the line indicates to $\text{T}_{\text{E}}\text{X}$ that you want one and a quarter spacing. To get double spacing, for example, you would change the 1.25 to 2. (Note that this line is not strictly required, but this choice along with the 12 point font makes the output a bit easier for the reader to read.) The other two lines define macros to let $\text{T}_{\text{E}}\text{X}$ know that you want to write the proof of a **Statement** or an explanation for an **Exercise**. The `*` after `\newtheorem` tells $\text{T}_{\text{E}}\text{X}$ that you do not want it to number things automatically. The word in the first set of curly braces is the keyword that $\text{T}_{\text{E}}\text{X}$ will use to reference the `amsthm` package and the word in the second set of curly braces is the text that will be printed when $\text{T}_{\text{E}}\text{X}$ creates your output. For example, if you wanted to write up a recipe you might include the line

```
\newtheorem*{recipe}{Recipe}
```

in your preamble.¹

¹Why you would want to include a recipe in your homework is anybody's guess, but isn't it nice to know that you can do it?

In order to streamline this process, it is a good idea to create a template document and open it each time you need to T_EX something. Then you can save your document each time with a meaningful name like `homework1.tex` or `statement4_34.tex`.

C.3 The Text

Now that you have the formatting part of your document set up you're ready to begin creating some text. We'll start with the input to include your name and the date. After the `\begin{document}` command you would type:

```
\begin{flushright}
\textbf{your name}\\
MAT ??? HW \# ?\
\today
\end{flushright}
```

The `\begin{flushright}...\end{flushright}` pair causes all of the intermediate text to be right justified. The middle three lines output your name, class and homework number and the date respectively, where the question marks are placeholders for the class number and homework number. The `\today` command will find the current date from your computer and put it in your document. (That was probably pretty obvious, right? But didn't we say that T_EX was intuitive?) The double backslashes `\\` at the end of each line indicate to T_EX that you want a new line. More about this later.

Now you're ready to add some content to the body of your document. For your homework, you will usually be turning in a proof of a **Statement**. In Section C.2 we created a macro for each of these types of assignments. (In case you are also turning in Exercises or Questions,² you can create the corresponding macros in the preamble and modify the following to fit.) For each proof in your homework, the source code will look like:

```
\begin{assignment type}[number]{statement to prove}
\begin{proof}
Your proof goes here.
\end{proof}
\end{assignment type}
\vspace{1in}
```

In place of `assignment type`, you would type the keyword for your `amsthm` proof environment macro. (Most likely `st` from Section

²or recipes

C.2.) The `number` placeholder is, of course, where you would put the number of the assignment. Then you would write the statement being resolved in the space after the assignment number, so that your proof is self-contained. (For more about this, see Appendix A.) Then you would type your proof in the obvious place. The `\vspace{1in}` command creates an inch of blank space between successive assignments. This leaves room for the reader to provide helpful comments about your proofs.³

C.4 Formatting text

Since \LaTeX has a preconceived notion of what your document will look like, it doesn't care how many spaces you put between words. That is, a single space or a number of spaces or even one line return will be interpreted by \TeX as a single space. Hence the following two inputs

```
The quick brown fox jumped over the lazy dog.
```

```
The quick brown
fox jumped over the lazy dog.
```

both generate the same output:

The quick brown fox jumped over the lazy dog.

Notice that this means that a single newline is not enough to create a new paragraph. Paragraph breaks are accomplished by leaving a blank line in the input file or by typing `\par`. As in the case of \TeX interpreting many spaces as a single space, \TeX will also interpret many blank lines as a single blank line. A line break can be forced by typing `\\` or `\newline`.

In Section 0.4 there was a reference to marking text. This is how WYSIWYG word processors work, but, for the most part, the process is transparent. For example, to get MS Word to italicize a word you could highlight the word and click the italics button, or you could turn on italics with a set of keystrokes and then turn it off after you have finished typing what needs to be italicized. With \LaTeX there are codes for *italics*, **boldface** and *other typefaces*. The input for the previous sentence looks like:

```
With \LaTeX\ there are codes for \textit{italics},
\textbf{boldface} and \textsl{other typefaces}.
```

³Of course, you can omit this command after the last proof in your homework set.

Here is a short table of some of the different typefaces that are available in \LaTeX for plain text.

Command	Typeface	Example
<code>\textbf</code>	Boldface	Example
<code>\textit</code>	Italics	<i>Example</i>
<code>\textsc</code>	Small Caps	EXAMPLE
<code>\textsf</code>	Sans Serif text	Example
<code>\textsl</code>	Slanted text	<i>Example</i>
<code>\texttt</code>	Typewriter	Example

To use these text markings, just enclose the text you want marked in a set of curly braces `{...}` and put the appropriate formatting command in front, as above. This is an example of where the \TeX command is at least sort of intuitive. The `\text` part of the command tells \TeX that you want to do something to some text and the last two letters correspond nicely to the way you want the text formatted. We will see more of this intuitiveness when we discuss writing mathematics.

While \LaTeX will typeset pages to look like they would in a textbook, sometimes you want to alter the format a little bit. In addition to the `\baselinestretch` option that allows you to change the interline spacing, you can also add extra space between lines and even a little space between words on the same line. Here is an extremely short list of formatting codes:

Command	Used for
<code>\vspace{length}</code>	a space of the given <i>length</i> between lines of text
<code>\</code>	a space between words or characters
<code>\,</code>	a thin space between words or characters
<code>\quad</code>	a space the width of a capital M between characters
<code>\qquad</code>	a space the width of MM between characters

C.5 Typesetting mathematics

Some of this may seem like using a sledgehammer to open a walnut, and perhaps it is to a degree, but now we're ready jump into the beauty of \TeX and witness its power. This power is the ability to typeset mathematics in a professional looking way. To invoke this power, you need to tell \TeX when you want it to create mathematics. This is accomplished by enclosing the \TeX commands for mathematics in dollar signs ($\$$). Recall

the sample program from Section C.1. (Did you guess that the dollar signs had something to do with math?) Notice that the inline math (x, y) corresponded to $\$(x, y)\$$ and the displayed math corresponded to the input $\[d=\sqrt{(a-x)^2+(b-y)^2}\]$. Thus, to cause T_EX to create inline math you need to enclose all of the math content in dollar signs. You don't need a pair of them for each mathematical symbol. However, you do want to make sure that everything that is supposed to be mathematics is enclosed in dollar signs. Note the difference between a regular text x and an x in T_EX's math font.

Command	Used for
$\$. . . \$$	Inline mathematics
$\[. . . \]$	Displayed mathematics


Sometimes the mathematical part of your writing may be too large to fit nicely on the same baseline as the surrounding text. For example, recall the usual definition of the derivative given by $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$. This looks cramped as inline math, so it would be a good place to appeal to T_EX's displayed math option. Then we would get the nicer looking output:

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h},$$

from an input of $\[f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}\]$. (Can you figure out what the commands mean?) In the next section we provide a short list of commands for typesetting mathematics. Keep in mind that as you get more practice, the process of writing in L^AT_EX will become easier. Don't be anxious about all of the commands, it's really not hard to get the hang of.

C.6 L^AT_EX codes for common mathematical symbols

As you are writing your proofs you will need to include some math symbols. (This is a math class, right?) The following table contains a non-comprehensive list of codes that will come in handy when you are writing proofs for this class. The complete list of T_EX codes is quite long. Check out [22] for over 300 pages of more than 14,000 T_EX codes that can be used for everything from math ($\oint_{\Gamma} f(z) dz$) to astronomy (\odot) to music

(\bullet) to laundry (\la) and even the code for this man's face: . The

following tables contain some common mathematical symbols that you may find useful as you are preparing your homework.

Math accents

\hat{a}	<code>\hat{a}</code>	\tilde{a}	<code>\tilde{a}</code>
\bar{a}	<code>\bar{a}</code>	\vec{a}	<code>\vec{a}</code>

Lowercase Greek letters

α	<code>\alpha</code>	η	<code>\eta</code>
β	<code>\beta</code>	θ	<code>\theta</code>
γ	<code>\gamma</code>	ι	<code>\iota</code>
δ	<code>\delta</code>	κ	<code>\kappa</code>
ϵ	<code>\epsilon</code>	λ	<code>\lambda</code>
ε	<code>\varepsilon</code>	μ	<code>\mu</code>
ζ	<code>\zeta</code>	ν	<code>\nu</code>
ξ	<code>\xi</code>	υ	<code>\upsilon</code>
\omicron	<code>\omicron</code>	ϕ	<code>\phi</code>
π	<code>\pi</code>	φ	<code>\varphi</code>
ρ	<code>\rho</code>	χ	<code>\chi</code>
σ	<code>\sigma</code>	ψ	<code>\psi</code>
τ	<code>\tau</code>	ω	<code>\omega</code>

Upper case Greek letters

Γ	<code>\Gamma</code>	Λ	<code>\Lambda</code>
Δ	<code>\Delta</code>	Ξ	<code>\Xi</code>
Θ	<code>\Theta</code>	Π	<code>\Pi</code>
Σ	<code>\Sigma</code>	Ψ	<code>\Psi</code>
Υ	<code>\Upsilon</code>	Ω	<code>\Omega</code>
Φ	<code>\Phi</code>		

Binary relations

\neq	<code>\neq</code>	\approx	<code>\approx</code>
\equiv	<code>\equiv</code>	\preceq	<code>\preceq</code>
$<$	<code><</code>	\leq	<code>\leq</code>
$>$	<code>></code>	\geq	<code>\geq</code>
\in	<code>\in</code>	\notin	<code>\notin</code>

Binary relations (cont.)

\subset	<code>\subset</code>	\subseteq	<code>\subseteq</code>
\supset	<code>\supset</code>	\supseteq	<code>\supseteq</code>
\mid	<code>\mid</code>	\sim	<code>\sim</code>
\simeq	<code>\simeq</code>	\cong	<code>\cong</code>
\parallel	<code>\parallel</code>	\perp	<code>\perp</code>

Binary operators

$+$	<code>+</code>	$-$	<code>-</code>
\pm	<code>\pm</code>	\mp	<code>\mp</code>
\times	<code>\times</code>	\cdot	<code>\cdot</code>
\cup	<code>\cup</code>	\cap	<code>\cap</code>
\bigcup	<code>\bigcup</code>	\bigcap	<code>\bigcap</code>
\sum	<code>\sum</code>	\int	<code>\int</code>
\vee	<code>\vee</code>	\wedge	<code>\wedge</code>
\circ	<code>\circ</code>	\setminus	<code>\setminus</code>

Arrows

\leftarrow	<code>\leftarrow</code>
\rightarrow	<code>\rightarrow</code> or <code>\to</code>
\Leftarrow	<code>\Leftarrow</code>
\Rightarrow	<code>\Rightarrow</code>
\leftrightarrow	<code>\leftrightarrow</code>
\Leftrightarrow	<code>\Leftrightarrow</code>
\longleftarrow	<code>\longleftarrow</code>
\longrightarrow	<code>\longrightarrow</code>
\Longleftarrow	<code>\Longleftarrow</code>
\Longrightarrow	<code>\Longrightarrow</code>
\longleftrightarrow	<code>\longleftrightarrow</code>
\Longleftrightarrow	<code>\Longleftrightarrow</code>

Delimiters

$($	<code>(</code>	$)$	<code>)</code>	$[$	<code>[</code>	$]$	<code>]</code>
$\{$	<code>\{</code>	$\}$	<code>\}</code>	\langle	<code>\langle</code>	\rangle	<code>\rangle</code>
$/$	<code>/</code>	\backslash	<code>\backslash</code>	$ $	<code> </code>	\parallel	<code>\parallel</code>

Miscellaneous symbols

◇	<code>\diamondsuit</code>	♥	<code>\heartsuit</code>
♣	<code>\clubsuit</code>	♠	<code>\spadesuit</code>
∀	<code>\forall</code>	∞	<code>\infty</code>
∃	<code>\exists</code>	∄	<code>\nexists</code>
∅	<code>\varnothing</code>	ℓ	<code>\ell</code>
ℕ	<code>\aleph</code>	℄	<code>\complement</code>
□	<code>\square</code>	○	<code>\bigcirc</code>
◇	<code>\lozenge</code>	△	<code>\triangle</code>
▷	<code>\triangleright</code>	◁	<code>\triangleleft</code>
▽	<code>\triangledown</code>	∠	<code>\angle</code>
*	<code>\ast</code>	•	<code>\bullet</code>
...	<code>\dots</code>	⋯	<code>\cdots</code>
⋮	<code>\vdots</code>	⋱	<code>\ddots</code>
★	<code>\bigstar</code>		

To create the blackboard bold font for the number systems you use the code `\mathbb` along with the letter of the corresponding set of numbers. For example, to indicate that n is a natural number you would write `n\in\mathbb{N}` to get $n \in \mathbb{N}$. Whereas a table in section C.4 listed typefaces for plain text, the following lists the typefaces available for mathematical text.

Command	Typeface	Example
<code>\mathbb</code>	Blackboard bold	\mathbb{N}
<code>\mathbf</code>	Math boldface	$\mathbf{x} + 2$

Notice that all of the symbols listed above are just one character. However, in mathematics we don't often use just one character to get our point across. Thus, we conclude this section with a few examples of mathematics with more than one character. The following table shows the \LaTeX codes for the output, but not the delimiters for inline or displayed math.

\LaTeX code	Output
<code>\frac{a}{b}</code>	$\frac{a}{b}$
<code>p_{n}</code>	p_n
<code>x^{n}</code>	x^n
<code>\sum_{i=1}^5</code>	$\sum_{i=1}^5$
<code>\int_{a}^b f(x) \, dx</code>	$\int_a^b f(x) dx$

Keep in mind that all of the preceding symbols are math symbols and as such need to be enclosed between dollar signs or a `\[... \]` pair. As a reminder, the code for symbols found in definitions is included at the end of the corresponding definition as follows: [*symbol* \leftrightarrow *code*].

C.7 Tables

Since the text starts with truth tables, it will be useful to know how to typeset a table in \LaTeX . Again, the syntax is fairly obvious and so it is not too hard to understand the `tabular` environment. For example, to typeset the table

Currency	Exchange rate to US dollars
Yen (¥)	1 Yen = 0.00916151 USD
Euro (€)	1 Euro = 1.22938 USD

you would type the following \TeX code.

```
\begin{center}
\begin{tabular}{|c|p{2.5in}|}
\hline
\textbf{Currency} & \textbf{Exchange
rate to US dollars}\\
\hline
Yen (\yen)&1 Yen = 0.00916151 USD\\
\hline
Euro (\euro)&1 Euro = 1.22938 USD\\
\hline
\end{tabular}
\end{center}
```

The following code is used for one of the truth tables in Section 1.2.

```
\begin{center}
\begin{tabular}{c|c|c}
\mathcal{P}\mathcal{Q}\mathcal{P}\wedge\mathcal{Q}\\
\hline
T&T&T\\
T&F&F\\
F&T&F\\
F&F&F\\
\end{tabular}
\end{center}
```

Here the `\begin{center}` and `\end{center}` commands cause \TeX to center everything in between. The `\begin{tabular}` command tells \TeX that you want to typeset a table. There are several options that go with the `tabular` environment. In the example above we have used the options `|`, `c` and `p{width}`. There are two more options: `r` and `l`. Each of the letter options denotes some kind of column in your table, and you will need one of these for each column you want in your table. The vertical bar indicates to \TeX that you want it to create a vertical bar in the table at that position. The following table gives the meaning of each of the column-type options.

Option	Meaning
<code>c</code>	center text in a variable width column
<code>l</code>	left justify text in a variable width column
<code>r</code>	right justify text in a variable width column
<code>p{width}</code>	create a column of the given width and add text in a paragraph format

Finally, the `\hline` command creates a horizontal line between rows of your table and the double backslash `\\` tells \TeX when to end a row of the table.

C.8 Arrays with reasons

Eventually you will be typesetting some razzmatazz. Sometimes you may even want to add notes or reasons to justify how you got from one step to the next. This can be done by using the `array` environment. In this environment you can create as many columns as you need and have them justified appropriately. It is like the `tabular` environment, but you don't need to indicate math text in every cell of the array. Here is an example from a proof by induction that may look familiar.

$$\begin{aligned}
 1 + 2 + \cdots + k + (k + 1) &= \frac{k(k + 1)}{2} + k + 1 && \text{(Why?)} \\
 &= \frac{k^2 + k}{2} + \frac{2k + 2}{2} && \text{(Why?)} \\
 &= \frac{k^2 + 3k + 2}{2} && \text{(Why?)} \\
 &= \frac{(k + 1)(k + 2)}{2} && \text{(Why?)} \\
 &= \frac{(k + 1)((k + 1) + 1)}{2}.
 \end{aligned}$$

Here is the \LaTeX code to get the equation array with comments.⁴

```
\[
\begin{array}{rclcl}
1+2+\cdots+k+(k+1) &=& & & \\
&\displaystyle\frac{k(k+1)}{2}+k+1 & & & \\
&&\mbox{(Why?)} && \\
&=&\displaystyle\frac{k^2+k}{2}+\frac{2k+2}{2} & & \\
&&\mbox{(Why?)} && \\
&=&\displaystyle\frac{k^2+3k+2}{2} & & \\
&&\mbox{(Why?)} && \\
&=&\displaystyle\frac{(k+1)(k+2)}{2} & & \\
&&\mbox{(Why?)} && \\
&=&\displaystyle\frac{(k+1)\left((k+1)\right.}{ & & \\
&&\left.+1\right)}{2}. & & \\
\end{array}
\]
```

The `\displaystyle` command causes the mathematics to be printed in the display style, rather than inline. Even though the array is in the `\[... \]` to begin with, this only causes the array to be centered in the page in math mode. The default display within the array is inline math. The `\mbox` command allows you to include text inside the math environment that isn't in math italics. However, if you want math italics inside your comment, you'll need to include some `\$s` to tell \TeX that you want to typeset something in math.

Notice that the `array` environment looks a lot like the `tabular` environment. In the array above, there are five columns. The first column is right justified so that the expression lines up against the next column. The second column is center justified so that the equal signs — or whatever symbol you happen to be using — line up, the third column is left justified so that the stuff on the right hand side of the “=” is lined up against the “=”, the fourth column is a buffer to leave a little space between the end of the longest expression and the column of reasons and the fifth column is left justified so that the reasons you want to include are lined up.

In the last line of the array you see a `\left` and a `\right`. These two commands tell \TeX that you're putting parentheses — or other grouping symbol — around something that may be larger than a standard line

⁴The lines that begin with some blank space do not need to be indented as they are shown. This unusual spacing is a result of the page width of the book you are reading.

of text. In this case, \TeX will adjust the size of the parentheses accordingly. Try putting a `\left ... \right` pair around one of the fractions to see how it would work for a much larger mathematical object.

C.9 Making lists (Checking them twice is a good idea.)

There are various reasons that you might want a list in a \LaTeX document. There are also various kinds of lists and sublists. In this section we will describe the `itemize` and `enumerate` environments. These are not the only choices for lists, but they should suffice for our purposes here. For example, there is a bulleted list where you have no need for the items to be numbered.

- This is an item on my list.
- *The Elements of Cantor Sets: With Applications* by Robert Vallin
- chocolate chip cookie dough ice cream

There are also numbered lists. This kind of list can be useful if you want to reference the items in the list by number later on. For example, here are three problems that might have appeared on an exam.

1. Prove that every order topology is Hausdorff.
2. Show that the Fano plane admits a characterization in terms of subtractive color arithmetic.
3. Prove that every even number greater than 2 can be written as the sum of two primes.

These list environments can be nested within each other, if necessary. For example, you might be a teacher writing an exam where a question may have several parts. In this case, you might have a list that looks like the following:

1. Let $K_1 = 7_2$ and $K_2 = 7_4$. Use Seifert's algorithm to sketch Seifert surfaces for K_1 and K_2 .
 - (a) How do you know that Seifert's algorithm produces a minimal genus Seifert surface?
 - (b) Sketch the associated Seifert graphs and explain how the graph can tell you $g(K_1)$ and $g(K_2)$.

Notice that the sublists have different enumeration schemes. Further nestings would produce Roman numerals and capital letters as the item "numbers." In a nested `itemized` list the bullets would change appearances on each level of nesting.

Here is the code for the bulleted list above. To create a numbered list, you would change the `itemize` environment to `enumerate`. To nest a list inside another list, you would put a copy of this whole code after the item you wanted to have a sublist.

```
\begin{itemize}
\item This is an item on my list.
\item \textit{The Elements of Cantor Sets:
      With Applications} by Robert Vallin
\item chocolate chip cookie dough ice cream
\end{itemize}
```

C.10 An example of a homework assignment in \LaTeX

Here is an example of how a homework assignment might look.

Ron Taylor

MAT 799 HW # 1

August 10, 2000

Theorem (1.7). If M is a proper subspace of $B(X)$, then $B(X) \setminus M$ is norm-dense in $B(X)$, and hence SOT-dense.

Proof. It suffices to show that there exists a sequence of operators (A_n) in $B(X) \setminus M$ that converges to an arbitrary operator in M .

Let A be an arbitrary operator in M and choose an arbitrary operator B in the set $B(X) \setminus M$. For each $n \in \mathbb{N}$, define $A_n = A + \frac{1}{n}B$. Since M is a vector subspace of $B(X)$, and is closed under vector addition, we have that $A_n \in B(X) \setminus M$. Now, we see that

$$\|A - A_n\| = \left\| \frac{1}{n}B \right\| = \frac{1}{n}\|B\| \rightarrow 0 \quad \text{as } n \rightarrow \infty.$$

Therefore, $A_n \rightarrow A$ and $B(X) \setminus M$ is dense in $B(X)$ and, hence, also SOT-dense. \square

C.11 \TeX Source Code for the example

Here is the source code for the example in Section C.10. Note the use of $\$$ for inline math and the $\[\dots \]$ pair for displayed math. See if there are any other commands you can find that are used to make the mathematics look more like you might see it in a textbook. Note the

use of the $\text{\mbox}\{...\}$ command to include non-math text inside the displayed math.

```

\documentclass[12pt]{article}
\usepackage{amssymb, amsthm, amsmath, fullpage}
\renewcommand{\baselinestretch}{1.25}
\newtheorem*{thm}{Theorem}

\begin{flushright}
\textbf{Ron Taylor}\
MAT 799 HW \# 1\
August 10, 2000
\end{flushright}

\begin{thm}[1.7] If  $M$  is a proper subspace of  $B(X)$ ,
then  $B(X) \setminus M$  is norm-dense in  $B(X)$ , and
hence SOT-dense.
\end{thm}

\begin{proof} It suffices to show that there exists a
sequence of operators  $(A_n)$  in  $B(X) \setminus M$ 
that converges to an arbitrary operator in  $M$ .

Let  $A$  be an arbitrary operator in  $M$  and choose an
arbitrary operator  $B$  in the set  $B(X) \setminus M$ .
For each  $n \in \mathbb{N}$ , define  $A_n = A + \frac{1}{n}B$ .
Since  $M$  is a vector subspace of  $B(X)$ ,
and is closed under vector addition, we have that
 $A_n \in B(X) \setminus M$ . Now, we see that
\[\|A - A_n\| = \left\| \frac{1}{n}B \right\|
= \frac{1}{n}\|B\| \rightarrow 0
\quad \text{\mbox{as } } n \rightarrow \infty.\]
\]
Therefore,  $A_n \rightarrow A$  and  $B(X) \setminus M$ 
is dense in  $B(X)$  and, hence, also SOT-dense.
\end{proof}

```

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A Texas Style Introduction to Proof by Ron Taylor and Patrick X. Rault is truly delightful, full of humanizing charm that softens the hard edge of mathematical rigor. It is gentle, lively, clear, and warm. Students who read (or, more accurately, actively engage with) this book are cheerfully invited to enjoy the adventure of learning to prove theorems for themselves. It regularly connects the mathematical precision of logic and proofs with whimsical examples outside math. From this book, students and their instructors will find many proofs of the joy of mathematics.

**—Michael Starbird, University Distinguished Teaching Professor
of Mathematics at The University of Texas at Austin and 2007
Haimo Award Winner**

This is an IBL textbook designed for a one-semester course on proofs (the “bridge course”) that also introduces TeX as a tool students can use to communicate their work. The pedagogy (and personality) is what one would expect from a pair of award winning teachers, Ron Taylor (2018 Haimo Award) and Patrick Rault (2015 Alder Award).

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