Topological Semifields and their Applications to General Topology

by

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and
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TABLE OF CONTENTS

Translator's note
Author's preface to the American edition
Chapter I. Boolean algebras
§1. Abstract spectra and their structure spaces
§2. Least upper and greatest lower bounds
§3. Boolean algebras
§4. Partitions of the identity and the structure space
§5. Boolean rings and Stone's theorem
§6. Complete Boolean algebras
§7. Topological Boolean algebras
§8. Direct products of Boolean algebras
§9. Dyadic Boolean algebras

Chapter II. Topological semifields
§10. The axioms for a topological semifield
§11. Inequalities
§12. The existence of a unit and of inverse elements
§13. Properties of least upper and greatest lower bounds
§14. Positive and negative parts; the modulus
§15. Disjointness
§16. Properties of idempotents
§17. The axis of a topological semifield
§18. Supports of elements
§19. Continuity of the modulus
§20. Topology of the axis

Chapter III. The universal semifield
§21. Improper functions
§22. Functions on the structure space
§23. Topology of a certain space of functions
§24. The universal semifield

Chapter IV. The classification theorem
§25. Auxiliary constructions
§26. Embedding an arbitrary semifield in the universal semifield
§27. The classification theorem
§28. Uniqueness of the cone $K$

Chapter V. Subsemifields
§29. Definition of a subsemifield
§30. Direct products of subsemifields
## CONTENTS

Chapter VI. Metric spaces over Tihonov semifields .................................................. 94
  §31. The Tihonov semifield ......................................................................................... 94
  §32. Metric spaces over $R^\Delta$ ............................................................................. 96
  §33. The metrization theorem .................................................................................... 98
  §34. Metrization of uniform structures ...................................................................... 98
  §35. Proper metrics and metrization of proximity spaces ....................................... 100
  §36. Convergence in metric spaces .......................................................................... 103
  §37. Completeness ..................................................................................................... 104
  §38. Ultracompleteness ............................................................................................. 106
  §39. The closed graph theorem ................................................................................ 110

Chapter VII. Homomorphisms of Tihonov semifields ............................................... 115
  §40. Definition of homomorphisms ......................................................................... 115
  §41. The continuity problem for homomorphisms ............................................... 120
  §42. Two-valued countably additive measures ..................................................... 122
  §43. The Stone-Čech compactification .................................................................. 128
  §44. Hewitt spaces .................................................................................................... 130
  §45. Topological reductions of the problem of measurability of cardinal numbers .......................................................... 133
  §46. Certain geometric properties of a Tihonov semifield ..................................... 134

Bibliography .................................................................................................................. 141
TRANSLATOR'S NOTE

The present translation is based on the following:

A. Topological Boolean algebras (Izdat. Akad. Nauk UzSSR. Tashkent, 1963 (MR 34 #5718));

B. A sketch of the theory of topological semifields (Uspehi Mat. Nauk 21 (1966), vyp. 4(130), 185–218 (MR 35 #832));


Monograph A and article B were written by all three authors; article C by Professors Antonovskii and Boltjanskii. The authors, as remarked in their preface, have kindly submitted a number of revisions to the translator, which have been incorporated in the translation. The translator has also supplied some revisions, as noted below. The bibliography is new. Thus the present monograph is not a mere reproduction of material already in print in the Russian language.

Chapters I–V are a faithful rendering of monograph A. Chapter VI is a faithful rendering of article B, with only small changes to conform with Chapters I–V. Chapter VII contains the substance of article C, with a number of revisions made by the translator. These have the advantages of brevity and, it is hoped, of perspicuity. Misprints and minor flaws have been tacitly corrected where found. One apparent error is pointed out in a note. The original terminology has been altered at a few points to conform with Western usage.

The translator records his gratitude to Laura Larsson for checking the bibliography and to Betty Underhill for typing the translation.

Edwin Hewitt
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AUTHORS’ PREFACE TO THE AMERICAN EDITION

This book consists for the most part of results that are not strictly new. Our goal is to bring a series of heterogeneous questions and problems in general topology under one unifying concept. This concept we have named topological semifields. We study in some detail the concept of a semifield, and also the concept of a topological Boolean algebra, which is closely connected with it. The three monographs in the Soviet literature on which this translation is based have been considerably edited and amplified.

The authors express their gratitude to the American Mathematical Society for including our work in its translation program. We especially wish to thank Professor Edwin Hewitt for his assistance in translating and editing the work. We are also grateful to the other persons who have contributed their labor to the preparation of this edition. We shall be glad if our monograph proves useful to the English-reading mathematical public.

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<table>
<thead>
<tr>
<th>Page</th>
<th>BIBLIOGRAPHY</th>
</tr>
</thead>
<tbody>
<tr>
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</tbody>
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Translated by H. HEWITT