Nonlinear Evolution Equations

N. N. Uraltseva
Editor
Recent Titles in This Series

164  N. N. Uraltseva, Editor, Nonlinear Evolution Equations
162  S. G. Gindikin, Editor, Applied Problems of Radon Transform
161  Katsumi Nomizu, Editor, Selected Papers on Analysis, Probability, and Statistics
160  K. Nomizu, Editor, Selected Papers on Number Theory, Algebraic Geometry, and Differential Geometry
159  O. A. Ladyzhenskaya, Editor, Proceedings of the St. Petersburg Mathematical Society, Volume II
157  M. Sh. Birman, Editor, Wave Propagation. Scattering Theory
156  V. N. Gerasimov, N. G. Nesterenko, and A. I. Valitskas, Three Papers on Algebras and Their Representations
154  V. A. Artamonov et al., Selected Papers in $K$-Theory
153  S. G. Gindikin, Editor, Singularity Theory and Some Problems of Functional Analysis
152  H. Draškovičová et al., Ordered Sets and Lattices II
150  S. G. Gindikin, Editor, Spectral Theory of Operators
149  V. S. Afraimovich et al., Thirteen Papers in Algebra, Functional Analysis, Topology, and Probability, Translated from the Russian
147  I. G. Bashmakova et al., Nine Papers from the International Congress of Mathematicians, 1986
146  L. A. Aizenberg et al., Fifteen Papers in Complex Analysis
145  S. G. Dalalyan et al., Eight Papers Translated from the Russian
144  S. D. Berman et al., Thirteen Papers Translated from the Russian
143  V. A. Belonogov et al., Eight Papers Translated from the Russian
142  M. B. Abalovich et al., Ten Papers Translated from the Russian
141  H. Draškovičová et al., Ordered Sets and Lattices
140  V. I. Bernik et al., Eleven Papers Translated from the Russian
139  A. Ya. Aizenshtat et al., Nineteen Papers on Algebraic Semigroups
138  I. V. Kovalishina and V. P. Potapov, Seven Papers Translated from the Russian
137  V. I. Arnol’d et al., Fourteen Papers Translated from the Russian
136  L. A. Aksent’ev et al., Fourteen Papers Translated from the Russian
135  S. N. Artemov et al., Six Papers in Logic
134  A. Ya. Aizenshtat et al., Fourteen Papers Translated from the Russian
133  R. R. Suncheleev et al., Thirteen Papers in Analysis
132  I. G. Dmitriev et al., Thirteen Papers in Algebra
131  V. A. Zmorovich et al., Ten Papers in Analysis
130  M. M. Lavrent’ev, K. G. Reznitskaya, and V. G. Yakhno, One-dimensional Inverse Problems of Mathematical Physics
129  S. Ya. Khavinson, Two Papers on Extremal Problems in Complex Analysis
128  I. K. Zhuk et al., Thirteen Papers in Algebra and Number Theory
127  P. L. Shabalin et al., Eleven Papers in Analysis
126  S. A. Akhmedov et al., Eleven Papers on Differential Equations

(Continued in the back of this publication)
This page intentionally left blank
Nonlinear Evolution Equations
Nonlinear Evolution Equations

N. N. Uraltseva
Editor
# Contents

Foreword ix

Hölder Estimates of Solutions to Initial-Boundary Value Problems for Parabolic Equations of Nondivergent Form with Wentzel Boundary Condition
D. E. APUSHKINSKAYA and A. I. NAZAROV 1

Reverse Hölder Inequalities with Boundary Integrals and $L_p$-Estimates for Solutions of Nonlinear Elliptic and Parabolic Boundary-Value Problems
A. A. ARKHIPOVA 15

Quasilinear Parabolic Equations with Small Parameter in a Hilbert Space
YA. BELOPOL’SKAYA 43

On the Stability of Solitary Waves for Nonlinear Schrödinger Equations
V. S. BUSLAEV and G. S. PERELMAN 75

On Semigroups Generated by Initial-Boundary Value Problems Describing Two-Dimensional Visco-Plastic Flows
O. LADYZHENSKAYA and G. SEREGIN 99

Elliptic Differential Inequalities, Embedding Theorems, and Variational Problems
V. A. MALYSHEV 125

Long Time Behavior of Flows Moving by Mean Curvature
V. I. OLIKER and N. N. URALTSEVA 163

Bifurcation Problem for Nonlinear Second Order Equations in Variable Regions
V. G. OSMOLOVSKII and A. V. SIDOROV 171

Existence of a Weak Solution of the Minimax Problem Arising in Coulomb-Mohr Plasticity
S. REPIN and G. SEREGIN 189
This page intentionally left blank
Foreword

The Mathematical Physics Seminar has been working in Leningrad (St. Petersburg) since September, 1947. Many experts in various areas of mathematics, theoretical mechanics, hydrodynamics and gasdynamics, diffraction theory, and mathematical problems in theoretical physics have participated in it. The seminar is devoted to the study of differential equations (mainly partial differential equations) and their applications both in mathematics itself and in the other disciplines mentioned above.

At the beginning linear problems dominated the seminar discussions, but since the mid-fifties results on nonlinear problems have often been reported as well. A number of mathematicians of our city and the rest of the country, as well as many well-known mathematicians from America, Asia, and Europe, have given talks at the seminar.

From its beginning until 1974 the seminar worked under the supervision of V. I. Smirnov, and after his death it was named the Smirnov Seminar.

Most of the papers in this book are devoted to nonlinear problems. The article by Ladyzhenskaya and Seregin deals with the first initial-boundary value problem for nonlinear systems describing visco-elastic flows. It is proved that their resolving operators form a continuous compact semigroup possessing a compact minimal $B$-attractor.

The existence of weak solutions to stationary variational problems in plasticity theory is proved under the Coulomb–Mohr fluidity conditions in the paper of Repin and Seregin. The Cauchy problem for an infinite-dimensional quasilinear parabolic equation with a small parameter is studied in the article of Belopol’skaya. It is proved that the solutions are of wave character, and the wave front velocity is computed. The probabilistic approach is used to obtain results in the infinite-dimensional case.

Arkhipova’s article is a review of her results on inverse Hölder inequalities and their applications to the study of the regularity of generalized solutions to the Neumann problem for systems of quasilinear elliptic and parabolic equations.

Nonlinear Schrödinger equations with initial data located near a soliton solution are studied by Buslaev and Perelman. They prove that if the spectrum of the problem, linearized on this soliton, is real, then the large-time asymptotics of these solutions is described by moving solitons and scattering states that obey the free Schrödinger equation.

Bifurcations of solutions of fully nonlinear elliptic equations in variable regions are investigated by Ōsmolovskii and Sidorov. Necessary and sufficient conditions are proved to ensure that the region is a bifurcation region for the equation under consideration.

The Wentzell problem for quasilinear parabolic equations is studied by Apushkinskaya and Nazarov. They obtain a priori estimates for Hölder norms of solutions under optimal restrictions on the data.
A new concept of nonlinear imbedding theorems is developed in Malyshev's article. Unlike the standard imbedding theorems, nonlinear ones are valid for some convex sets in Banach spaces, but not for the spans of those sets. Classical theorems about the convergence of convex, monotone, or subharmonic functions may be regarded as examples of nonlinear imbedding theorems.

N. N. Uraltseva
Recent Titles in This Series  

(Continued from the front of this publication)

125  D. V. Anosov et al., Seven Papers in Applied Mathematics
124  B. P. Allahverdiev et al., Fifteen Papers on Functional Analysis
123  V. G. Maz'ya et al., Elliptic Boundary Value Problems
122  N. U. Arakelyan et al., Ten Papers on Complex Analysis
121  V. D. Mazurov, Yu. I. Merzlyakov, and V. A. Churkin, Editors, The Kourovka Notebook: Unsolved Problems in Group Theory
120  M. G. Kreín and V. A. Jakubovič, Four Papers on Ordinary Differential Equations
119  V. A. Dem'janenko et al., Twelve Papers in Algebra
118  Ju. V. Egorov et al., Sixteen Papers on Differential Equations
117  S. V. Bočkarëv et al., Eight Lectures Delivered at the International Congress of Mathematicians in Helsinki, 1978
116  A. G. Kuśnirenko, A. B. Katok, and V. M. Alekseev, Three Papers on Dynamical Systems
115  I. S. Belov et al., Twelve Papers in Analysis
114  M. Š. Birman and M. Z. Solomjak, Quantitative Analysis in Sobolev Imbedding Theorems and Applications to Spectral Theory
113  A. F. Lavrik et al., Twelve Papers in Logic and Algebra
112  D. A. Gudkov and G. A. Utkin, Nine Papers on Hilbert's 16th Problem
111  V. M. Adamjan et al., Nine Papers on Analysis
110  M. S. Budjanu et al., Nine Papers on Analysis
109  D. V. Anosov et al., Twenty Lectures Delivered at the International Congress of Mathematicians in Vancouver, 1974
108  Ja. L. Geronimus and Gábor Szegő, Two Papers on Special Functions
107  A. P. Miščina and L. A. Skornjakov, Abelian Groups and Modules
106  M. Ja. Antonovskii, V. G. Boltjanskii, and T. A. Sarymsakov, Topological Semifields and Their Applications to General Topology
105  R. A. Aleksandrian et al., Partial Differential Equations, Proceedings of a Symposium Dedicated to Academician S. L. Sobolev
103  M. S. Brodskii et al., Nine Papers in Analysis
102  M. S. Budjanu et al., Ten Papers in Analysis
101  B. M. Levitan, V. A. Marcenko, and B. L. Roždestvenskii, Six Papers in Analysis
100  G. S. Ceitín et al., Fourteen Papers on Logic, Geometry, Topology and Algebra
99  G. S. Ceitín et al., Five Papers on Logic and Foundations
98  G. S. Ceitín et al., Five Papers on Logic and Foundations
97  B. M. Budak et al., Eleven Papers on Logic, Algebra, Analysis and Topology
96  N. D. Filippirov et al., Ten Papers on Algebra and Functional Analysis
95  V. M. Adamjan et al., Eleven Papers in Analysis
94  V. A. Baranskii et al., Sixteen Papers on Logic and Algebra
93  Ju. M. Berezanskiĭ et al., Nine Papers on Functional Analysis
92  A. M. Ančikov et al., Seventeen Papers on Topology and Differential Geometry
91  L. I. Barklon et al., Eighteen Papers on Analysis and Quantum Mechanics
90  Z. S. Agranovič et al., Thirteen Papers on Functional Analysis
89  V. M. Alekseev et al., Thirteen Papers on Differential Equations
88  I. I. Eremin et al., Twelve Papers on Real and Complex Function Theory
87  M. A. Aizerman et al., Sixteen Papers on Differential and Difference Equations, Functional Analysis, Games and Control

(See the AMS catalog for earlier titles)