The Interplay between Differential Geometry and Differential Equations

V. V. Lychagin
Editor
Recent Titles in This Series

167 V. V. Lychagin, Editor, The Interplay between Differential Geometry and Differential Equations
166 O. A. Ladyzhenskaya, Editor, Proceedings of the St. Petersburg Mathematical Society, Volume III
165 Yu. Ilyashenko and S. Yakovenko, Editors, Concerning the Hilbert 16th Problem
164 N. N. Uraltseva, Editor, Nonlinear Evolution Equations
163 L. A. Bokut', M. Hazewinkel, and Yu. G. Reshetnyak, Editors, Third Siberian School "Algebra and Analysis"
162 S. G. Gindikin, Editor, Applied Problems of Radon Transform
161 Katsumi Nomizu, Editor, Selected Papers on Analysis, Probability, and Statistics
160 K. Nomizu, Editor, Selected Papers on Number Theory, Algebraic Geometry, and Differential Geometry
159 O. A. Ladyzhenskaya, Editor, Proceedings of the St. Petersburg Mathematical Society, Volume II
157 M. Sh. Birman, Editor, Wave Propagation. Scattering Theory
156 V. N. Gerasimov, N. G. Nesterenko, and A. I. Valitkas, Three Papers on Algebras and Their Representations
154 V. A. Artamonov et al., Selected Papers in K-Theory
153 S. G. Gindikin, Editor, Singularity Theory and Some Problems of Functional Analysis
152 H. Draškovičová et al., Ordered Sets and Lattices II
151 I. A. Aleksandrov, L. A. Bokut', and Yu. G. Reshetnyak, Editors, Second Siberian Winter School “Algebra and Analysis”
150 S. G. Gindikin, Editor, Spectral Theory of Operators
149 V. S. Afraimovich et al., Thirteen Papers in Algebra, Functional Analysis, Topology, and Probability, Translated from the Russian
147 I. G. Bashmakova et al., Nine Papers from the International Congress of Mathematicians, 1986
146 L. A. Aizenberg et al., Fifteen Papers in Complex Analysis
145 S. G. Dalalyan et al., Eight Papers Translated from the Russian
144 S. D. Berman et al., Thirteen Papers Translated from the Russian
143 V. A. Belonogov et al., Eight Papers Translated from the Russian
142 M. B. Abalovich et al., Ten Papers Translated from the Russian
141 H. Draškovičová et al., Ordered Sets and Lattices
140 V. I. Bernik et al., Eleven Papers Translated from the Russian
139 A. Ya. Aizenshtat et al., Nineteen Papers on Algebraic Semigroups
138 I. V. Kovalishina and V. P. Potapov, Seven Papers Translated from the Russian
137 V. I. Arnol'd et al., Fourteen Papers Translated from the Russian
136 L. A. Aksent'ev et al., Fourteen Papers Translated from the Russian
135 S. N. Artemov et al., Six Papers in Logic
134 A. Ya. Aizenshtat et al., Fourteen Papers Translated from the Russian
133 R. R. Suncheleev et al., Thirteen Papers in Analysis
132 I. G. Dmitriev et al., Thirteen Papers in Algebra
131 V. A. Zmorovich et al., Ten Papers in Analysis
130 M. M. Lavrent'ev, K. G. Reznitskaya, and V. G. Yakhno, One-dimensional Inverse Problems of Mathematical Physics
129 S. Ya. Khavinson, Two Papers on Extremal Problems in Complex Analysis

(See the AMS catalog for earlier titles)
This page intentionally left blank
The Interplay between Differential Geometry and Differential Equations
The Interplay between Differential Geometry and Differential Equations

V. V. Lychagin
Editor
ADVANCES IN THE MATHEMATICAL SCIENCES
EDITORIAL COMMITTEE

V. I. ARNOLD
S. G. GINDIKIN
V. P. MASLOV

1991 Mathematics Subject Classification. Primary 17B37, 35Axx, 58Axx, 58Gxx.

ABSTRACT. The purpose of the book is to emphasize the advantage of algebraic geometry approach to nonlinear differential equations, including applications of symplectic methods and the discussion of quantization problems. One of the common features for the majority of papers in the book is the systematic use of geometry of jet spaces. The book is useful to researchers and graduate students who are interested in nonlinear differential equations, differential geometry, quantum groups, and their applications.

Library of Congress Card Number 91-640741
ISBN 0-8218-0428-6
ISSN 0065-9290

Copying and reprinting. Material in this book may be reproduced by any means for educational and scientific purposes without fee or permission with the exception of reproduction by services that collect fees for delivery of documents and provided that the customary acknowledgment of the source is given. This consent does not extend to other kinds of copying for general distribution, for advertising or promotional purposes, or for resale. Requests for permission for commercial use of material should be addressed to the Assistant Director of Production, American Mathematical Society, P. O. Box 6248, Providence, Rhode Island 02940-6248. Requests can also be made by e-mail to reprint-permission@math.ams.org.

Excluded from these provisions is material in articles for which the author holds copyright. In such cases, requests for permission to use or reprint should be addressed directly to the author(s). (Copyright ownership is indicated in the notice in the lower right-hand corner of the first page of each article.)

© Copyright 1995 by the American Mathematical Society. All rights reserved.
The American Mathematical Society retains all rights except those granted to the United States Government.
Printed in the United States of America.

© The paper used in this book is acid-free and falls within the guidelines established to ensure permanence and durability.
-printed on recycled paper.
This volume was typeset by the authors using \LaTeX, the American Mathematical Society's \TeX macro system.

10 9 8 7 6 5 4 3 2 1 00 99 98 97 96 95
Contents

Foreword
V. Lychagin ix

Modeling Integro-Differential Equations and a Method for Computing their Symmetries and Conservation Laws
V. N. Chetverikov and A. G. Kudryavtsev 1

Braiding of the Lie Algebra $sl(2)$
J. Donin and D. Gurevich 23

Poisson–Lie Aspects of Classical $W$-Algebras
B. Enriquez, S. Khoroshkin, A. Radul, A. Rosly, and V. Rubtsov 37

On Symmetry Subalgebras and Conservation Laws for the $k – \varepsilon$ Turbulence Model and the Navier–Stokes Equations
N. G. Khor’kova and A. M. Verbovetsky 61

Graded Frölicher–Nijenhuis Brackets and the Theory of Recursion Operators for Super Differential Equations
P. H. M. Kersten and I. S. Krasil’chik 91

Symplectic Geometry of Mixed Type Equations
A. Kushner 131

Homogeneous Geometric Structures and Homogeneous Differential Equations
V. Lychagin 143

Geometry of Quantized Super PDE’s
Agostino Prástaro 165

Symmetries of Linear Ordinary Differential Equations
Alexey V. Samokhin 193

Foliations of Manifolds and Weighting of Derivatives
N. A. Shanahan 207

Higher Symmetry Algebra Structures and Local Equivalences of Euler–Darboux Equations
Valery E. Shemarulin 217

Hyperbolicity and Multivalued Solutions of Monge–Ampère Equations
D. V. Tunitsky 245
Singularities of Solutions of the Maxwell–Dirac Equation
L. ZILBERGLEIT 261

Characteristic Classes of Monge–Ampère Equations
L. ZILBERGLEIT 279
Foreword

This collection presents work concentrated mainly around the differential geometry approach to the theory of nonlinear differential equations.

Actually, differential geometry and differential equations are so closely related that it is practically impossible to draw a clear delimiting line between these two branches of mathematics.

Thus, whereas the connections between linear differential equations and differential geometry were few and far between, as were the connections between analytic geometry (in the elementary sense of the term) and geometry in the sense of Felix Klein, in contrast the theory of nonlinear differential equations is clearly a geometric theory, based on the special geometry of jet spaces.

Moreover, differential geometry, in its turn, may be presented as the part of the theory of nonlinear differential equations that studies differential equations that have no solutions. From this point of view, all the basic notions of differential geometry acquire their natural meaning. For example, various curvatures and torsion arise as obstructions to the solvability of the appropriate differential equations. In this sense the theory of differential equations in its classical understanding is the geometry of flat objects.

The papers comprising this collection may be conditionally subdivided into three groups.


Finally, the third group is devoted to quantization problems (I. Donin and D. Gurevich, B. Enriquez, S. Khoroshkin, A. Radul, A. Rosly, and V. Rubtsov).

The appearance of this last group of papers in a collection mainly concerned with differential equations understood in the classical sense, and not in the quantum one, nevertheless seems quite logical to me, since the true understanding of the problems of differential equations must unavoidably pass through the quantum domain.

V. Lychagin
Moscow, August 1994

Translated by A. B. SOSSINSKY
This page intentionally left blank