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179  V. M. Buchstaber and S. P. Novikov, Editors, Solitons, Geometry, and Topology: On the Crossroad
178  V. Kreinovich and G. Mints, Editors, Problems of Reducing the Exhaustive Search
177  R. L. Dobrushin, R. A. Minlos, M. A. Shubin, and A. M. Vershik, Editors, Topics in Statistical and Theoretical Physics (F. A. Berezin Memorial Volume)
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175  R. L. Dobrushin, R. A. Minlos, M. A. Shubin, and A. M. Vershik, Editors, Contemporary Mathematical Physics (F. A. Berezin Memorial Volume)
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149  V. S. Afraimovich et al., Thirteen Papers in Algebra, Functional Analysis, Topology, and Probability, Translated from the Russian
147  I. G. Bashmakova et al., Nine Papers from the International Congress of Mathematicians, 1986

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Solitons, Geometry, and Topology: On the Crossroad
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Solitons, Geometry, and Topology: On the Crossroad

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S. P. Novikov
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Secondary 16W30.

ABSTRACT. The present collection contains articles by participants of the S. P. Novikov seminar
on topology and mathematical physics in Moscow. The papers in the collection are devoted
to problems in geometry, topology, and mathematical physics, including some applications of
topology to physical problems. The book will be of interest to researchers and graduate students
working in the corresponding area of mathematics and mathematical physics.

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10 9 8 7 6 5 4 3 2 1 02 01 00 99 98 97
Preface ix

Hyperelliptic Kleinian functions and applications
V. M. Buchstaber, V. Z. Enolskii, and D. V. Leikin 1

Functionals of the Peierls–Fröhlich type and the variational principle for the
Whitham equations
Boris Dubrovin 35

Semiclassical motion of the electron. A proof of the Novikov conjecture in
general position and counterexamples
I. A. Dynnikov 45

An invariant of integral homology 3-spheres which is universal for all finite
type invariants
Thang T. Q. Le 75

Krichever–Novikov algebras and the cohomology of the algebra of
meromorphic vector fields
D. V. Millionshchikov 101

Exactly solvable two-dimensional Schrödinger operators and Laplace
transformations
S. P. Novikov and A. P. Veselov 109

Modified Novikov–Veselov equation and differential geometry of surfaces
Iskander A. Taimanov 133

Supermanifold forms and integration. A dual theory
Theodore Voronov 153

On hyperplane sections of periodic surfaces
Anton Zorich 173
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Preface

The present collection combines papers in topology and mathematical physics, including some applications of topology to physical problems. Such a combination is a characteristic of the S. P. Novikov seminar.

This collection is the direct continuation of the book *Topics in Topology and Mathematical Physics*, Advances in the Mathematical Sciences, S. P. Novikov (ed.) Amer. Math. Soc. Translations, Ser. 2, Vol. 170, 1995. As in the previous collection, this book contains articles by participants of the S. P. Novikov seminar. Here we shall not recall the history of the seminar, which begins back in 1965 (see the introduction to the previous volume).

Integration theory for various systems of mathematical physics, involving $\theta$-functions and Kleinian functions on Riemann surfaces, is the topic of the articles [1, 6, 7]. In [1], a series of new applications of the theory of Klein $\sigma$-functions on hyperelliptic Riemann surfaces is presented. This opens new possibilities for the theory of nonlinear equations of the KdV type and for linear operators. The paper [6] deals with exactly solvable two-dimensional Schrödinger operators in magnetic fields and with new methods using “Laplacian” chains. In [7] an elegant differential-geometric interpretation of integrable $2 + 1$ systems, well known in soliton theory, is discussed.

The article [2] is a study of the relationship between the minimization problem for functionals of the Peierls–Fröhlich type and evolution problems for multivalued functionals related to dispersive hydrodynamics.

The paper [5] deals with the homology theory of Lie algebras that appeared in the work of Novikov and Krichever in the quantum theory of boson strings. This homology has not been computed at present.

The paper [3] is devoted to the qualitative theory of one-dimensional Hamiltonian foliations on periodic surfaces in $\mathbb{R}^3$. The topological problems considered there arose in the quantum theory of solids. The paper [9] deals with the multidimensional generalization of this problem.

The paper [4] is about three-dimensional topology, specifically the theory of Vassiliev invariants for knots and links, as well as about the corresponding invariants for 3-manifolds.

The paper [8] is concerned with the geometry and topology of supermanifolds and the theory of differential forms on such manifolds.
References

1. V. M. Buchstaber, V. Z. Enolskii, and D. V. Leikin, Hyperelliptic Kleinian functions and applications.
2. Boris Dubrovin, Functionals of the Peierls–Fröhlich type and the variational principle for the Whitam equations.
4. Thang T. Q. Le, An invariant of integral homology 3-spheres which is universal for all finite type invariants.
6. S. P. Novikov and A. P. Veselov, Exactly solvable two-dimensional Schrödinger operators and Laplace transformations.
7. Iksander A. Taimanov, Modified Novikov–Veselov equation and differential geometry of surfaces.
8. Theodore Voronov, Supermanifold forms and integration. A dual theory.

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