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Preface

This is a collection of several papers that originally appeared in the journal Sugaku in Japanese. These translated articles would normally appear in the AMS journal Sugaku Expositions. In order to expedite publication, the AMS has chosen, with the consent of the Mathematical Society of Japan, to publish them as a volume of selected papers in the Society’s Translations, Series 2.

The classical representation theory for commutative Lie groups was developed in close connection with harmonic analysis (Fourier series and Fourier transforms). The paper by Kobayashi starts with a survey of the geometry of various homogeneous manifolds and the construction of standard unitary representations associated to elliptic orbits, and continues to $L^2$-harmonic analysis, in particular, discrete series representations in relation to the restrictions of unitary representations to reductive subgroups. The author has added a few remarks on very recent results and an update on references.

Harmonic maps, minimal submanifolds, Einstein metrics and Yang-Mills connections come from important variational problems in differential geometry with close ties to global analysis, topology and mathematical physics. The paper by Guest and Ohnita deals with harmonic maps as perhaps the most fundamental aspect, and discusses group actions and resulting deformations in the space of harmonic maps, utilizing loop groups, thus studying the geometry and topology of the space of harmonic maps.

The paper by Umeda gives a historical review on Capelli identities from their initial creation to the current revival of interest and new related research. Algebraic invariant theory was started by Boole and developed by Cayley about 150 years ago. Just as Hilbert’s new foundation was being developed, Capelli in 1887-90 introduced the idea of Capelli identity, which played a central role in H. Weyl’s Classical Groups (1939). This paper presents a new, deep understanding of the identities from the viewpoint of representation theory and other contemporary research in mathematics.

The paper by Koike is concerned with the representations of classical groups such as $GL(n, \mathbb{C})$, $SL(n, \mathbb{C})$, $O(n, \mathbb{C})$, $SO(n, \mathbb{C})$, and $Sp(2n, \mathbb{C})$ in terms of combinatorics on Young diagrams. The formula for the irreducible decompositions of the tensor product of two given irreducible representations of a classical group is treated, as well as the rules governing the restriction of a representation to subgroups.
The theory of elliptic integrals has long been an idealistic form and an inexhaustible source of mathematical research. This was basically the theory of cubic polynomials of two complex variables, and developed into transcendental elliptic integrals through the depth of the periods of integrals and the relations to the Gauss-Manin equation and the recent Hodge theory. Another interesting aspect is the so-called Jacobi inverse problem – the construction of the theta functions and the modular forms. The paper by Saito introduces the concept of regular system of weights (generalizing some of the familiar concepts of Lie group theory – the root system, the Coxeter transformation, the exponent, etc.) and a list of 49 special polynomials in three variables. Each of these gives a $K3$ surface and its polarization, and provides good material for research on the period maps.

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