American Mathematical Society
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Advances in the Mathematical Sciences

Moscow Seminar in Mathematical Physics

A. Yu. Morozov
M. A. Olshanetsky
Editors

American Mathematical Society

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Moscow Seminar in Mathematical Physics

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Advances in the Mathematical Sciences — 43

(*Formerly Advances in Soviet Mathematics*)

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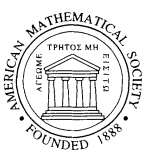

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EDITORIAL COMMITTEE

V. I. ARNOLD
S. G. GINDIKIN
V. P. MASLOV

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ABSTRACT. The volume contains articles resulting from talks given at the seminar in mathematical physics at Moscow Institute of Theoretical and Experimental Physics. The articles are mainly devoted to various aspects of Knizhnik-Zamolodchikov-Bernard connections and integrable models in two-dimensional quantum field theory.

The book is useful for researchers and graduate students working in various areas of mathematical and theoretical physics.

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Preface

ITEP: Proceedings of the Mathematical Physics Seminar

The Theory Department of the Institute of Theoretical and Experimental Physics was founded by I. Pomeranchuk in 1946 and is internationally recognized for achievements in various branches of theoretical physics. For many years, ITEP and especially its seminars have been among the main centers of scientific life in Moscow. The most remarkable feature of the best Moscow seminars is that these are places where science is done, not just reported.

This volume represents some results from the seminar on mathematical physics, which has been conducted at ITEP since 1983. This is not the only ITEP theoretical seminar. For this reason many of the famous results of the ITEP school are not reflected in this book. Another restriction on its content is the emphasis on the current activities in one particular field: integrable systems and their role in modern quantum theory. This has been one of the favorite topics of research at ITEP for many years. A lot of old results due to the ITEP theory group in integrability are beyond the scope of this volume. Still, it represents to a good extent the general style and one particular direction of our work. At the present time it attracts new attention because of the role played by integrability in describing nonperturbative phenomena in the framework of contemporary string theory.

Generally speaking, research topics included in the volume originally appeared in modern physics, and their transformation into well-defined mathematical problems is due to the intensive cooperation between physicists and mathematicians. The current mainstream of theoretical physics is becoming more and more geometrical. Thus, collaboration between physicists and mathematicians (with ITEP as one of the meeting points in Moscow) will undoubtedly produce new results in the same spirit in the future.

The seminar, although affiliated to ITEP, is in fact a meeting place which attracts people from different institutes. We have also been happy to see many mathematicians coming to the seminar. We are grateful to all those who have shared their time and inspiration with us during all these years. Moscow science could not have achieved its success without such scientific seminars, and it is important that they continue to work. We are glad that so many people feel the same way and are working toward this. Their support means a lot.

Most of the papers in this volume ([**1, 3, 4, 6, 8, 9**]) describe a certain universal structure in 2d quantum field theory, namely the Knizhnik–Zamolodchikov–Bernard connection and its far-reaching generalizations. The remaining papers are related to other aspects of 2d quantum field theory and integrable models. In [**7**] quantum Lax operators with elliptic dependence on a spectral parameter are analyzed by methods

of algebraic geometry. A family of current algebras associated with complex curves is investigated in [**2**]. In [**5**] the relationship between matrix models and integrable systems is described.

We appreciate the kind suggestion of Simon Gindikin that we should publish this book. We are also indebted to Sergey Kharchev and Alesha Rosly for their help in the work on the volume.

A. Morozov, M. Olshanetsky

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[1] G. Arutyunov, L. Chekhov and S. Frolov, *Quantum dynamical R-matrices*, this volume, pp. 1–32.

[2] B. Enriques and V. Rubtsov, *Some examples of quantum groups in higher genus*, this volume, pp. 33–65.

[3] V. Fock and A. Rosly, *Poisson structure on moduli of flat connections on Riemann surfaces and the r-matrix*, this volume, pp. 67–86.

[4] D. Ivanov and A. Losev, *KZB equations as a flat connection with spectral parameter*, this volume, pp. 87–117.

[5] S. Kharchev, *Kadomtsev–Petviashvili hierarchy and generalized Kontsevich model*, this volume, pp. 119–162.

[6] S. Khoroshkin, D. Lebedev, and S. Pakuliak, *Yangian algebras and classical Riemann problems*, this volume, pp. 163–198.

[7] I. Krichever and A. Zabrodin, *Vacuum curves of elliptic L-operators and representations of the Sklyanin algebra*, this volume, pp. 199–221.

[8] A. Levin and M. Olshanetsky, *Hierarchies of isomonodromic deformations and Hitchin systems*, this volume, pp. 223–262.

[9] N. Nekrasov, *Infinite-dimensional algebras, many-body systems and gauge theories*, this volume, pp. 263–299.

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