American Mathematical Society
TRANSLATIONS
Series 2 • Volume 202
Advances in the Mathematical Sciences

Topology, Ergodic Theory, Real Algebraic Geometry

Rokhlin's Memorial

V. Turaev
A. Vershik
Editors

American Mathematical Society

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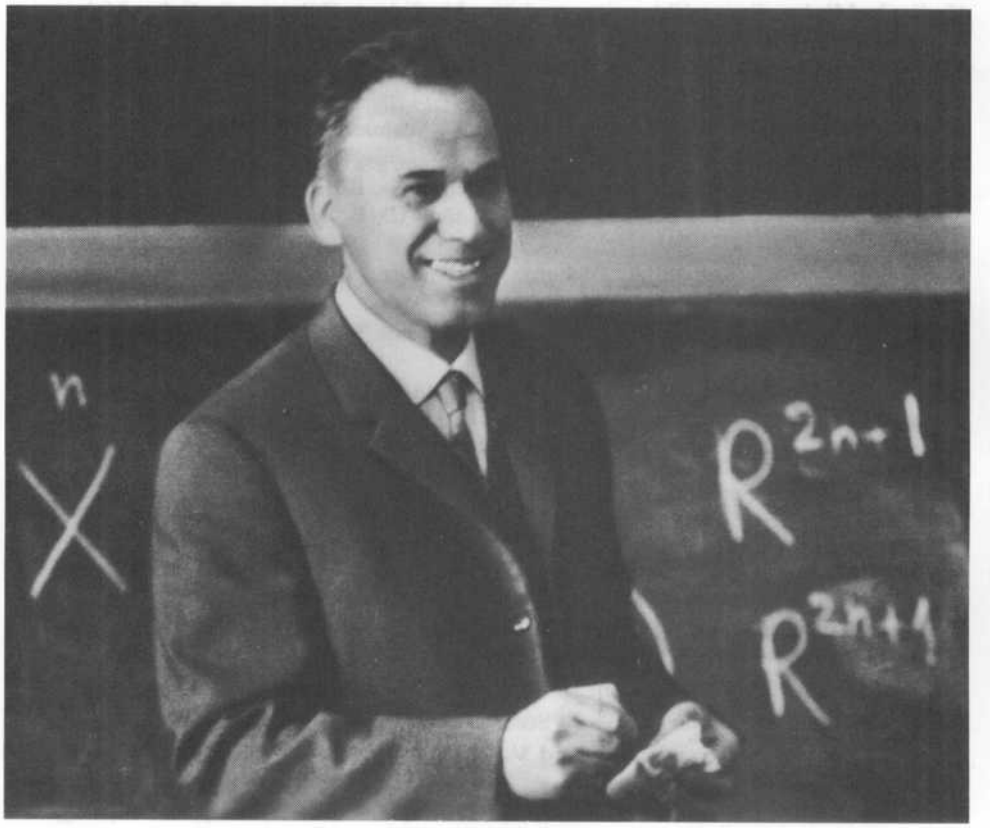

Vladimir Abramovich
ROKHLIN
1919–1984

Topology, Ergodic Theory, Real Algebraic Geometry

Rokhlin's Memorial

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TRANSLATIONS

Series 2 • Volume 202

Advances in the Mathematical Sciences—50

(*Formerly Advances in Soviet Mathematics*)

Topology, Ergodic Theory, Real Algebraic Geometry

Rokhlin's Memorial

V. Turaev
A. Vershik
Editors

American Mathematical Society
Providence, Rhode Island

ADVANCES IN THE MATHEMATICAL SCIENCES
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ABSTRACT. Vladimir Abramovich Rokhlin (August 23, 1919–December 4, 1984) was an outstanding Russian mathematician whose contributions to topology and ergodic theory have been internationally recognized. The present volume contains papers by Rokhlin's colleagues and students; the topics covered by these papers reflect the wide spectrum of Rokhlin's interests (topology, geometry, real algebraic geometry, dynamical systems).

The book is useful to researchers and graduate students working in topology, algebraic geometry, and dynamical systems.

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Foreword

Vladimir Abramovich Rokhlin (August 23, 1919–December 4, 1984) was an outstanding Russian mathematician. He considerably influenced the development of mathematics in Russia in the period from 1950 to the early '80's, especially in Moscow and Leningrad (St. Petersburg). Rokhlin's contributions to topology and ergodic theory have been internationally recognized.

Rokhlin studied mathematics at Moscow University in the late 1930's. Among his professors were P. S. Aleksandrov, I. M. Gel′fand, A. N. Kolmogorov, A. N. Plesner, and L. S. Pontryagin.

Rokhlin successfully worked in several areas of mathematics. His papers on three-dimensional and four-dimensional topology, cobordism theory, real algebraic geometry, and ergodic theory have become classics. For a discussion of his contributions to topology and dynamical systems, see Uspekhi Mat. Nauk (Russian Math. Surveys (2000), no. 4); Ergodic Theory and Dynamical Systems **9** (1989), no. 4; and the book *À la Recherche de la Topologie Perdue*, by L. Guillou and A. Marin (Birkhäuser, 1986). See also the papers in this volume.

First of all, Rokhlin was a geometer and topologist with a special taste for algebraic methods. His knowledge and understanding of various mathematical fields allowed him to see deep interconnections and the unity of mathematics. The work of Rokhlin is characterized by vast mathematical culture, a sense of the new, and an exquisite writing style. He always tried to pass these qualities on to his students and younger colleagues.

During the last 25 years of his life, V. A. Rokhlin worked at Leningrad University, where he created a strong school of topology, real algebraic geometry and dynamical systems. A number of his students now work at leading universities of Europe and America. Their papers are in diverse mathematical fields, often far removed from Rokhlin's own interests. Among his students and mathematicians personally influenced by him, let us mention L. Abramov, R. Belinskaya, Ya. Eliashberg, S. Finashin, M. Gromov, N. Ivanov, V. Kharlamov, N. Netsvetaev, V. Turaev, A. Vershik, O. Viro, S. Yuzvinsky, and A. Zhubr.

A set of important biographic materials, photos and recollections devoted to Rokhlin appeared in 1999 in a volume of his selected papers, edited by A. Vershik, with contributions by V. Arnold, S. Novikov, Ya. Sinaĭ, and A. Vershik (MCCME Publishers, Moscow, 1999). In this volume we reproduce Rokhlin's biography written by one of the editors. In August 1999 at the Euler Institute in St. Petersburg the conference "Topology and Dynamics: Rokhlin's Memorial" with more than 100 participants from many countries took place. Proceedings of this conference will appear in Zapiski Nauchnyh Seminarov POMI, **266**, **267** (English translation in

J. Math. Sci.). The issues of St. Petersburg Math. J. **11**, no. 5, and Proceedings of St. Petersburg Math. Soc. **7** also were devoted to V. A. Rokhlin.

Here we would like to say a few words about Rokhlin's life. He had to confront the most serious hardships and to suffer from both the Nazi and the Communist systems. His father was arrested and shot in the period of Stalinist purges. During World War II Rokhlin was incarcerated in a German concentration camp for almost three years. After the end of the war he was sent to a Soviet deportation camp for former prisoners of war. Even during the relatively prosperous period of his stay in Leningrad, Rokhlin had to suffer from permanent clashes with hostile scientific and party bureaucracy, which finally forced him to retire at the age of 60.

Soviet authorities never allowed Rokhlin to take part in scientific conferences abroad (not even in the countries belonging to the so-called "socialist camp"); he was prohibited from going outside the USSR. Rokhlin did not live to see the time when he would be able to accept invitations from foreign universities.

The influence of Rokhlin on the Leningrad mathematical community and on the mathematics department of the university was very strong despite all the obstacles. He initiated the first (in Russia) obligatory course in topology for undergraduate students. He was an excellent lecturer and an admirable scientific adviser.

In the present volume we have collected several papers by Rokhlin's students, followers, and their students. The topics covered by these papers reflect the wide spectrum of Rokhlin's interests (topology, geometry, real algebraic geometry, dynamical systems) as well as the evolution of his students since his death. We do not analyze the content of the papers here, leaving it to the reader to find the topics of his/her interest.

We sincerely hope that this volume will contribute to the memory of this remarkable scholar, to whom we both owe so much.

Vladimir Turaev, Anatoly Vershik

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