American Mathematical Society
TRANSLATIONS
Series 2 • Volume 204

Selected Papers on Classical Analysis

Katsumi Nomizu
Editor

American Mathematical Society

Selected Titles in This Series

204 **K. Nomizu, Editor,** Selected papers on classical analysis

203 **N. N. Uraltseva, Editor,** Proceedings of the St. Petersburg Mathematical Society, Volume VII

202 **V. Turaev and A. Vershik, Editors,** Topology, ergodic theory, real algebraic geometry. Rokhlin's memorial

201 **Michael Semenov-Tian-Shansky, Editor,** L. D. Faddeev's seminar on mathematical physics

200 **L. Lerman, G. Polotovskiĭ, and L. Shilnikov, Editors,** Methods of qualitative theory of differential equations and related topics

199 **N. N. Uraltseva, Editor,** Proceedings of the St. Petersburg Mathematical Society, Volume VI

198 **R. A. Minlos, Senya Shlosman, and Yu. M. Suhov, Editors,** On Dobrushin's way. From probability theory to statistical physics

197 **Vladimir Arnold, Maxim Kontsevich, and Anton Zorich, Editors,** Pseudoperiodic topology

196 **Ya. Eliashberg, D. Fuchs, T. Ratiu, and A. Weinstein, Editors,** Northern California symplectic geometry seminar

195 **O. V. Belegradek et al.,** Model Theory and Applications

194 **A. Astashkevich and S. Tabachnikov, Editors,** Differential Topology, Infinite-Dimensional Lie Algebras, and Applications (D. B. Fuchs' 60th Anniversary Collection)

193 **N. N. Uraltseva, Editor,** Proceedings of the St. Petersburg Mathematical Society, Volume V

192 **Lev Beklemishev, Mati Pentus, and Nikolai Vereshchagin,** Provability, Complexity, Grammars

191 **A. Yu. Morozov and M. A. Olshanetsky, Editors,** Moscow Seminar in Mathematical Physics

190 **S. Tabachnikov, Editor,** Differential and Symplectic Topology of Knots and Curves

189 **V. Buslaev, M. Solomyak, and D. Yafaev, Editors,** Differential Operators and Spectral Theory (M. Sh. Birman's 70th anniversary collection)

188 **O. A. Ladyzhenskaya, Editor,** Proceedings of the St. Petersburg Mathematical Society, Volume IV

187 **M. V. Karasev, Editor,** Coherent Transform, Quantization, and Poisson Geometry

186 **A. Khovanskiĭ, A. Varchenko, and V. Vassiliev, Editors,** Geometry of Differential Equations

185 **B. Feigin and V. Vassiliev, Editors,** Topics in Quantum Groups and Finite-Type Invariants (Mathematics at the Independent University of Moscow)

184 **Peter Kuchment and Vladimir Lin, Editors,** Voronezh Winter Mathematical Schools (Dedicated to Selim Krein)

183 **K. Nomizu, Editor,** Selected Papers on Harmonic Analysis, Groups, and Invariants

182 **V. E. Zakharov, Editor,** Nonlinear Waves and Weak Turbulence

181 **G. I. Olshanski, Editor,** Kirillov's Seminar on Representation Theory

180 **A. Khovanskiĭ, A. Varchenko, and V. Vassiliev, Editors,** Topics in Singularity Theory

179 **V. M. Buchstaber and S. P. Novikov, Editors,** Solitons, Geometry, and Topology: On the Crossroad

178 **V. Kreinovich and G. Mints, Editors,** Problems of Reducing the Exhaustive Search

177 **R. L. Dobrushin, R. A. Minlos, M. A. Shubin, and A. M. Vershik, Editors,** Topics in Statistical and Theoretical Physics (F. A. Berezin Memorial Volume)

176 **E. V. Shikin, Editor,** Some Questions of Differential Geometry in the Large

(*Continued in the back of this publication*)

This page intentionally left blank

Selected Papers on Classical Analysis

This page intentionally left blank

http://dx.doi.org/10.1090/trans2/204

American Mathematical Society

TRANSLATIONS

Series 2 • Volume 204

Selected Papers on Classical Analysis

Katsumi Nomizu
Editor

American Mathematical Society
Providence, Rhode Island

Translation edited by KATSUMI NOMIZU

2000 *Mathematics Subject Classification.* Primary 46J10, 34M55, 35K57, 35Q30, 57S30, 46Lxx.

Library of Congress Cataloging-in-Publication Data

Selected papers on classical analysis / Katsumi Nomizu, editor.
p. cm. — (American Mathematical Society translations, ISSN 0065-9290 ; ser. 2, v. 204)
Includes bibliographical references.
ISBN 0-8218-2780-4 (acid-free paper)
1. Mathematical analysis. I. Nomizu, Katsumi, 1924– II. Series.

QA3.A572 ser. 2, vol. 204
[QA300.5]
510 s—dc21 2001022373
[515]

Copying and reprinting. Material in this book may be reproduced by any means for educational and scientific purposes without fee or permission with the exception of reproduction by services that collect fees for delivery of documents and provided that the customary acknowledgment of the source is given. This consent does not extend to other kinds of copying for general distribution, for advertising or promotional purposes, or for resale. Requests for permission for commercial use of material should be addressed to the Assistant to the Publisher, American Mathematical Society, P. O. Box 6248, Providence, Rhode Island 02940-6248. Requests can also be made by e-mail to `reprint-permission@ams.org`.

Excluded from these provisions is material in articles for which the author holds copyright. In such cases, requests for permission to use or reprint should be addressed directly to the author(s). (Copyright ownership is indicated in the notice in the lower right-hand corner of the first page of each article.)

© 2001 by the American Mathematical Society. All rights reserved.
The American Mathematical Society retains all rights
except those granted to the United States Government.
Printed in the United States of America.

⊗ The paper used in this book is acid-free and falls within the guidelines
established to ensure permanence and durability.
Visit the AMS home page at URL: `http://www.ams.org/`

10 9 8 7 6 5 4 3 2 1 03 02 01

Contents

Preface ix

Free Products of Operator Algebras and Free Probability Theory
FUMIO HIAI 1

Dynamics of Kleinian Groups—The Hausdorff Dimension of Limit Sets
KATSUHIKO MATSUZAKI 23

Topological Methods in Stability Analysis of Travelling Waves
SHUNSAKU NII 45

Extension of Almost Periodic Functions and Analyticity on Flows
JUN-ICHI TANAKI 63

Painlevé Equations in the Past 100 Years
HIROSHI UMEMURA 81

The Navier-Stokes Equation in Various Function Spaces
MASAO YAMAZAKI 111

This page intentionally left blank

Preface

This is a collection of several papers that originally appeared in the journal *Sūgaku* in Japanese. These translated articles would normally appear in the AMS journal *Sugaku Expositions*. In order to expedite publication, the AMS has chosen, with the consent of the Mathematical Society of Japan, to publish them as a volume of selected papers in the *Translations Series* 2 of the AMS.

The volume contains papers in the general area of mathematical analysis as it pertains to Free Probability, built based on operator algebras, in **Hiai**'s paper, and in **Matsuzaki**'s paper, to dynamics of Kleinian groups that originally goes back to the classical work by Fricke, Klein, and Poincaré and continues to Ahlfors and Bers, and develops into the recent work by Sullivan.

Nii's paper discusses stability analysis of travelling waves, and **Tanaka**'s article deals with extension of almost periodic functions and analyticity on related flows. One problem proposed by Forelli in 1970 on whether the function ring introduced by a minimal flow is a Dirichlet algebra is negatively answered.

Umemura's article discusses the Painlevé equations (known as P_I, P_{II}, P_{III}, P_{IV}, P_V, P_{VI}) that Painlevé discovered around 1890. Regarding whether they are irreducible and are really special functions, Poincaré had serious doubt. The last question was settled for P_I only in 1988. The validity in the general case is now convincing thanks to results related to representation theory—about 100 years after the original problem was posed.

Finally, **Yamazaki**'s article is summarized as follows: As an example of the close relationship between real analysis and nonlinear partial differential equations, the author surveys the recent work on the Navier-Stokes equations employing various function spaces, in particular, the work by Miyakawa on the decay of weak solutions, and that of Kozono and Yamazaki on the Cauchy problem with distributions as initial data.

Katsumi Nomizu

This page intentionally left blank

Selected Titles in This Series

(*Continued from the front of this publication*)

175 **R. L. Dobrushin, R. A. Minlos, M. A. Shubin, and A. M. Vershik, Editors,** Contemporary Mathematical Physics (F. A. Berezin Memorial Volume)

174 **A. A. Bolibruch, A. S. Merkur′ev, and N. Yu. Netsvetaev, Editors,** Mathematics in St. Petersburg

173 **V. Kharlamov, A. Korchagin, G. Polotovskiĭ, and O. Viro, Editors,** Topology of Real Algebraic Varieties and Related Topics

172 **K. Nomizu, Editor,** Selected Papers on Number Theory and Algebraic Geometry

171 **L. A. Bunimovich, B. M. Gurevich, and Ya. B. Pesin, Editors,** Sinai's Moscow Seminar on Dynamical Systems

170 **S. P. Novikov, Editor,** Topics in Topology and Mathematical Physics

169 **S. G. Gindikin and E. B. Vinberg, Editors,** Lie Groups and Lie Algebras: E. B. Dynkin's Seminar

168 **V. V. Kozlov, Editor,** Dynamical Systems in Classical Mechanics

167 **V. V. Lychagin, Editor,** The Interplay between Differential Geometry and Differential Equations

166 **O. A. Ladyzhenskaya, Editor,** Proceedings of the St. Petersburg Mathematical Society, Volume III

165 **Yu. Ilyashenko and S. Yakovenko, Editors,** Concerning the Hilbert 16th Problem

164 **N. N. Uraltseva, Editor,** Nonlinear Evolution Equations

163 **L. A. Bokut′, M. Hazewinkel, and Yu. G. Reshetnyak, Editors,** Third Siberian School "Algebra and Analysis"

162 **S. G. Gindikin, Editor,** Applied Problems of Radon Transform

161 **K. Nomizu, Editor,** Selected Papers on Analysis, Probability, and Statistics

160 **K. Nomizu, Editor,** Selected Papers on Number Theory, Algebraic Geometry, and Differential Geometry

159 **O. A. Ladyzhenskaya, Editor,** Proceedings of the St. Petersburg Mathematical Society, Volume II

158 **A. K. Kelmans, Editor,** Selected Topics in Discrete Mathematics: Proceedings of the Moscow Discrete Mathematics Seminar 1972–1990

157 **M. Sh. Birman, Editor,** Wave Propagation. Scattering Theory

156 **V. N. Gerasimov, N. G. Nesterenko, and A. I. Valitskas,** Three Papers on Algebras and Their Representations

155 **O. A. Ladyzhenskaya and A. M. Vershik, Editors,** Proceedings of the St. Petersburg Mathematical Society, Volume I

154 **V. A. Artamonov et al.,** Selected Papers in K-Theory

153 **S. G. Gindikin, Editor,** Singularity Theory and Some Problems of Functional Analysis

152 **H. Draškovičová et al.,** Ordered Sets and Lattices II

151 **I. A. Aleksandrov, L. A. Bokut′, and Yu. G. Reshetnyak, Editors,** Second Siberian Winter School "Algebra and Analysis"

150 **S. G. Gindikin, Editor,** Spectral Theory of Operators

149 **V. S. Afraĭmovich et al.,** Thirteen Papers in Algebra, Functional Analysis, Topology, and Probability, Translated from the Russian

148 **A. D. Aleksandrov, O. V. Belegradek, L. A. Bokut′, and Yu. L. Ershov, Editors,** First Siberian Winter School "Algebra and Analysis"

147 **I. G. Bashmakova et al.,** Nine Papers from the International Congress of Mathematicians, 1986

For a complete list of titles in this series, visit the AMS Bookstore at **www.ams.org/bookstore/**.

ISBN 0-8218-2780-4

9 780821 827802

TRANS2/204

AMS *on the* Web
www.ams.org