American Mathematical Society

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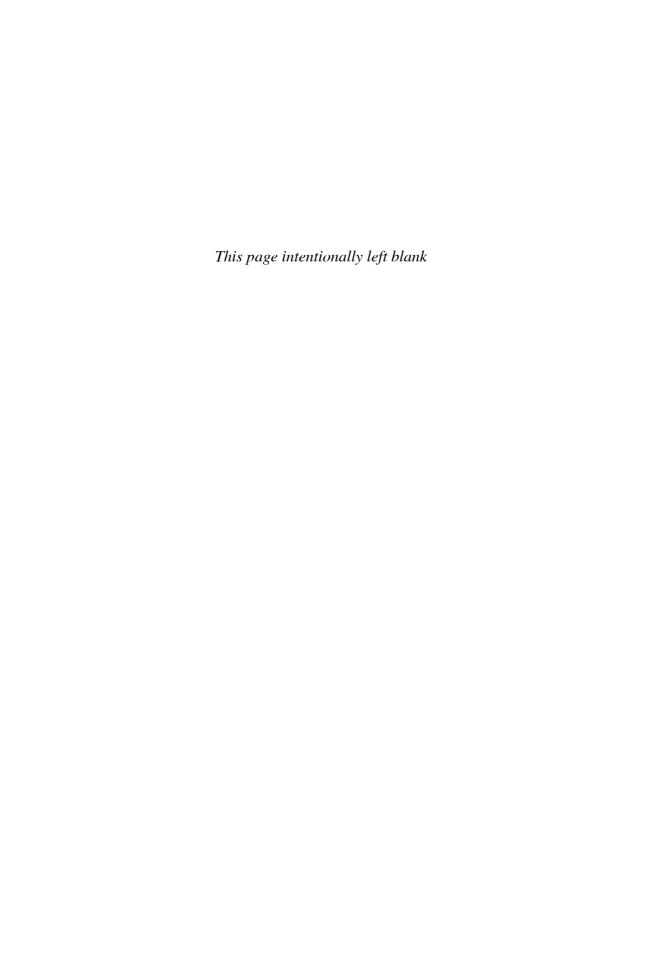
Selected Papers on Classical Analysis

Katsumi Nomizu Editor

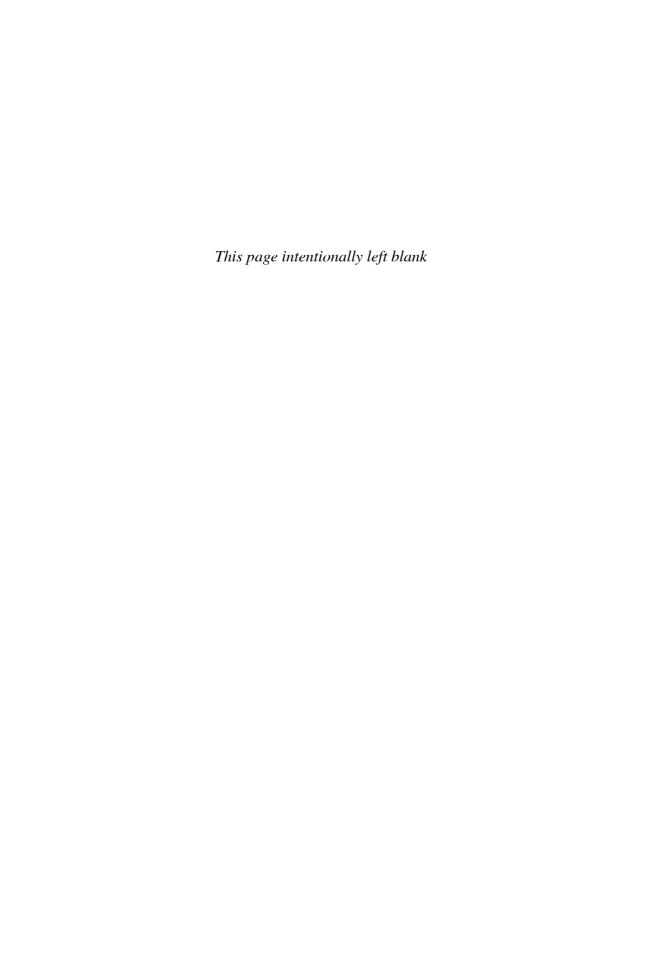
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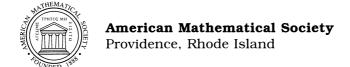
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Katsumi Nomizu Editor



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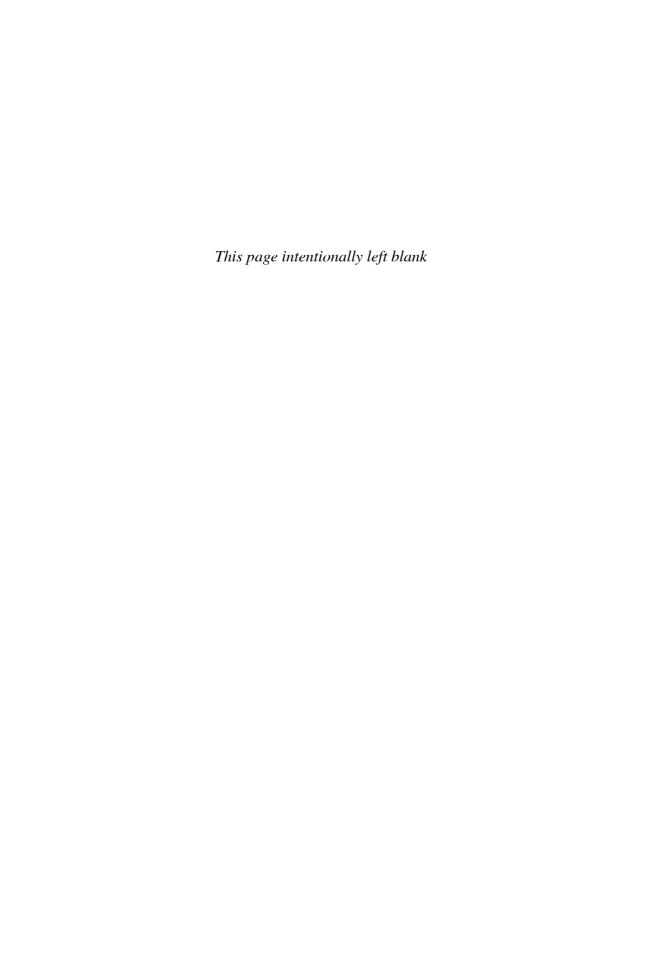
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Preface

This is a collection of several papers that originally appeared in the journal $S\bar{u}gaku$ in Japanese. These translated articles would normally appear in the AMS journal Sugaku Expositions. In order to expedite publication, the AMS has chosen, with the consent of the Mathematical Society of Japan, to publish them as a volume of selected papers in the Translations Series 2 of the AMS.

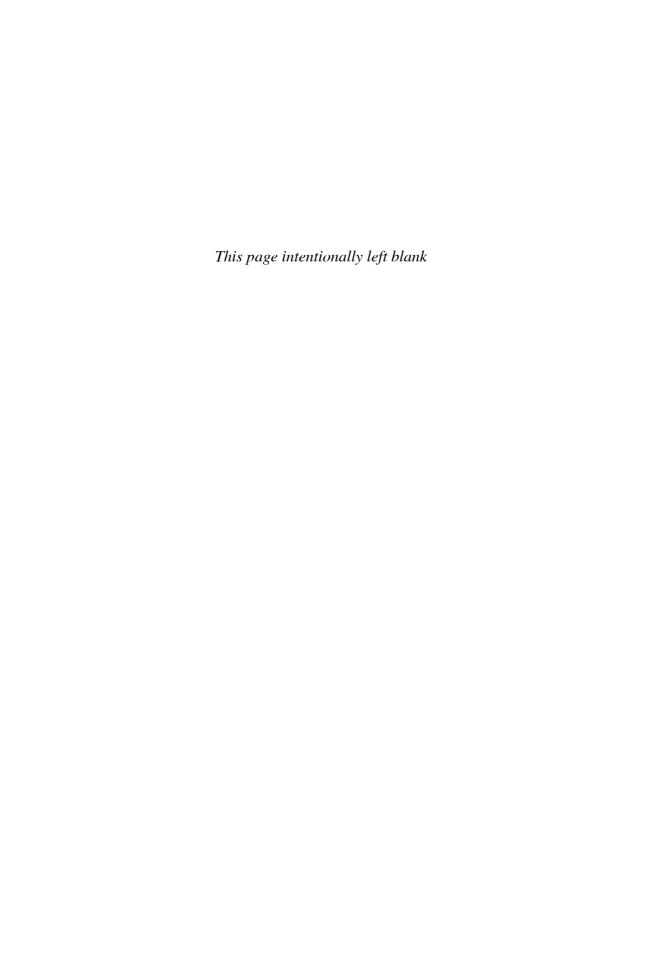
The volume contains papers in the general area of mathematical analysis as it pertains to Free Probability, built based on operator algebras, in **Hiai**'s paper, and in **Matsuzaki**'s paper, to dynamics of Kleinian groups that originally goes back to the classical work by Fricke, Klein, and Poincaré and continues to Ahlfors and Bers, and develops into the recent work by Sullivan.

Nii's paper discusses stability analysis of travelling waves, and Tanaka's article deals with extension of almost periodic functions and analyticity on related flows. One problem proposed by Forelli in 1970 on whether the function ring introduced by a minimal flow is a Dirichlet algebra is negatively answered.

Umemura's article discusses the Painlevé equations (known as P_I , P_{II} , P_{III} , P_{IV} , P_V , P_V) that Painlevé discovered around 1890. Regarding whether they are irreducible and are really special functions, Poincaré had serious doubt. The last question was settled for P_I only in 1988. The validity in the general case is now convincing thanks to results related to representation theory—about 100 years after the original problem was posed.

Finally, Yamazaki's article is summarized as follows: As an example of the close relationship between real analysis and nonlinear partial differential equations, the author surveys the recent work on the Navier-Stokes equations employing various function spaces, in particular, the work by Miyakawa on the decay of weak solutions, and that of Kozono and Yamazaki on the Cauchy problem with distributions as initial data.

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