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In Memory of Fridrikh Karpelevich

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Analytic Methods in Applied Probability

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Foreword

This volume is dedicated to the memory of Fridrikh Karpelevich (1927–2000) and is one of two books planned to commemorate his contribution to various areas of mathematics. As can be seen from the title, this volume focuses on Karpelevich's work in and its impact on applied probability.

The volume opens with a review by Kreinin and Suhov of Karpelevich's contribution to applied probability theory. It also contains a bibliography of his works in this area. An article by Boxma, Schlegel and Yechiali follows, where an M/G/1 queue is analyzed, with the server taking subsequent "vacations" at the end of each busy period. The authors produce a variety of formulas for the Laplace–Stieltjes transform of related probability distributions. Next, a paper by Foss and Zachary deals with asymptotics, as $y \to \infty$, of probability $\mathbb{P}\Big(\sup_n \sum_{i=1}^n \xi_i^{X_i} > y\Big)$. Here X_1, $X_2, \ldots$ is a Markov chain with a discrete state space $\mathcal{X}$ and $\{\xi_n^x,\, x \in \mathcal{X}\}_{n=1,2,\ldots}$ is a sequence of independent families of random variables ξ_n^x. The main results of the paper generalize the well-known formula

$$\mathbb{P}\Big(\sup_n \sum_{i=1}^n \xi_i > y\Big) \sim \frac{-1}{a} \int_y^\infty \mathrm{d}t \mathbb{P}(\xi_\bullet > y)$$

valid for i.i.d. random variables $\xi_1, \xi_2, \ldots$ with $a = \mathbb{E}\xi_\bullet < 0$ and $\mathbb{E}e^{t\xi_\bullet} \equiv 0 \; \forall \; t > 0$ (the right heavy-tail condition).

A conceptual article by Harrison addresses various principal questions of the applicability of queueing network theory, particularly in the form of heavy traffic approximation (fluid models), to industrial manufacturing and processing. It contains an extensive up-to-date review of the fluid model results.

In a paper by Kalashnikov there are several new facts established on the classical problem of stability (continuity) of a queueing model regime with respect to perturbations in distributions of its parameters, viz., the interarrival and/or service time in a GI/GI/1 queue. Sadly, Vladimir Kalashnikov, whom I knew personally for many years, died shortly after submitting his contribution to this volume. The next paper, by Karpelevich, Malyshev, Petrov, Pirogov, and Rybko, provides further development of the mathematical theory of random grammars. To my knowledge, this is the last work coauthored by Karpelevich.

Kelbert, Rachev, and Suhov consider in their paper a version of risk theory on a tree and apply to this situation various methods developed in a previous cycle of papers with Karpelevich. This is a modification of standard risk theory where an insurance (or investment) company repeatedly has to make a decision, after exponential time, which affects the company's balance by the time of the next decision. Standard one-dimensional time is replaced here by a tree-like structure

and a point process (of claims) by a process indexed by a tree. The authors study properties of the potential ruin time and show a number of unexpected results.

Two articles, by Martin and by Mitzenmacher and Vöcking, contain results on the so-called join-the-shortest-queue (JSQ) models. The original idea here was developed independently in 1996 by Vvedenskaya, Dobrushin, and Karpelevich (VDK) and by Mitzenmacher. These authors considered a system with N single servers where each arriving task choses d queues at random and then joins the shortest queue from the sample, breaking ties at random. The result was that the tail of the queue length distribution at any given server decays superexponentially. Later, Martin and Suhov applied the JSQ principle to Jackson-type networks and coined the term "fast Jackson networks". In the paper by Mitzenmacher and Vöcking, the above result of VDK and Mitzenmacher is pushed further forward by allowing an asymmetric choice in the case of ties; this leads to the so-called Fibonacci exponential decrease in the tail of the queue length distribution. Martin's paper deals with the arrival flow at a given queue in a node of the fast Jackson network. Here the author establishes stochastic bounds for the queue length similar to Massey's 1984 bounds for standard Jackson networks.

Next, a paper by Menshikov and Petritis studies a class of Markov chains in a nonnegative quadrant of $\mathbb{Z}^2$ with so-called excitable boundaries where a state of the chain comprises an additional component describing "excitation". The authors use an assumption of partial spatial homogeneity on the jump probabilities and a number of auxiliary assumptions under which they establish conditions for recurrence and transience of the chain and discuss applications of this result to various queueing models. The problem of stability is also the topic of a paper by Rybko, Stolyar, and Suhov. Here, a network with a LIFO (Last-In-First-Out) discipline is proposed and a natural stability condition is verified (that the average load at every node is less than its service rate). For different models, a similar question is analyzed in papers by Shakkottai and Stolyar and by Thomas.

A paper by Shur focuses on a classical topic of limit theorems for a Markov chain with a general state space. These theorems take the form of a ratio convergence.

As the reader can see, most of the papers in this volume are related to queueing and queueing network theory, a domain in which Karpelevich's work made perhaps the strongest impact. I am sure we will witness in the future many more results inspired by his profound ideas and extremely fine techniques that impressed so many of us on so many occasions.

Yuri Suhov

Cambridge, 8 June 2002

This volume is dedicated to F. I. Karpelevich, an outstanding Russian mathematician who made important contributions to applied probability theory. The book contains original papers focusing on several areas of applied probability and its uses in modern industrial processes, telecommunications, computing, mathematical economics, and finance.

It opens with a review of Karpelevich's contributions to applied probability theory and includes a bibliography of his works. Other articles discuss queueing network theory, in particular, in heavy traffic approximation (fluid models).

The book is suitable for graduate students, theoretical and applied probabilists, computer scientists, and engineers.

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