American Mathematical Society
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Selected Papers on Analysis and Differential Equations

Katsumi Nomizu
Editor

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Selected Papers on Analysis and Differential Equations

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American Mathematical Society
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Preface

This is a collection of several papers that originally appeared in the journal *Sūgaku* in Japanese. These translated articles would normally appear in the AMS journal *Sugaku Expositions*. In order to expedite publication, the AMS has chosen, with the consent of the Mathematical Society of Japan, to publish them as a volume of selected papers in the *Translations Series* 2 of the AMS. To give the reader a quick idea of the contents, we give brief quotations from the introduction to each article, with occasional emendations.

The first paper is *Van Vleck formula for Wiener integrals and Jacobi fields*, by Nobuyuki Ikeda. This survey is based on a lecture presented at the annual meeting of the Mathematical Society of Japan in 1996, which in turn was based on joint work with Shigeo Kusuoka and Shojiro Manabe. The key words and phrases include: M. Kac, Feynman path integrals, Levy's formula, Gaussian integrals, Van Vleck formula, Jacobian of a transform, and Selberg's trace formula.

Ruishi Kuwabara wrote on *Spectral geometry for Schrödinger operators in a magnetic field focusing on geometrical and dynamical structures on manifolds*. The aim of this article is to present some topics and results on the relationships among the spectrum of $\hat{H}$, geometry of (M, g, A), and mechanics in $\mathcal{H}_{cl}$ in the case where M is a compact manifold without boundary and the scalar potential $V = 0$. In the framework of symplectic structures and their quantized objects we will consider how the magnetic field Θ or its vector potential A affects the quantum spectrum from the geometric and dynamical viewpoints. The techniques used here are based on the theory of Hamiltonian mechanics, the geometry of connections, and the theory of pseudodifferential and Fourier integral operators. The main results in this survey were obtained around 1990, and we cannot cover the many arguments that have recently been deeply and widely developed om this subject.

In Kengo Matsymoto's paper, *Symbolic dynamics and $C*$-algebras*, the author will deal with two different types of objects. One is discrete dynamical systems, consisting of the shifts on bi-infinite sequences of finite symbols, whose theory is mainly part of topological dynamical systems. The other one is operator algebras, consisting of infinite dimensional algebras of mutually noncommutative operators, whose theory is mainly part of functional analysis. These two objects look very different. In this paper, the author will introduce an interesting relationship between them. Many concepts and interesting applications of the two subjects will indeed come into play; for example, we mention how Hadamard's old ideas were instrumental to the origin of symbolic dynamics.

In *Inverse problems for elasticity*, Gen Nakamura considers such problems with and without residual stress. The inverse problem with residual stress may not be

very familiar, but it is very interesting and important in applications. In addition to covering known results, including his own, the author points out some of the open problems, and makes conjectures on possible future results.

Yoshihiro Shibata contributes a paper on *Time-global solutions of nonlinear evolution equations and their stability.* In it he considers a certain problem on stability of stationary solutions to evolution equations, concentrating on his own recent results. The equation to be treated is

$$u_t + A(u) = f,$$

where the subscript t denotes partial differentiation with respect to time.

Kazuya Tachizawa writes on *Wavelets and eigenvalues of Schrödinger operators.* He starts by explaining the basics of wavelet theory, and goes on to consider their applications to eigenvalues of Schrödinger operators. One of the main topics of Fourier analysis is the decomposition of a function; a typical example is the Fourier series decomposition of a periodic function. In such a decomposition it is important to investigate the relation between the norm of the function and its components.

Finally, Eiji Yanagida and Shoji Yotsutani write on *Recent topics in nonlinear partial differential equations: Structure of radial solutions for semilinear elliptic equations.* Over the past two decades there has been a considerable development in the theory of nonlinear PDEs, and many interesting facts have been discovered for various equations. In this article the authors focus on the structure of radial solutions for semilinear elliptic equations with power semilinearity, and related topics, and try to survey the recent progress as clearly as possible.

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This volume contains translations of papers that originally appeared in the Japanese journal, *Sugaku*. The papers range over a variety of topics, including nonlinear partial differential equations, C^*-algebras, and Schrödinger operators.

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