

American Mathematical Society

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Series 2 • Volume 213

Advances in the Mathematical Sciences

Lie Groups and Invariant Theory

Ernest Vinberg
Editor



American Mathematical Society



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The leaders of the Seminar on Lie Groups, A. L. Onishchik and E. B. Vinberg, with guests of the seminar A. G. Elashvili, B. Kostant, V. Kac, A. A. Kirillov, A. M. Vershik, and A. Dzhumadildaev (June of 1990).

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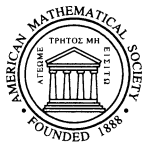
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Lie Groups and Invariant Theory

Ernest Vinberg
Editor



American Mathematical Society
Providence, Rhode Island

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Preface

This is a collection of papers written by participants of or people closely related to the Moscow seminar on Lie groups and invariant theory headed by E. B. Vinberg and A. L. Onishchik. Started about 1960 as a seminar on Lie groups, it was conceived as a continuation of E. B. Dynkin's seminar on Lie groups. (About the latter, see the remarkable preface by S. G. Gindikin to "Lie Groups and Lie Algebras: E. B. Dynkin's Seminar", AMS Translations, Series 2, vol. 169, 1995.) It inherited some traditions of Dynkin's seminar, in particular, producing results and conjectures on a solid experimental basis, for which the classification of semisimple Lie groups and their representations provides an excellent material.

In the 1967/1968 academic year, the seminar was devoted to studying the theory of algebraic groups and to developing the theory of semisimple Lie groups on this basis (an idea going back to C. Chevalley). This resulted in a book "Seminar on Algebraic Groups and Lie Groups" by E. B. Vinberg and A. L. Onishchik, which first appeared in a mimeographed form in 1969. The authors used preliminary drafts of talks prepared by the participants of the seminar E. M. Andreev, V. G. Kac, B. N. Kimelfeld, and A. K. Tolpygo. (The tables of tensor decompositions were compiled with the assistance of A. G. Elashvili, V. G. Kac, B. N. Kimelfeld, B. O. Makarevich, and V. L. Popov.) A considerably revised and expanded version of the book was published in Russian in 1988. Its English translation was published by Springer-Verlag under the title "Lie Groups and Algebraic Groups" in 1990.

The invariant-theoretic component of the seminar was based on the previous work of E. B. Vinberg, A. G. Elashvili, E. M. Andreev, V. L. Popov, V. G. Kac, D. I. Panyushev, and P. I. Katsylo. For several years a separate seminar on invariant theory was headed by E. B. Vinberg and V. L. Popov. In 1986, it was combined with the seminar on Lie groups, and in 1986–2000 V. L. Popov was a codirector of this joint seminar.

A collection of papers by participants of the seminar was first published by the American Mathematical Society in the series "Advances in Soviet Mathematics" under the title "Lie Groups, Their Discrete Subgroups, and Invariant Theory" in 1992. The present volume is the second such collection. It consists mainly of original papers with new results in geometry and topology of homogeneous spaces of Lie groups, structure of Lie algebras, algebraic transformation groups, supermanifolds and Lie superalgebras.

We have also included translations of two old papers of topical interest, by A. Alexeevski and by E. B. Vinberg, which are almost unknown in the West because they were published only in Russian.

Many former participants of the seminar now live abroad but retain close contacts with us. In particular, when visiting Russia, they often give talks at our seminar. Still considering them to be participants of the seminar, we invited some of them to contribute to this volume. We are grateful to D. V. Alekseevsky, A. G. Elashvili, S. G. Gindikin, Yu. Khakimjanov, V. Kac, D. Leites, and V. Serganova, who kindly agreed to participate.

Many papers of this volume are devoted to A. L. Onishchik, who reached his 70th birthday on November 14, 2003. All the participants of the seminar wish him good health and long years of further activity as a scientist and teacher.

E. B. Vinberg

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This volume, devoted to the 70th birthday of A. L. Onishchik, contains a collection of articles by participants in the Moscow Seminar on Lie Groups and Invariant Theory headed by E. B. Vinberg and A. L. Onishchik. The book is suitable for graduate students and researchers interested in Lie groups and related topics.

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