

American Mathematical Society

# TRANSLATIONS

Series 2 • Volume 220

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Advances in the Mathematical Sciences

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## Nonlinear Equations and Spectral Theory

M. S. Birman  
N. N. Uraltseva  
Editors



American Mathematical Society



# Nonlinear Equations and Spectral Theory



**Ol'ga Aleksandrovna Ladyzhenskaya**  
(March 7, 1922–January 12, 2004)

American Mathematical Society

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Series 2 • Volume 220

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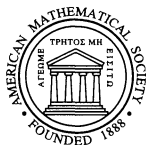
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*(Formerly Advances in Soviet Mathematics)*

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**American Mathematical Society**  
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## Preface

The famous mathematician Olga Aleksandrovna Ladyzhenskaya was well known for her remarkable scientific achievements as well as for her activity in the promotion of science. For many years Olga Aleksandrovna ran the Saint Petersburg Seminar on mathematical physics. This seminar became the basis of the scientific school she created. Many of the participants in the seminar, including her own students, later became well-known mathematicians. For several years Olga Aleksandrovna served as the president of the Saint Petersburg Mathematical Society. Her personal warmth and very high moral prestige had a profound impact on the mathematical community of Saint Petersburg.

The scientific interests of Olga Aleksandrovna were mainly concentrated around the theory of multi-dimensional boundary problems (both linear and nonlinear) and the mathematical aspects of hydrodynamics. More details about her scientific and personal biography can be found in the article “Ol’ga Aleksandrovna Ladyzhenskaya (on her 80th birthday)” by G. A. Serëgin and N. N. Ural’tseva, *Russian Math. Surveys* **58** (2003), 395–425.

The authors of articles in the present volume are students and colleagues of Olga Aleksandrovna Ladyzhenskaya, in particular in her position as the head of the seminar. The papers in the volume are mainly devoted to boundary problems and spectral problems.

*The editors*



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This volume is devoted to the memory of the famous Saint Petersburg mathematician Olga Aleksandrovna Ladyzhenskaya. For many years she ran the Saint Petersburg Seminar on mathematical physics, which became a basis for the scientific school she created. The ten articles in the volume, written by students and colleagues of O. A. Ladyzhenskaya, are mainly devoted to boundary value problems for partial differential equations and to spectral problems for differential operators.

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