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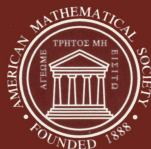
TRANSLATIONS

Series 2 • Volume 221

Advances in the Mathematical Sciences

Moscow Seminar on Mathematical Physics, II

Yu. Neretin
M. Olshanetsky
A. Rosly
Editors



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Moscow Seminar on
Mathematical Physics, II

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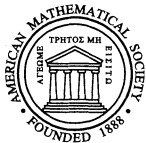
Series 2 • Volume 221

Advances in the Mathematical Sciences—60

(Formerly Advances in Soviet Mathematics)

Moscow Seminar on Mathematical Physics, II

Yu. Neretin
M. Olshanetsky
A. Rosly
Editors



American Mathematical Society
Providence, Rhode Island

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2000 *Mathematics Subject Classification*. Primary 00B25; Secondary 37–06.

Library of Congress Card Number 91-640741
ISBN 0-8218-4371-0
ISSN 0065-9290

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Mathematical Physics Seminar in ITEP

This volume contains proceedings of seminars of the Mathematical Physics Group of the Institute for Theoretical and Experimental Physics (ITEP). From its initiation by Alexei Morozov in 1990, the Mathematical Physics Group continues to be an original research center and an important place of collaboration between physicists and mathematicians. This volume is the second such collection published by the American Mathematical Society, the first being *Moscow Seminar in Mathematical Physics* (A. Yu. Morozov and M. A. Olshanetsky, editors), Amer. Math. Soc., Providence, RI, 1999.

Papers of this collections are devoted to various mathematical topics that strongly influenced modern physics. These topics are quite diverse. We hope, however, that the choice of topics shows why the ITEP seminar on mathematical physics has been known to and was attended by a large group of participants. We are glad to use this opportunity to cordially thank all who participated in our seminar.

Let us give a brief description of the papers in this volume.

Benjamin Enriquez and Vladimir Rubtsov construct an explicit parametrization for the moduli space of 2-dimensional vector bundles over an algebraic curve and apply it to the Hitchin and Knizhnik-Zamolodchikov-Bernard systems and Beilinson-Drinfeld operators.

The subject of the work of Boris Feigin, Anatol Kirillov, and Sergei Loktev is “Weyl modules” over the algebras of polynomial currents in two variables.

The paper by Vladimir Fock shows how many aspects of 2-dimensional geometry come to one’s mind by looking at the cosh-Gordon equation (when one looks).

The work of Anton Gerasimov, Sergei Kharchev, Dmitry Lebedev, and Sergei Oblezin is devoted to a realization of one class of representations of Lie algebras and quantum groups based on Gelfand-Tsetlin schemes.

Alexei Gorodentsev, Anton Khoroshkin, and Alexei Rudakov investigate syzygies of flag manifolds from the cohomology of certain infinite-dimensional Lie superalgebras. Their approach allows them to compute the syzygies explicitly in some special cases.

The subject of the paper by Alexei Gorodentsev and Sergei Kuleshov is Harder-Narasimhan filtrations on categories.

The paper by Dmitry Kaledin is aimed to clarify the Gabber theorem on the involutivity of the singular support of D -modules.

The work of Sergei Khoroshkin and Stanislav Pakuliak is devoted to the Borel subalgebras in the quantum affine algebras and the method of Drinfeld projectors. It is also a survey of the subject.

The paper by Andrey Levin and Andrei Zotov is devoted to various forms of the Painlevé VI equation.

In the first note of Yuri Neretin, the author constructs a canonical embedding of the space L^2 with respect to a determinantal point stochastic process (as the Dyson sine-process) to the fermionic Fock space.

In the second note, Neretin discusses a realization of the space of functions of a real variable as a space of distributions on the adelic product of all the p -adic fields.

Misha Verbitsky considers a certain class of hypercomplex manifolds and gives a criterion that the holonomy of a canonical torsion-free connection is contained in the quaternionic SL.

We are grateful to the American Mathematical Society and especially to Simon Gindikin for an opportunity to publish this collection. We also thank Andrey Levin and Andrei Marshakov, who helped us in our work on this volume. The work is supported by the Federal Agency for Atomic Energy of Russia, and by the grants SciSch-8004.2006.2, SciSch-8065.2006.2.

Yu. Neretin,¹ M. Olshanetsky, and A. Rosly

¹Also affiliated with Vienna University, grant FWF, project 19064.

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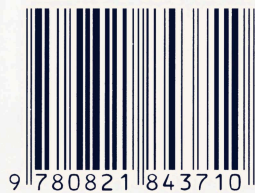
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The Institute for Theoretical and Experimental Physics (ITEP) is internationally recognized for achievements in various branches of theoretical physics. For many years, the seminars at ITEP have been among the main centers of scientific life in Moscow. This volume is a collection of articles by participants of the seminar on mathematical physics that has been held at ITEP since 1983. This is the second such collection; the first was published in the same series, AMS Translations, Series 2, vol. 191.

The papers in the volume are devoted to several mathematical topics that strongly influenced modern theoretical physics. Among these topics are cohomology and representations of infinite Lie algebras and superalgebras, Hitchin and Knizhnik-Zamolodchikov-Bernard systems, and the theory of D -modules.

The book is intended for graduate students and research mathematicians working in algebraic geometry, representation theory, and mathematical physics.

ISBN 978-0-8218-4371-0



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