

American Mathematical Society

TRANSLATIONS

Series 2 • Volume 223

Selected Papers on Analysis and Related Topics



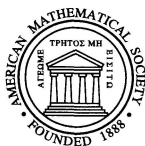
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Foreword

This is a collection of papers originally published in Japanese in the journal *Sūgaku*. Normally translations of these papers would have appeared in the journal *Sugaku Expositions* published by the American Mathematical Society. To speed up the publication, the AMS decided to publish them as a volume in its series *American Mathematical Society Translations Series 2*.

The volume begins with the article “Entrance to Operator Algebras” by Masamichi Takesaki. The author, one of the leading experts in the theory of operator algebras, presents some basic facts of the theory and puts them in historic perspective. In particular, he pays special attention to the von Neumann theory of factors and to the relations of the theory C^* -algebras to the theory of representations of noncompact groups. The article also provides interesting facts about the history of the research on C^* -algebras in Japan.

Another article devoted to operator algebras is “Classification of C^* -Algebras” by Masaki Izumi. Here the author presents a very readable account of the classification theory of C^* -algebras (the so-called Elliott program). He also outlines some applications of the classification theory to the general theory of C^* -algebras.

The article “Weighted Hardy Spaces and Jacobi Series”, by Akihiko Miyachi, presents important results about operators in weighted Hardy spaces of functions on domains in \mathbf{R}^n . The main result of the article shows that various singular integral operators defined as multiplier operators with respect to certain sequences of Jacobi polynomials are bounded as operators between appropriate Hardy spaces.

The article “Legacy of J. Marcinkiewicz to Real Analysis in the 20th Century”, by Satoru Igari, describes a relatively unknown part of the history of mathematical analysis of the first half of the 20th century. The Polish mathematician Józef Marcinkiewicz (1910–1940?) was a student of A. Zygmund. After receiving his Ph.D., Marcinkiewicz worked mainly at the University of Wilna. In 1939, he was taken by Germany as a prisoner of war, and is believed to have died in the spring of 1940. During his short scientific career, Marcinkiewicz published 55 papers devoted to such diverse areas as real functions, trigonometric series, interpolation by trigonometric polynomials, operators in functional spaces, orthogonal series, complex functions, and probability. In his article, S. Igari reviews Marcinkiewicz’s results in all these areas and describes their influence on further research.

The article “The Kakeya Conjecture” by Hitoshi Tanaka is devoted to the so-called Kakeya conjecture, which was originally generated by a problem in geometry, but lately has been found to be related with several difficult problems in other fields of mathematics, especially in analysis. The original “needle problem” was formulated by S. Kakeya in 1917. It asks *what is the smallest area required to rotate a unit line segment (“needle”) by 180° on the plane?* In 1927, A. Besicovitch

established that, surprisingly, *one can rotate the needle inside a compact set of an arbitrarily small area*. In the present paper the author describes relations of the Kakeya problem to problems in Fourier analysis and the theory of maximal functions. He also outlines some recent work of J. Bourgain, N. Katz, T. Tao, and others on problems originated from and related to the Kakeya conjecture.

The paper “Recent Developments of Analysis on Fractals”, by Takashi Kumagai, describes some aspects of the analysis on fractal spaces. Instead of discussing a general fractal space, the author limits the presentation to the example of the so-called Sierpinski gasket. The article begins with the overview of the construction of the Sierpinski gasket and the Brownian motion on the gasket. The author continues with the presentation of recent developments of analysis on fractals.

In the article “Symbolic Dynamical System and Number Theoretical Tilings”, Shigeki Akiyama studies symbolic dynamical systems associated with a class of simplest aperiodic one-sided infinite sequences, the so-called Sturmian sequences. A Sturmian sequence ξ is defined by the condition that the number $P_\xi(n)$ of different n -length subwords of ξ equals $n + 1$. The condition $P_\xi(1) = 2$ implies that a Sturmian sequence ξ is necessarily binary. The article describes various number-theoretic and geometric constructions associated with Sturmian sequences. In particular, it turns out that Sturmian sequences are related with the so-called Pisot representation of real numbers. In turn, Pisot representations lead to quite interesting aperiodic tilings on the plane, called Pisot tilings and dual Pisot tilings. All this, and more, is explained in this short but informative article.

The article “Notions of Independence in Quantum Probability and Spectral Analysis of Graphs”, by Nobuaki Obata, begins with the review of some basic notions of quantum probability theory. Then the author discusses various approaches to the definition of independence in quantum probability and the corresponding formulations of the central limit theorem. The article concludes with a section devoted to applications of quantum probability theory to spectral analysis on graphs.

As is clear from the title, the article “On Various Applications of the Wavelet Analysis to Statistics”, by Katsuto Tanaka, discusses statistical aspects of the theory of the wavelet by comparing the wavelet method with usual time and domain methods, and explores various applications of the wavelet method to time series analysis. Among these applications are the estimation and testing problems associated with the ARFIMA (Fractionally Integrated Autoregressive Moving Average) models, testing for the presence of noise, unit root tests for testing the random walk hypothesis, testing in the state space model, etc. The author also presents some results of simulations and uses them to compare the wavelet method with the standard statistical methods.

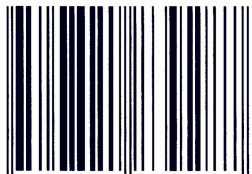
The article “Bhattacharyya Type Inequalities”, by Hidekazu Tanaka, studies the problem of estimating an unknown parameter by using a sample from a probability distribution depending on this unknown parameter. The Bhattacharyya inequality (a generalization of the classical Cramer-Rao inequality) gives a sharp lower bound to the variance of the estimate. The article surveys recent results in this direction, including the study of a distribution attaining the Bhattacharyya bound, as well as various examples and generalizations.

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This volume contains translations of papers that originally appeared in the Japanese journal *Sūgaku*. The papers range over a variety of topics, including operator algebras, analysis, and statistics.

This volume is suitable for graduate students and research mathematicians interested in analysis and its applications.

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