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Advances in the Mathematical Sciences

Geometry, Topology, and Mathematical Physics

S. P. Novikov's Seminar:

2006–2007

V. M. Buchstaber

I. M. Krichever

Editors



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S. P. Novikov

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Preface

Sergey Petrovich Novikov is one of the most outstanding mathematicians of our time. The articles in this book, written by members of his famous seminar and covering many aspects of seemingly unrelated areas of modern mathematics and mathematical physics, reflect the breadth of Novikov's scientific interests and are dedicated to him on the occasion of his 70th birthday.¹

We briefly describe the papers included in this volume.

In the paper by Alexeevskii and Natanzon, an analog of classical Hurwitz numbers for regular coverings of surfaces with marked points by seamed surfaces is introduced. Seamed surfaces are not actual surfaces. A simple example of a seamed surface is a book-like seamed surface: several rectangles are glued by edges like sheets in a book. The authors show that such numbers define a new example of Klein Topological Field Theory. It is known that such theories are in one-to-one correspondence with a certain class of algebras called Cardy–Frobenius algebras. The algebras corresponding to coverings with finite group action are described in terms of the group and its subgroups. As a result, an algebraic formula for the proposed generalization of Hurwitz numbers is obtained.

The work of Buchstaber and Terzić is on the border of algebraic topology and complex geometry. An effective computation of the universal toric genus of the complex and stable complex structures on homogeneous spaces is provided. As an application, explicit formulas for the cobordism classes and characteristic numbers of the flag manifolds, Grassmann manifolds and some other interesting examples are obtained. In particular, they completely solved the problem that characteristic Chern numbers may differ for complex structures on the same homogeneous space. It is well known that Pontryagin numbers do not do this. The first result in that problem was obtained by Borel and Hirzebruch. In 1958, they gave an example of two complex structures on 10-dimensional homogeneous space having different numbers c_1^5 .

S. P. Novikov is one of the founders of cobordism theory. In 1967, he published a paper which helped to determine the future directions of cobordism theory. The paper of Buchstaber and Terzić is one more demonstration of the effectiveness of the methods developed by S. P. Novikov.

¹The three previous volumes of Novikov's seminar were published by the American Mathematical Society in the series *Advances in the Mathematical Sciences*, Amer. Math. Soc. Translations, Ser. 2.

1. "Topics in Topology and Mathematical Physics", vol. 170, 1995.
2. "Solitons, Geometry and Topology: On the Crossroad", vol. 179, 1997.
3. "Geometry, Topology, and Mathematical Physics, S. P. Novikov's Seminar: 2002–2003", vol. 212, 2004.

The paper of Feigin and Veselov is devoted to the study of a geometry of certain collections of vectors in a linear space called V -systems. They were introduced earlier by the second named author in relation with a certain class of solutions of the generalized Witten–Dijkgraaf–Verlinder–Verlinder (WDVV) or associativity equations, playing an important role in $2D$ topological field theories and supersymmetric gauge theories. It is shown that these systems are closed under the natural operations of restriction and taking subsystems. A detailed analysis of a special class of V -systems related to generalized root systems and basic classical Lie superalgebras is presented.

Gusein-Zade, Luengo and Melle-Hernández in their paper introduce an equivariant version of the classical monodromy zeta function. A number of invariants of singularities have an equivariant version for singularities invariant with respect to an action of a finite group G . For example, an equivariant version of the Milnor number is an element of the ring of virtual representations of the group. It is not so clear what should be considered as the equivariant version of the monodromy zeta function. The two main ingredients of the definition proposed in the paper are equivariant Lefschetz numbers and the λ structure on the Grothendieck ring of finite G -sets.

The work of Taimanov and Grinevich is devoted to the study of a certain class of nonlinear equations considered in the framework of soliton theory in which Novikov has made many profound contributions. The authors show that the characteristic feature of these equations, often called equations with self-consistent sources, is a special type of deformation of the spectral curves of auxiliary linear operators. It turns out that despite the fact that the spectral curves are not preserved, they still provide many conservation laws for the system.

Dubrovin's paper analyzes the singularities of solutions to the systems of first order quasilinear PDEs and their perturbations containing higher derivatives. The work is an essential step in the further development of the author's approach to the classification of integrable systems. The study is focused on the subclass of Hamiltonian PDEs with one spatial dimension. For the systems of order one or two the local structure of singularities of a generic solution to the unperturbed system near the point of "gradient catastrophe" is described in terms of standard objects of classical singularity theory. The author argues that their perturbed companions must be given by certain special solutions of Painlevé equations and their generalizations.

The main goal of the work of Hamilton and Lazarev is to establish the existence of the so-called string structures on the ordinary and equivariant homology of the free loop space on a manifold or, more generally, a formal Poincaré duality space. The authors' method is based on the obstruction theory of certain algebras and rational homotopy theory. It is shown that the resulting string topology operations are manifestly homotopy invariant.

Krichever and Shiota in their paper introduce a general notion of the abelian solutions of soliton equations. The theory of such solutions unifies the ideas of the theory of the Calogero–Moser type systems connected with the theory of pole systems of meromorphic solutions of a variety of soliton equations, and the ideas underlying Novikov's remarkable conjecture: the Jacobians of curves are exactly the indecomposable principally polarized abelian varieties whose theta-functions provide explicit solutions of the KP equation. This conjecture was proved by the

second named author and until recently has remained the most effective solution of the famous Riemann–Schottky problem. The main result of the present paper is that all solutions of the KP equation expressible in terms of holomorphic sections of a line bundle on an abelian (not necessarily principally polarized) variety are algebraic-geometrical. Evidence for the existence of a new type of integrable system on the spaces of higher order theta-functions is provided.

Mokhov’s paper is devoted to a further study of interconnections between classical differential geometry and the Hamiltonian theory of hydrodynamic-type systems originated by Dubrovin and Novikov. It is shown that the associativity equations of two-dimensional topological quantum field theory can be identified with certain reductions of the fundamental nonlinear equations of the theory of submanifolds in pseudo-Euclidean spaces. A notion of potential submanifolds is introduced. It is proved that each semisimple Frobenius manifold can be locally represented as a potential submanifold.

Maltsev in his paper presents a detailed study of deformations of Whitham-type systems in the almost linear case. The Whitham equations are a cornerstone of the perturbation theory of algebraic-geometrical solutions of soliton equations. The general structure of these systems is well understood by now. Nevertheless, it turns out that in the case when the initial data are close to the linear case, the deformation approach should be modified in order to make all the constructions stable in the linear limit.

Pavlov’s paper is devoted to a construction of particular solutions for the famous Benney hydrodynamical chain. Connections between the famous Löwner equation and the Gibbons–Tsarev systems that are special reductions of the Benney chain are discussed.

The main goal of the paper by Pogrebkov is to explore further an earlier author’s observation that some well-known integrable systems are in mutual correspondence with some simple commutator identities in associative algebras. It turns out that under some general assumptions, the nonlinear equations and their Lax pairs can be reconstructed from representations of these algebras. In the present paper the construction is generalized for the lattice case.

The purpose of Sheinman’s work is to present recent results on Lax operator algebras and their applications. These new types of almost graded algebras represent further developments of ideas going back to a theory of high rank solutions of integrable equations developed by Novikov and Krichever.

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