American Mathematical Society

TRANSLATIONS

Series 2 • Volume 225

Advances in the Mathematical Sciences

Spectral Theory of Differential Operators

M. Sh. Birman 80th Anniversary Collection

T. Suslina D. Yafaev Editors



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(Formerly Advances in Soviet Mathematics)

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American Mathematical Society Providence, Rhode Island

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Mikhail Shlemovich Birman

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This volume is dedicated to Professor M. Sh. Birman in honor of his eightieth birthday. It contains original articles in spectral and scattering theory of differential operators, in particular, Schrödinger operators, and in homogenization theory. All articles are written by members of M. Sh. Birman's research group who are affiliated with different universities all over the world. A specific feature of the majority of the papers is a combination of traditional methods with new modern ideas.



