

American Mathematical Society

# TRANSLATIONS

Series 2 • Volume 226

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Advances in the Mathematical Sciences

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## Linear and Complex Analysis

Dedicated to V. P. Havin on the  
Occasion of His 75th Birthday

Alexei Alexandrov  
Anton Baranov  
Sergey Kislyakov  
Editors



American Mathematical Society



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Victor Havin near the main building  
of St. Petersburg State University.

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**American Mathematical Society**  
Providence, Rhode Island

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## Preface

Victor Petrovich Havin was born in Leningrad on 7 March, 1933. His academic career is linked entirely with the Department of Mathematics and Mechanics of St. Petersburg State University, which he entered as a student in 1950. (On many occasions he talked about the dramatic choice between linguistics and mathematics he faced at that time, and about the role of his father who strongly dissuaded him from linguistics — because to be a mathematician was much safer in the Soviet era, especially under the Stalin regime.) He got his Ph.D. degree in 1958 under supervision of L. V. Kantorovich and (informally) V. I. Smirnov; he obtained the Doctor of Science degree in 1969 and full professorship in 1970. He headed the division of mathematical analysis at St. Petersburg State University from 1997 until 2004. V. P. Havin supervised 27 Ph.D. theses (in particular, two of us were his students at different times; see Mathematics Genealogy Project site <http://genealogy.math.ndsu.nodak.edu> for more details) and some uncontrollable number of master theses. Three of his former students have won the Salem Prize. He received several honorary awards and degrees from his home university and from some foreign universities. His publication list totals more than 100 items, among which there are at least 70 research papers, a seminal book on the uncertainty principle in harmonic analysis (written jointly with Burglind Jöricke, also his former student), an introductory university course of Analysis (*not* to be named “calculus”), etc.

However, this bare listing of facts hides something. It only reflects an ordinary career of a very successful university professor. In reality, it is to a great extent due to various kinds of activity of Victor Petrovich and to his personality that Mathematical Analysis in St. Petersburg has developed to its present state. A brilliant scientist and absolutely brilliant teacher, he has succeeded in making difficult issues of modern analysis attractive to many young researchers. Since 1963, he has been running the Linear and Complex Analysis Seminar (joint for the University and the V. A. Steklov Mathematical Institute in St. Petersburg; before 1991, this responsibility was shared with N. K. Nikolski). For many years, this seminar has been the center of activity of an informal (and very large) research group; it plays this role even now, when many members of this group live and work elsewhere.

This volume is a gift, made by the contributors and editors for Victor Petrovich Havin on the occasion of his 75th birthday.

The list of Havin’s publications follows this Preface. We refrain from a detailed analysis of his scientific achievements, which are well known. We only indicate



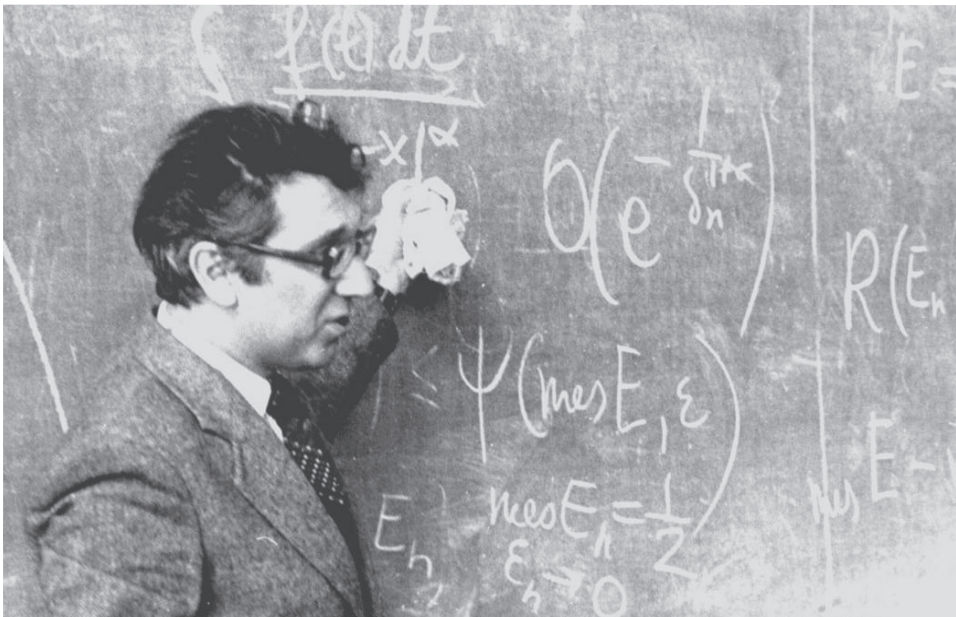
some fields of his interest usually emphasized by Havin himself (we do not name coauthors and collaborators):

- separation of singularities of (bounded) analytic functions;
- divisibility by the inner factor in certain classes of analytic functions, and the drop of the smoothness of a function relative to that of its modulus;
- integral analogs of Vitushkin's theorem on uniform approximation by analytic functions;
- nonlinear potential theory;
- free interpolation;
- approximation by harmonic vector fields;
- uncertainty principle in harmonic analysis;
- questions related to the Beurling–Malliavin theorem.

Alexei Alexandrov  
Anton Baranov  
Sergey Kislyakov



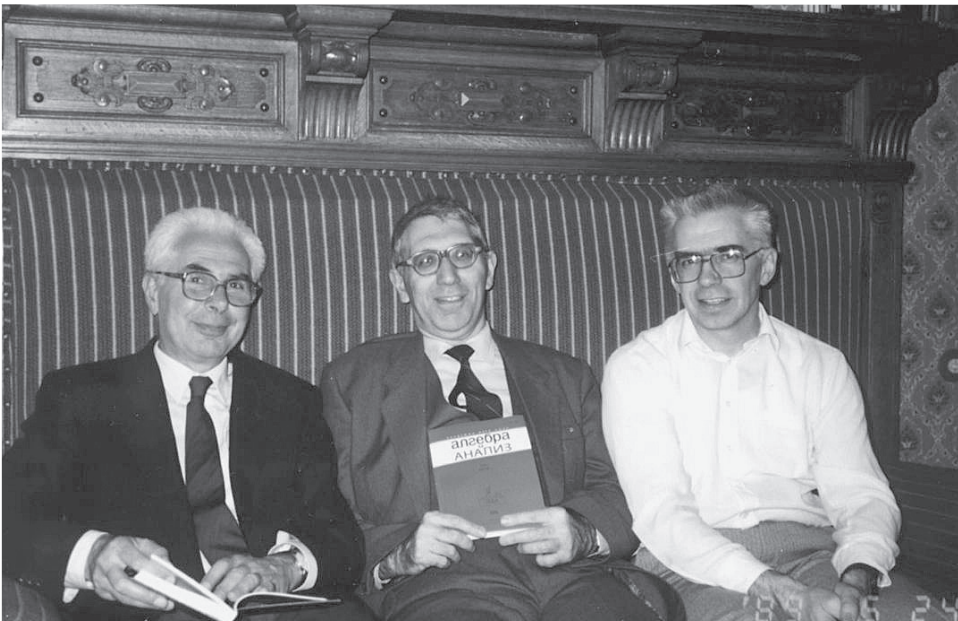
Victor Havin with his fellow students near the Department of Mathematics and Mechanics. Leningrad, 1953.



Giving a lecture at a conference. Ufa, 1980.



Doctor Honoris Causa. Linköping, 1993.



With Jean-Pierre Kahane and Nikolai Nikolski.

## Publications Authored by V. P. Havin

### Books

- [1] *The basic of mathematical analysis, Part 1. Differential and integral calculus of functions of one real variable.* Leningrad. Univ., Leningrad, 1989. MR1007979 (90j:00001)
- [2] *The uncertainty principle in harmonic analysis* (with B. Jöricke), *Ergebnisse der Mathematik und ihrer Grenzgebiete* (3), **28**, Springer-Verlag, Berlin, 1994. MR1303780 (96c:42001)

### Research papers

- [1] *On analytic functions representable by an integral of Cauchy–Stieltjes type*, *Vestnik Leningrad. Univ. Ser. Mat. Meh. Astr.* **13** (1958), no. 1, 66–79. MR0095256 (20 #1762)
- [2] *The separation of the singularities of analytic functions*, *Dokl. Akad. Nauk SSSR* **121** (1958), 239–242. MR0098168 (20 #4630)
- [3] *Analytic continuation of power series and Faber polynomials*, *Dokl. Akad. Nauk SSSR (N.S.)* **118** (1958), 879–881. MR0098175 (20 #4637)
- [4] *On a problem of V. V. Golubev*, *Dokl. Akad. Nauk SSSR* **126** (1959), 511–513. MR0106292 (21 #5026)
- [5] *The rate of growth of functions of the  $H_p$  class and the multiplication of integrals of the Cauchy–Stieltjes type*, *Dokl. Akad. Nauk SSSR* **127** (1959), 757–759. MR0107010 (21 #5738)
- [6] *On the norms of some operations in the space of polynomials*, *Vestnik Leningrad. Univ.* **14** (1959), no. 19, 47–59. MR0111826 (22 #2686)
- [7] *On the space of bounded regular functions*, *Dokl. Akad. Nauk SSSR* **131** (1960), 40–43; translated in *Soviet Math. Dokl.* **1** (1960), 202–204. MR0120525 (22 #11277)
- [8] *Some estimates of analytic capacity* (with S. Ja. Havinson), *Dokl. Akad. Nauk SSSR* **138** (1961), 789–792. MR0131529 (24 #A1379)
- [9] *On the space of bounded regular functions*, *Sibirsk. Mat. Z.* **2** (1961), 622–638. MR0138985 (25 #2425)
- [10] *Relations between certain classes of functions regular in the unit circle*, *Vestnik Leningrad. Univ.* **17** (1962), no. 1, 102–110. MR0152660 (27 #2635)
- [11] *Analytic representation of linear functionals in spaces of harmonic and analytic functions which are continuous in a closed region*, *Dokl. Akad. Nauk SSSR* **151** (1963), 505–508. MR0152661 (27 #2636)
- [12] *Boundary properties of integrals of Cauchy type and of conjugate harmonic functions in regions with rectifiable boundary*, *Mat. Sb. (N.S.)* **68 (110)** (1965), 499–517. MR0192059 (33 #286)
- [13] *Removability of singularities of analytic functions and transfer of masses* (with R. E. Val’skii), *Sibirsk. Mat. Z.* **7** (1966), 55–60. MR0203441 (34 #3293)
- [14] *Duality of spaces of analytic functions and certain of its applications*, *Contemporary Problems in Theory Anal. Functions* (Proc. Internat. Conf., Erevan, 1965), “Nauka”, Moscow, 1966, pp. 311–314. MR0208356 (34 #8166)
- [15] *Continuity in  $L_p$  of an integral operator with the Cauchy kernel*, *Vestnik Leningrad. Univ.* **22** (1967), no. 7, 103–108. MR0218537 (36 #1623)
- [16] *Approximation in the mean by harmonic functions* (with V. G. Maz’ya), *Zap. Naučn. Sem. Leningrad. Otdel. Mat. Inst. Steklov. (LOMI)* **5** (1967), 196–200. MR0226046 (37 #1636)
- [17] *Approximation by analytic functions in the mean*, *Dokl. Akad. Nauk SSSR* **178** (1968), 1025–1028. MR0224830 (37 #429)
- [18] *Approximation in the mean by analytic functions* (with V. G. Maz’ya), *Vestnik Leningrad. Univ.* **23** (1968), no. 13, 62–74. MR0235131 (38 #3443)

- [19] *The Cauchy problem for Laplace's equation* (with V. G. Maz'ya), Vestnik Leningrad. Univ. **23** (1968), no. 7, 146–147. MR0236534 (38 #4829)
- [20] *Polynomial approximation in the mean in certain non-Carathéodory regions. I*, Izv. Vyssh. Uchebn. Zaved. Matematika **9** (76) (1968), 86–93. MR0237796 (38 #6077)
- [21] *Polynomial approximation in the mean in certain non-Carathéodory regions. II*, Izv. Vyssh. Uchebn. Zaved. Matematika **10** (77) (1968), 87–94. MR0247104 (40 #373)
- [22] *On the uniqueness theorem of L. Carleson for analytic functions with finite Dirichlet integral* (with V. G. Maz'ya), Problems of Math. Anal. **2**: Linear Operators and Operator Equations, 153–156, Izdat. Leningrad. Univ., Leningrad, 1969. MR0269843 (42 #4738)
- [23] *A nonlinear analogue of the Newtonian potential, and metric properties of  $(p, l)$ -capacity* (with V. G. Maz'ya), Dokl. Akad. Nauk SSSR **194** (1970), 770–773. MR0273045 (42 #7926)
- [24] *Analytic functions with a Lipschitzian modulus of the boundary values* (with F. A. Shamyoyan), Zap. Naučn. Sem. Leningrad. Otdel. Mat. Inst. Steklov. (LOMI) **19** (1970), 237–239. MR0289784 (44 #6971)
- [25] *Multipliers and divisors of Cauchy–Stieltjes type integrals* (with S. A. Vinogradov and M. G. Goluzina), Zap. Naučn. Sem. Leningrad. Otdel. Mat. Inst. Steklov. (LOMI) **19** (1970), 55–78. MR0291471 (45 #562)
- [26] *A generalization of the Privalov–Zygmund theorem on the modulus of continuity of the conjugate function*, Izv. Akad. Nauk Armjan. SSR Ser. Mat. **6** (1971), no. 2–3, 252–258; *ibid.* **6** (1971), no. 4, 265–287. MR0302923 (46 #2066)
- [27] *Impossibility of the Carleman approximation of functions continuous on the unit circle by the boundary values of functions analytic and uniformly continuous in the unit disk*, Zap. Naučn. Sem. Leningrad. Otdel. Mat. Inst. Steklov. (LOMI) **22** (1971), 161–170. MR0291813 (45 #903)
- [28] *The factorization of analytic functions that are smooth up to the boundary*, Zap. Naučn. Sem. Leningrad. Otdel. Mat. Inst. Steklov. (LOMI) **22** (1971), 202–205. MR0289783 (44 #6970)
- [29] *Approximation in the mean by harmonic functions* (with V. G. Maz'ya), Investigations on linear operators and the theory of functions, III. Zap. Naučn. Sem. Leningrad. Otdel. Mat. Inst. Steklov. (LOMI) **30** (1972), 91–105. MR0328107 (48 #6449)
- [30] *On the theory of nonlinear potentials and  $(p, l)$ -capacity* (with V. G. Maz'ya), Vestnik Leningrad. Univ. Mat. Meh. Astronom. **13** (1972), no. 3, 46–51. MR0342720 (49 #7465)
- [31] *A nonlinear potential theory* (with V. G. Maz'ya), Uspehi Mat. Nauk **27** (1972), no. 6, 67–138. MR0409858 (53 #13610)
- [32] *Weak completeness of the space  $L^1/H_0^1$* , Vestnik Leningrad. Univ. Mat. Meh. Astronom. **13** (1973), no. 3, 77–81. MR0344863 (49 #9602)
- [33] *Application of the  $(p, l)$ -capacity to certain problems of the theory of exceptional sets* (with V. G. Maz'ya), Mat. Sb. (N.S.) **90** (132) (1973), 558–591. MR0367207 (51 #3449)
- [34] *The spaces  $H^\infty$  and  $L^1/H_0^1$* , Investigations on linear operators and the theory of functions, IV. Zap. Naučn. Sem. Leningrad. Otdel. Mat. Inst. Steklov. (LOMI) **39** (1974), 120–148. MR0348467 (50 #965)
- [35] *The solutions of the Cauchy problem for the Laplace equation (uniqueness, normality, approximation)* (with V. G. Maz'ya), Trudy Moskov. Mat. Obshch. **30** (1974), 61–114. MR0385135 (52 #6000)
- [36] *Free interpolation in  $H^\infty$  and in certain other classes of functions. I* (with S. A. Vinogradov), Investigations on linear operators and the theory of functions, V. Zap. Naučn. Sem. Leningrad. Otdel. Mat. Inst. Steklov. (LOMI) **47** (1974), 15–54. MR0393497 (52 #14307)
- [37] *Approximation in  $L^p$  by the solutions of certain systems of linear differential equations*, Collection of articles dedicated to the memory of Academician V. I. Smirnov. Vestnik Leningrad. Univ. Mat. Meh. Astronom. **1** (1975), no. 1, 150–158. MR0390752 (52 #11575)
- [38] *Free interpolation in  $H^\infty$  and in certain other classes of functions. II* (with S. A. Vinogradov), Investigations on linear operators and theory of functions, VI. Zap. Naučn. Sem. Leningrad. Otdel. Mat. Inst. Steklov. (LOMI) **56** (1976), 12–58. MR0492283 (58 #11425)
- [39] *On the Carleson formula for the Dirichlet integral of an analytic function* (with A. B. Aleksandrov and A. E. Dzrbasjan), Vestnik Leningrad. Univ. Mat. Mekh. Astronom. **1979**, no. 4, 8–14. MR0564567 (81d:30051)

- [40] *The uncertainty principle for operators that commute with a shift*, I (with B. Jöricke), Investigations on linear operators and the theory of functions, IX. Zap. Nauchn. Sem. Leningrad. Otdel. Mat. Inst. Steklov. (LOMI) **92** (1979), 134–170. MR0566746 (81e:42021)
- [41] *On a class of uniqueness theorems for convolutions* (with B. Jöricke), Complex analysis and spectral theory (Leningrad, 1979/1980), Lecture Notes in Math., **864**, Springer, Berlin-New York, 1981, 143–170, MR0643382 (83c:44008)
- [42] *The uncertainty principle for operators that commute with a shift*. II (with B. Jöricke), Investigations on linear operators and the theory of functions, XI. Zap. Nauchn. Sem. Leningrad. Otdel. Mat. Inst. Steklov. (LOMI) **113** (1981), 97–134. MR0629836 (84j:47050)
- [43] *The uncertainty principle for one-dimensional Riesz potentials*, Dokl. Akad. Nauk SSSR **264** (1982), no. 3, 559–563. MR0659763 (84j:31002)
- [44] *Free interpolation and the Dirichlet problem* (with A. O. Derviz), Vestnik Leningrad. Univ. Mat. Mekh. Astronom. **1983**, 84–86. MR0702634 (84j:31001)
- [45] *Analogues of the Carleman–Goluzin–Krylov interpolation formula* (with I. V. Videnskii and E. M. Gavurina), Operator theory and function theory, no. 1, pp. 21–32, Leningrad. Univ., Leningrad, 1983. MR0768773 (86a:30057)
- [46] *Free interpolation and the Dirichlet problem* (with A. O. Derviz), Investigations on linear operators and the theory of functions, XIII. Zap. Nauchn. Sem. Leningrad. Otdel. Mat. Inst. Steklov. (LOMI) **135** (1984), 51–65. MR0741694 (85j:31002)
- [47] *Equivalent norms in  $L^p$ -spaces of analytic functions and free interpolation of harmonic functions* (with B. Jöricke), Vestnik Leningrad. Univ. Mat. Mekh. Astronom. **1984**, no. 4, 103–105. MR0776964 (86d:30080)
- [48] *Traces of harmonic functions and comparison of  $L^p$ -norms of analytic functions* (with B. Jöricke), Math. Nachr. **123** (1985), 225–254. MR0809347 (87e:31003)
- [49] *The uncertainty principle for operators that commute with a shift*. III (with B. Jöricke), Geometric questions in the theory of functions and sets, Kalinin. Gos. Univ., Kalinin, 1985, pp. 62–80. MR0829929 (88c:47053)
- [50] *A remark on Taylor series of harmonic functions*, Multidimensional complex analysis, Akad. Nauk SSSR Sibirsk. Otdel., Inst. Fiz., Krasnoyarsk, 1985, pp. 192–197. MR0899349 (88g:31007)
- [51] *The uncertainty principle in harmonic analysis* (with B. Jöricke), Proc. GDR mathematicians' congress (Rostock, 1986), pp. 72–74, Wilhelm-Pieck-Univ., Rostock, 1986. MR0845290
- [52] *The length of a harmonic vector* (with M. Giesecke), Vestnik Leningrad. Univ. Mat. Mekh. Astronom. **1987**, no. 3, 33–38. MR0928158 (88m:58175)
- [53] *The Poisson kernel is the only approximate identity that is asymptotically multiplicative on  $H^\infty$*  (with H. Wolff), Zap. Nauchn. Sem. Leningrad. Otdel. Mat. Inst. Steklov. (LOMI) **170** (1989), 82–89; translation in J. Soviet Math. **63** (1993), no. 2, 159–163. MR1039574 (91i:46058)
- [54] *Approximation properties of harmonic vector fields and differential forms* (with A. Presa Sague), Methods of approximation theory in complex analysis and mathematical physics (Leningrad, 1991), pp. 149–156, Lecture Notes in Math., 1550, Springer, Berlin. MR1322297
- [55] *Weighted approximation by trigonometric sums and the Carleman–Goluzin–Krylov formula* (with V. A. Bart), Zap. Nauchn. Sem. St. Petersburg. Otdel. Mat. Inst. Steklov. (POMI) **206** (1993), Issled. po Linein. Oper. i Teor. Funktsii. **21**, pp. 5–14; translation in J. Math. Sci. **80** (1996), no. 4, 1873–1879. MR1255311 (95f:42004)
- [56] *Szegő–Kolmogorov–Krein theorems on weighted trigonometric approximation, and Carleman-type formulas* (with V. A. Bart), Ukrain. Mat. Zh. **46** (1994), no. 1-2, 100–127; translation in Ukrainian Math. J. **46** (1994), no. 1-2, 101–132. MR1294817 (95k:42004)
- [57] *Uniqueness and free interpolation for logarithmic potentials and the Cauchy problem for the Laplace equation in  $R^2$*  (with A. Aleksandrov, J. Bourgain, M. Giesecke, and Yu. Vymenets), Geom. Funct. Anal. **5** (1995), no. 3, 529–571. MR1339816 (96f:31007)
- [58] *Uniform approximation by harmonic differential forms in Euclidean space* (with A. Presa Sague), Algebra i Analiz **7** (1995), no. 6, 104–152; translation in St. Petersburg Math. J. **7** (1996), no. 6, 943–977. MR1381980 (97b:58003)
- [59] *Approximation properties of harmonic vector fields and differential forms*, Potential theory—ICPT **94** (Kouty, 1994), pp. 91–102, de Gruyter, Berlin, 1996. MR1404702 (97f:58005)

- [60] *Uniform approximation by harmonic differential forms. A constructive approach* (with E. V. Malinnikova), *Algebra i Analiz* **9** (1997), no. 6, 156–196; translation in *St. Petersburg Math. J.* **9** (1998), no. 6, 1149–1180. MR1610176 (99g:58005)
- [61] *Approximation and extension problems for some classes of vector fields* (with S. K. Smirnov), *Algebra i Analiz* **10** (1998), no. 3, 133–162; translation in *St. Petersburg Math. J.* **10** (1999), no. 3, 507–528. MR1628034 (99f:41046)
- [62] *Bounded separation of singularities of analytic functions* (with A. H. Nersessian), *Entire functions in modern analysis* (Tel-Aviv, 1997), pp. 149–171, *Israel Math. Conf. Proc.*, 15, Bar-Ilan Univ., Ramat Gan, 2001. MR1890536 (2002m:31018)
- [63] *On the uncertainty principle for M. Riesz potentials* (with D. B. Beliaev), *Ark. Mat.* **39** (2001), no. 2, 223–243. MR1861059 (2003a:31002)
- [64] *Admissible majorants for model subspaces of  $H^2$ , I. Slow winding of the generating inner function* (with J. Mashreghi), *Canad. J. Math.* **55** (2003), no. 6, 1231–1263. MR2016246 (2004i:30029a)
- [65] *Admissible majorants for model subspaces of  $H^2$ , II. Fast winding of the generating inner function* (with J. Mashreghi), *Canad. J. Math.* **55** (2003), no. 6, 1264–1301. MR2016247 (2004i:30029b)
- [66] *Separation of singularities of analytic functions with preservation of boundedness*, *Algebra i Analiz* **16** (2004), no. 1, 293–319; translation in *St. Petersburg Math. J.* **16** (2005), no. 1, 259–283. MR2068355 (2005d:30081)
- [67] *On a theorem of I. I. Privalov on the Hilbert transform of Lipschitz functions* (with Yu. S. Belov), *Mat. Fiz. Anal. Geom.* **11** (2004), no. 4, 380–407. MR2114001 (2005k:26006)
- [68] *The Beurling–Malliavin multiplier theorem: the seventh proof* (with J. Mashreghi and F. L. Nazarov), *Algebra i Analiz* **17** (2005), no. 5, 3–68; translation in *St. Petersburg Math. J.* **17** (2006), no. 5, 699–744. MR2241422 (2007g:42028)
- [69] *Admissible majorants for model subspaces and arguments of inner functions* (with A. D. Baranov), *Funktsional. Anal. i Prilozhen.* **40** (2006), no. 4, 3–21; translation in *Funct. Anal. Appl.* **40** (2006), no. 4, 249–263. MR2307699 (2008c:30058)
- [70] *Majorants of meromorphic functions with fixed poles* (with A. D. Baranov and A. A. Borichev), *Indiana Univ. Math. J.* **56** (2007), no. 4, 1595–1628. MR2354693 (2008i:30034)
- [71] *Uniform estimates in the Poincaré–Aronszajn theorem on the separation of singularities of analytic functions* (with A. H. Nersessian and J. Ortega-Cerdà), *J. Anal. Math.* **101** (2007), 65–93. MR2346540 (2008k:30061)

### Surveys or similar

- [1] *Spaces of analytic functions*, *Math. Analysis*, Akad. Nauk SSSR Inst. Naučn. Inform., Moscow, 1966, pp. 76–164. MR0206694 (34 #6512)
- [2] P. Koosis, *Introduction to  $H^p$  spaces. With an appendix on Wolff’s proof of the corona theorem*. Translated from the English by V. V. Peller and A. G. Tumarkin. Translation edited and with a preface and appendixes by V. P. Havin. “Mir”, Moscow, 1984. MR0777505 (86b:30053)
- [3] *Methods and structure of commutative harmonic analysis*, *Commutative harmonic analysis*, I. Current problems in mathematics. Fundamental directions, **15**, pp. 6–133, *Vsesoyuz. Inst. Nauchn. i Tekhn. Inform.*, Moscow, 1987. Translation in *Encyclopaedia Math. Sci.*, **15**, pp. 1–111, Springer-Verlag, Berlin, 1991. MR1134136
- [4] *The uncertainty principle in harmonic analysis* (with B. Jöricke). *Commutative harmonic analysis*, III, Current problems in mathematics. Fundamental directions, pp. 181–260, *Vsesoyuz. Inst. Nauchn. i Tekhn. Inform.*, Moscow, 1991. Translation in *Encyclopaedia Math. Sci.*, **72**, pp. 177–259, Springer-Verlag, Berlin, 1995. MR1129019 (93e:42001)
- [5] *On some potential theoretic themes in function theory*, *The Mazya anniversary collection*, Vol. 1 (Rostock, 1998), pp. 99–110, *Oper. Theory Adv. Appl.*, **109**, Birkhäuser, Basel, 1999. MR1747866 (2001f:31007)
- [6] P. Koosis, *Introduction to  $H_p$  spaces*. Second edition. With two appendixes by V. P. Havin. *Cambridge Tracts in Mathematics*, **115**. Cambridge University Press, Cambridge, 1998. MR1669574 (2000b:30052)
- [7] *On the uncertainty principle in harmonic analysis*, *Twentieth century harmonic analysis—a celebration* (Il Ciocco, 2000), pp. 3–29, *NATO Sci. Ser. II Math. Phys. Chem.*, **33**, Kluwer Acad. Publ., Dordrecht, 2001. MR1858777 (2002k:42015)

### Translations and editing

- [1] W. K. Hayman, *Multivalent functions*. Translated from the English by V. P. Havin. Edited by I. E. Bazilevic. Izdat. Inostr. Lit., Moscow, 1960. MR0121492 (22 #12230)
- [2] J. A. Jenkins, *Univalent functions and conformal mapping*. Translated from the English by V. P. Havin. Izdat. Inostr. Lit., Moscow, 1962.
- [3] L. Carleson, *Selected problems in the theory of exceptional sets*. Translated from the English by V. P. Havin. Edited by V. G. Maz'ya. "Mir", Moscow, 1971. MR0352445 (50 #4932)
- [4] W. Rudin, *Principles of mathematical analysis*. Translated from the English by V. P. Havin. "Mir", Moscow, 1976.
- [5] *Investigations on linear operators and the theory of functions. 99 unsolved problems in linear and complex analysis*. Compiled and edited by N. K. Nikolskii, V. P. Havin, and S. V. Hrushchev. Zap. Nauchn. Sem. Leningrad. Otdel. Mat. Inst. Steklov. (LOMI) **81** (1978). "Nauka", Leningrad, 1978. MR0524720 (82e:46002)
- [6] *Complex analysis and spectral theory*. Papers from the Seminar held in Leningrad, 1979/1980. Edited by V. P. Havin and N. K. Nikolskii. Lecture Notes in Mathematics, **864**. Springer, Berlin, 1981. MR0643379 (82m:46001)
- [7] J. Garnett, *Bounded analytic functions*. Translated from the English by E. M. Dyn'kin. Translation edited and with a preface by V. P. Havin. "Mir", Moscow, 1984. MR0767451 (85j:30072)
- [8] *Linear and complex analysis problem book. 199 research problems*. Edited by V. P. Havin, S. V. Hrushchev, and N. K. Nikolskii. Lecture Notes in Mathematics, **1043**, Springer-Verlag, Berlin, 1984. MR0734178 (85k:46001)
- [9] V. I. Smirnov, *Selected works. Complex analysis. Mathematical theory of diffraction*. With a biography of Smirnov by O. A. Ladyzhenskaya. Compiled and with supplementary material by V. M. Babich, N. K. Nikolskii and V. P. Havin. Leningrad. Univ., Leningrad, 1988. MR1064111 (91g:01040)
- [10] *Commutative harmonic analysis, I. General survey. Classical aspects*. Current problems in mathematics. Fundamental directions, **15**. Vsesoyuz. Inst. Nauchn. i Tekhn. Inform., Moscow, 1987. Translation by D. Khavinson and S. V. Kislyakov. Translation edited by V. P. Havin and N. K. Nikolskii. Encyclopaedia of Mathematical Sciences, **15**, Springer-Verlag, Berlin, 1991. MR1134135 (93b:42001)
- [11] *Commutative harmonic analysis, II. Group methods in commutative harmonic analysis*. Current problems in mathematics. Fundamental directions, **25**, Vsesoyuz. Inst. Nauchn. i Tekhn. Inform., Moscow, 1988. Translated by D. Dynin and S. Dynin. Translation edited by V. P. Havin N. K. Nikolskii. Encyclopaedia of Mathematical Sciences, **25**. Springer-Verlag, Berlin, 1998. MR1622489 (99h:43002)
- [12] *Commutative harmonic analysis., IV. Harmonic analysis in  $R^n$* . Current problems in mathematics. Fundamental directions. **42**. Vsesoyuz. Inst. Nauchn. i Tekhn. Inform., Moscow, 1989. Translation by J. Peetre. Translation edited by V. P. Havin and N. K. Nikolskii. Encyclopaedia of Mathematical Sciences, **42**, Springer-Verlag, Berlin, 1992. MR1299534 (95d:42001)
- [13] *Commutative harmonic analysis, III. Generalized functions. Applications*. Current problems in mathematics. Fundamental directions. Vsesoyuz. Inst. Nauchn. i Tekhn. Inform., Moscow, 1991. Translation by R. Cooke. Translation edited by V. P. Havin and N. K. Nikolskii. Encyclopaedia of Mathematical Sciences, **72**, Springer-Verlag, Berlin, 1995. MR1447630 (98b:42001)
- [14] *Complex analysis, I. Entire and meromorphic functions. Polyanalytic functions and their generalizations*. Akad. Nauk SSSR, Vsesoyuz. Inst. Nauchn. i Tekhn. Inform., Moscow, 1991. Translation by V. I. Rublinskii and V. A. Tkachenko. Translation edited by A. A. Gonchar, V. P. Havin, and N. K. Nikolskii. Encyclopaedia of Mathematical Sciences, **85**, Springer-Verlag, Berlin, 1997. MR1464197 (98d:30002)
- [15] *Linear and complex analysis. Problem book 3. Part I*. Edited by V. P. Havin and N. K. Nikolskii. Lecture Notes in Mathematics, **1573**, Springer-Verlag, Berlin, 1994. MR1334345 (96c:00001a)
- [16] *Linear and complex analysis. Problem book 3. Part II*. Edited by V. P. Havin and N. K. Nikolskii. Lecture Notes in Mathematics, **1574**, Springer-Verlag, Berlin, 1994. MR1334346 (96c:00001b)



- [17] *Complex analysis, operators, and related topics. The S. A. Vinogradov memorial volume.* Edited by V. P. Havin and N. K. Nikolskii. Operator Theory: Advances and Applications, **113**, Birkhäuser Verlag, Basel, 2000. MR1771746 (2001b:00031)
- [18] S. A. Vinogradov, *Interpolation problems for analytic functions continuous in the closed disk and for functions whose sequence of coefficients is in  $\ell^p$ .* Translated from the 1968 Russian original by V. P. Havin. Oper. Theory Adv. Appl., **113**, Complex analysis, operators, and related topics, pp. 23–29, Birkhäuser, Basel, 2000. MR1771749
- [19] S. A. Vinogradov, *Free interpolation in spaces of analytic functions.* Translated from the 1983 Russian original by V. P. Havin. Oper. Theory Adv. Appl., **113**, Complex analysis, operators, and related topics, pp. 31–42, Birkhäuser, Basel, 2000. MR1771750

### Biographical notes

- [1] *Boris Zaharovič Vulih (on the occasion of his sixtieth birthday)* (with D. A. Vladimirov, M. K. Gavurin, G. I. Natanson, I. V. Romanovskii), Vestnik Leningrad. Univ. Mat. Meh. Astronom. **7** (1973), no. 2, 158–159. MR0325338 (48 #3685)
- [2] *Grigorii Mikhailovich Fikhtengolts (on the 100th anniversary of his birth)* (with S. A. Vinogradov, D. A. Vladimirov, M. K. Gavurin, S. M. Ermakov, B. M. Makarov, G. I. Natanson, M. Z. Solomyak, and D. K. Faddeev), Vestnik Leningrad. Univ. Mat. Mekh. Astronom. **1988**, no. 3, 3–6. MR0974784 (89m:01112)
- [3] *Gleb Pavlovich Akilov* (with A. D. Aleksandrov, A. M. Vershik, V. V. Ivanov, A. G. Kusraev, S. S. Kutateladze, B. M. Makarov, and Yu. G. Reshetnyak), Uspekhi Mat. Nauk **43** (1988), no. 1, 181–182; translation in Russian Math. Surveys **43** (1988), no. 1, 221–223. MR0937023 (89b:01031)
- [4] *Recollections of G. P. Akilov (Leningrad, the 1950s and 1960s)* (with A. I. Veksler, D. A. Vladimirov, and B. M. Makarov), Optimizatsiya **48** (65) (1990), 35–40. MR1177106 (93g:01080b)
- [5] *Stanislav Aleksandrovich Vinogradov, his life and mathematics* (with N. K. Nikolskii), Complex analysis, operators, and related topics, pp. 1–18, Oper. Theory Adv. Appl., **113**, Birkhäuser, Basel, 2000. MR1771747 (2001b:01025a)
- [6] *Viktor Solomonovich Videnskii (on the occasion of his eightieth birthday)* (with V. V. Zhuk, V. N. Malozemov and G. I. Natanson), Uspekhi Mat. Nauk **57** (2002), no. 5 (347), 182–186; translation in Russian Math. Surveys **57** (2002), no. 5, 1033–1038. MR1992100 (2004d:01042)
- [7] *Garald Isidorovich Natanson* (with O. L. Vinogradov, V. V. Zhuk, and V. L. Fainshmidt), Proceedings of the St. Petersburg Mathematical Society, Vol. X, pp. 237–252, Amer. Math. Soc. Transl. Ser. 2, **214**, Amer. Math. Soc., Providence, RI, 2005. MR2181517
- [8] *Semyon Yakovlevich Khavinson (May 17, 1927–January 30, 2003)*, Selected topics in complex analysis, pp. 1–22, Oper. Theory Adv. Appl., **158**, Birkhäuser, Basel, 2005. MR2147585

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