

American Mathematical Society

# TRANSLATIONS

Series 2 • Volume 227

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## Selected Papers on Probability and Statistics



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**American Mathematical Society**  
Providence, Rhode Island

2000 *Mathematics Subject Classification*. Primary 60–06, 62–06.

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**Library of Congress Cataloging-in-Publication Data**

Selected papers on probability and statistics.

p. cm. — (American Mathematical Society translations ; ser. 2, v. 227)

English translation of 10 articles originally published in Japanese in the journal *Sūgaku*.

Includes bibliographical references.

ISBN 978-0-8218-4821-0 (alk. paper)

1. Probabilities. 2. Mathematical statistics. I. American Mathematical Society.

QA273.S447 2009

519.2—dc22

2009004070

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10 9 8 7 6 5 4 3 2 1 14 13 12 11 10 09

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## Preface

This is a collection of papers originally published in Japanese in the journal *Sūgaku*. Normally translations of these papers would have appeared in the journal *Sugaku Expositions* published by the American Mathematical Society. To speed up the publication, the AMS decided to publish them as a volume in its series *American Mathematical Society Translations Series 2*. This book contains English translations of 10 articles devoted to various topics of stochastic processes and statistics.

The volume starts with the article “Noises, stochastic flows, and  $E_O$ -semigroups” by Jirō Akahori, Masaki Izumi, and Shinzo Watanabe. The authors consider the approach, introduced by B. Tsirelson, to the description of a stochastic process as a functional of some fundamental process, called a noise. Two standard examples of the noise process (in continuous time) are the Wiener process (white noise) and the Poisson point process (Poisson noise). In the paper the authors describe a class of other noise processes (called non-classical noises) using the theory of topological semigroups, in particular semigroups of automorphisms of certain von Neumann algebras.

The article “Volume of tubes and distribution of the maxima of Gaussian random fields”, by Satoshi Kuriki and Akimichi Takemura, describes some geometric and analytic properties of smooth Gaussian random fields on Riemannian manifolds.

In the article “Stochastic analysis on large scale interacting systems”, the author Tadahisa Funaki analyses the problem of phase separation and interface surfaces in multi-phase physical systems (such as fluids at low temperature), using the scaling limit approach that connects descriptions of a system on the macroscopic and microscopic levels. Two special types of models analyzed in the paper are the lattice gas model and  $\nabla\varphi$  interphase model.

In “Optimal transportation problem as stochastic mechanics”, the author Toshio Mikami introduces the Monge-Kantorovich optimal transportation problem and describes its applications to various problems of the theory of stochastic processes, such as the problem of constructing a stochastic process with given marginal distribution (the marginal problem) and the Markov optimal control problem.

In the article “Quantum Estimation and the Quantum Central Limit Theorem”, Masahito Hayashi presents a mathematical model of quantum estimation related to problems of quantum communication theory. The main unifying approach in the article is the use of multi-mode quantum Gaussian system and an analog of the central limit theorem (called quantum central limit theorem) for such systems. The article also contains a rather detailed introduction to the history of quantum information processing and an extensive bibliography.



The next three articles are devoted to various questions of statistics. In “Statistics and Gröbner bases – The origin and development of computational algebraic statistics”, Satoshi Aoki and Akimichi Takemura present an overview of a new area of statistics called computational algebraic statistics. In computational algebraic statistics statistical models are described as sets of solutions of certain polynomial equations and are analysed using algebraic methods. The main emphasis of the article is on the use of the Gröbner basis theory in the analysis of various types of contingency tables.

The survey article “Analysis of square contingency tables in statistics” by Sadao Tomizawa is devoted to the analysis of square contingency tables in statistics. With many examples and a long bibliography, the paper can serve as a good introduction to the subject.

The article “The structure of higher order asymptotic theory of statistical estimation” by Masafumi Akahira is devoted to the study of the maximal likelihood estimator (MLE)  $\hat{\theta}$  of an unknown parameter  $\theta$  from  $n$  independent identically distributed observations with the distribution belonging to a family of distributions depending on  $\theta$ . The author describes some new developments in the study of higher order asymptotics for the rate of convergence of  $\hat{\theta}$  to  $\theta$  as  $n \rightarrow \infty$ .

The last two articles deal with applications to financial and actuarial mathematics. The paper “On an asymptotic expansion approach to numerical problems in finance” by Akihiko Takahashi reviews the approach to numerical problems of pricing financial assets and securities based on the study of asymptotic expansions of solutions of the corresponding stochastic differential equations.

The main topic of the paper “Actuarial mathematics, basic theory and current issues in Japan”, by Koji Kuroda and Naoki Matsuyama, is clear from the title. The authors present the survey of mathematical problems related to traditional life insurance practices and describe current actuarial issues in Japan, concentrating, in particular, on possible integration of insurance and finance.

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This volume contains translations of papers that originally appeared in the Japanese journal *Sūgaku*. The papers range over a variety of topics in probability theory, statistics, and applications.

This volume is suitable for graduate students and research mathematicians interested in probability and statistics.

ISBN 978-0-8218-4821-0



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