American Mathematical Society

# TRANSLATIONS

Series 2 • Volume 231

Advances in the Mathematical Sciences

Operator Theory and Its Applications

In Memory of V. B. Lidskii (1924–2008)

Michael Levitin Dmitri Vassiliev Editors



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(Formerly Advances in Soviet Mathematics)

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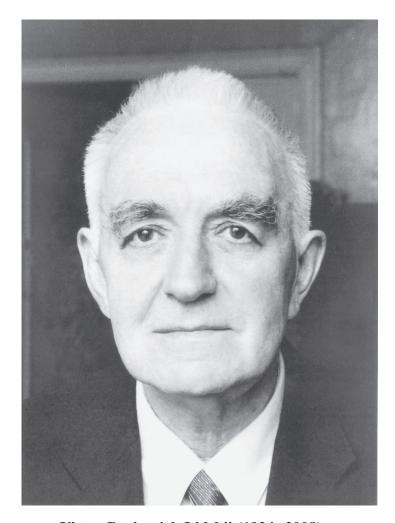
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Victor Borisovich Lidskii (1924–2008)

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This book is a collection of articles devoted to the theory of linear operators in Hilbert spaces and its applications. The subjects covered range from the abstract theory of Toeplitz operators to the analysis of very specific differential operators arising in quantum mechanics, electromagnetism, and the theory of elasticity; the stability of numerical methods is also discussed. Many of the articles deal with spectral problems for not necessarily selfadjoint operators. Some of the articles are surveys outlining the current state of the subject and presenting open problems.



