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Lars Gårding



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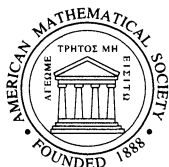
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Some Points of Analysis
and Their History

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ABSTRACT. The purpose of these lectures is to give historical background and leisurely accounts of some important results in analysis in this century. Most of them belong to the classical analysis and the theory of partial differential operators and are associated with Swedish mathematicians, but there is also the Tarski-Seidenberg theorem and Wiener's classical results in harmonic analysis.

The book is of interest to all specialists and students in analysis and partial differential equations.

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Author's Preface

The purpose of these lectures is to give historical background and leisurely accounts of some important results in analysis in this century. Most of the results in classical analysis and the theory of partial differential operators are associated with Swedish mathematicians, but we also include the Tarski-Seidenberg theorem and Wiener's classical results in harmonic analysis, which have demonstrated over time that simple things may lie behind problems that were once very famous and that engendered much work.

It goes without saying that the circle of problems treated here represent just a tiny fraction of the thousands of important results in analysis. Personal affinity rather than systematic selection has determined my sample.

The inspiration for the lectures was an invitation to join the centenary of Wuhan University in 1993. For various reasons I was not able to attend at that time, but I gave some of the lectures when I visited Nankai, Wuhan, Fudan, Jilin and Beijing universities a year later. I want to express my gratitude for the courtesy extended to me by all these universities.

I also thank Professor Li Da-Tsien for arranging the printing of my lectures and their translation into Chinese. The present expanded version, including lectures on Picard's great theorem, Nevanlinna theory, and a personal essay on the impact of distributions in analysis, has been accepted by the American Mathematical Society.¹ Finally, I thank Jana Madjarova for careful proofreading, Natałya Pluzhnikov for expert editing, and Sven Spanne for helping me with a tricky font.

Lars Gårding

Lund, 1997

¹The original title, "Some problems of analysis and their history", has now been changed to "Some points of analysis and their history".

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