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Lectures on Hilbert Schemes of Points on Surfaces

Hiraku Nakajima

American Mathematical Society

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Providence, Rhode Island

1991 *Mathematics Subject Classification.* Primary 14C05; Secondary 14F05, 14J17, 14J60, 17B65, 17B69, 16G20, 53C25, 81R10, 81T30.

Abstract. In this book, the author discusses the Hilbert scheme of points $X^{[n]}$ on a complex surface X from various points of view. It inherits structures of X, e.g. it is a nonsingular complex manifold, it has a holomorphic symplectic form if X has one, it has a hyper-Kähler metric if $X = \mathbb{C}^2$, and so on. A new structure is revealed when we study the homology group of $X^{[n]}$. The generating function of Poincaré polynomials has a very nice expression. The direct sum $\bigoplus_n H_*(X^{[n]})$ is a representation of the Heisenberg algebra.

Part of this book was written while the author enjoyed the hospitality of the Institute for Advanced Study. His stay was supported by National Science foundation Grant #DMS97-29992.

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Preface

This book is based on courses of lectures which I delivered at University of Tokyo, Nagoya University, Osaka University and Tokyo Institute of Technology between 1996 and 1998.

The purpose of the lectures was to discuss various properties of the Hilbert schemes of points on surfaces. This object is originally studied in algebraic geometry, but as it has been realized recently, it is related to many other branches of mathematics, such as singularities, symplectic geometry, representation theory, and even to theoretical physics. The book reflects this feature of Hilbert schemes. The subjects are analyzed from various points of view. Thus this book tries to tell the harmony between different fields, rather than focusing attention on a particular one.

These lectures were intended for graduate students who have basic knowledge on algebraic geometry (say chapter 1 of Hartshorne: "Algebraic Geometry", Springer) and homology groups of manifolds. Some chapters require more background, say spectral sequences, Riemannian geometry, Morse theory, intersection cohomology (perverse sheaves), etc., but the readers who are not comfortable with these theories can skip those chapters and proceed to other chapters. Or, those readers could get some idea about these theories before learning them in other books.

I have tried to make it possible to read each chapter independently. I believe that my attempt is almost successful. The interdependence of chapters is outlined in the next page. The broken arrows mean that we need only the statement of results in the outgoing chapter, and do not need their proof.

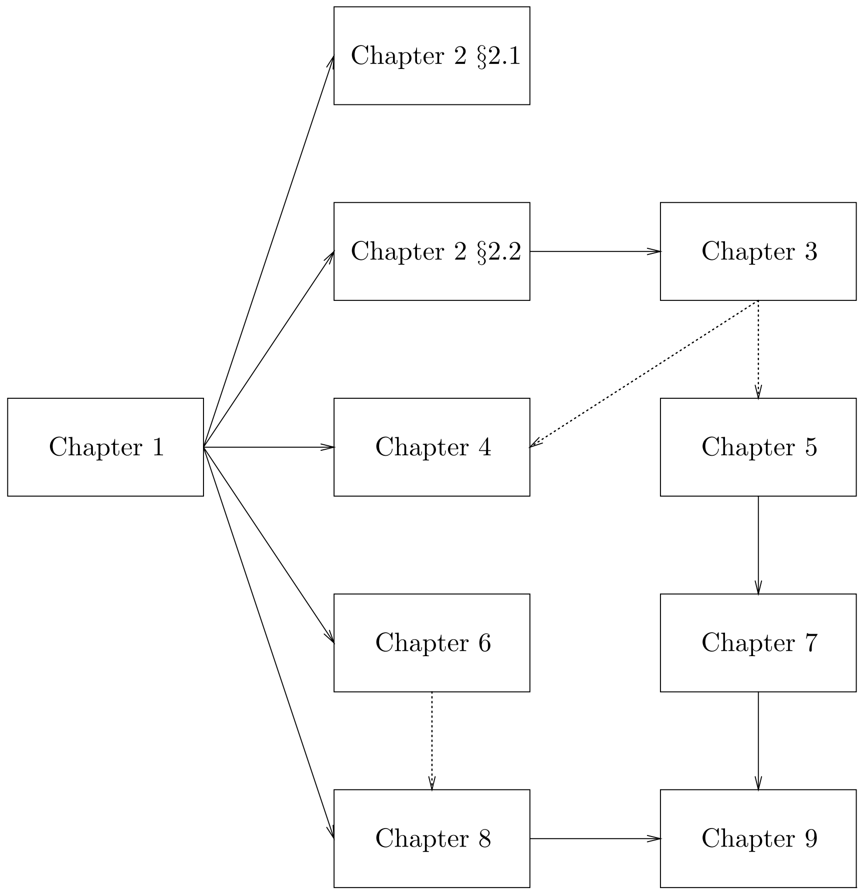
Sections 9.1, 9.4 are based on A. Matsuo's lectures at the University of Tokyo. His lectures contained Monster and Macdonald polynomials. I regret omitting these subjects. I hope to understand these by Hilbert schemes in the future.

The notes were prepared by T. Gocho and N. Nakamura. I would like to thank them for their efforts. I am also grateful to A. Matsuo and H. Ochiai for their comments throughout the lectures. A preliminary version of this book has been circulating since 1996. Thanks are due to all those who read and reviewed it, in particular to V. Baranovsky, P. Deligne, G. Ellingsrud, A. Fujiki, K. Fukaya, M. Furuta, V. Ginzburg, I. Grojnowski, K. Hasegawa, N. Hitchin, Y. Ito, A. King, G. Kuroki, M. Lehn, S. Mukai, I. Nakamura, G. Segal, S. Strømme, K. Yoshioka, and M. Verbitsky. Above all I would like to express my deep gratitute to M. A. de Cataldo for his useful comments throughout this book.

February, 1999

Hiraku Nakajima

Interdependence of the Chapters



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This beautifully written book deals with one shining example: the Hilbert schemes of points on algebraic surfaces … The topics are carefully and tastefully chosen … The young person will profit from reading this book.

—Mathematical Reviews

The Hilbert scheme of a surface X describes collections of n (not necessarily distinct) points on X. More precisely, it is the moduli space for 0-dimensional subschemes of X of length n. Recently it was realized that Hilbert schemes originally studied in algebraic geometry are closely related to several branches of mathematics, such as singularities, symplectic geometry, representation theory—even theoretical physics. The discussion in the book reflects this feature of Hilbert schemes.

One example of the modern, broader interest in the subject is a construction of the representation of the infinite-dimensional Heisenberg algebra, i.e., Fock space. This representation has been studied extensively in the literature in connection with affine Lie algebras, conformal field theory, etc. However, the construction presented in this volume is completely unique and provides an unexplored link between geometry and representation theory.

The book offers an attractive survey of current developments in this rapidly growing subject. It is suitable as a text at the advanced graduate level.

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