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Oscillating Patterns in Image Processing and Nonlinear Evolution Equations

The Fifteenth Dean Jacqueline B. Lewis Memorial Lectures

Yves Meyer

American Mathematical Society

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2000 *Mathematics Subject Classification.* Primary 35Q30, 42C40, 65T60, 76D05, 76D06.

ABSTRACT. This book is aimed at bridging a gap between signal processing, image processing, and nonlinear differential equations. More precisely the role played by oscillating patterns will be emphasized. These patterns occur as the textured components of images. They also occur in frequency modulated signals or chirps. What is more surprising is the relevance of such patterns in partial differential equations.

The first half of this essay can be used as a textbook, since fully detailed proofs are given.

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Contents

Preface

This book consists of three chapters in which some seemingly unrelated scientific problems will be considered. The first chapter is devoted to **image processing** and more precisely to image compression and denoising. This research is motivated by the upcoming standard for still image compression. This standard will be unveiled in March 2001 and is known as JPEG-2000. In the second chapter, a few new results on the **Navier-Stokes** equations and other nonlinear evolution equations will be discussed and, in the third chapter, **frequency modulated signals** will be analyzed. Motivation comes from the Virgo program of detection of gravitational waves.

How could these distinct themes possibly be studied from the same perspective? An answer is found in the contents of the three chapters.

Analyzing the performances of a compression algorithm requires a model for still images. In the first chapter, our discussion will be based on the Osher-Rudin model. This model originated in a joint paper between Stanley Osher, Leonid Rudin and Emad Fatemi [**81**]. It was then developed by Osher and Rudin and that is why it will be named the Osher-Rudin model. It amounts to splitting an image f into a sum $f = u + v$ between two components u and v. The first component u represents the 'objects' which are contained inside the given image f. It is then natural to assume that u belongs to the space BV of functions with bounded variation. Wavelet expansions are quite effective for analyzing BV functions (Theorem 14, Section 21, Chapter 1). On the other hand, we will prove that the 'texture + noise' component v is an 'oscillating pattern' (Theorem 3, Section 14, Chapter 1). Here 'oscillating patterns' will be defined by some Besov norm estimates. This discussion will imply that a 'wavelet thresholding' wipes away this v component.

The second chapter starts with an interesting sharpening of Gagliardo-Nirenberg estimates (Theorem 16, Section 2). It continues with an improvement on Poincaré's inequality (Theorem 21, Section 3). These advances rely heavily on Theorem 14 of the first chapter. The same Besov space which was used for modeling textures will be the heart of the matter here.

This specific Besov space and some related functional spaces will again be important in this second chapter. We are alluding here to a new result on the Navier-Stokes equations which was obtained by Herbert Koch and Daniel Tataru (Theorem 32, Section 6, Chapter 2).

Feature extraction is pivotal in fluid dynamics. One would like to detect and extract the elusive 'coherent structures'. These structures are specific patterns which appear both in experimental work in fluid dynamics and in numerical simulations. In order to study coherent structures, one needs to investigate localized and oscillating solutions of the Navier-Stokes equations. This will also be done in Chapter 2.

Einstein's general relativity implies the existence of gravitational waves. The theory predicts that such waves might be created by gigantic gravitational disasters. In the case of a collapse of a pair of binary stars, these waves would be frequency modulated signals, also named chirps. There is no doubt that detecting such signals relies on describing and analyzing oscillating patterns. This important topic is discussed in Chapter 3.

The unity between our three chapters is now clear. Beyond Fourier analysis and its variants, new tools are available. These tools help in understanding and modeling oscillating patterns. Such patterns are present in many natural images. They help in explaining nonlinear evolutions. They are pivotal in chirps and frequency modulated signals.

Let me express my deep thanks to the Department of Mathematics of Rutgers University. I was proud and happy to give these three Jacqueline Lewis Lectures. Richard Falk and his colleagues have been kind and helpful.

This book is not a treatise but rather an expanded version of my informal talks. It is not a textbook. Instead it is aimed at describing a scientific vision.

Am I the sole writer of this book? This is not clear since most of the fine results which will be described or proved belong to my students or my colleagues. Moreover the 'grand unification' which is unveiled in this essay can be traced back to a program which was launched in the early forties by the 'Institute for the Unity of Science' (see Section 2, Chapter 1).

Let me thank Albert Cohen and his collaborators for letting me include some still unpublished material and for his encouraging remarks. Naoki Saito read a first version of this book and suggested many improvements. The anonymous referee was kind and helpful.

I would like to dedicate my work to Jacqueline Lewis. Let me also remember Alberto Calderón. I miss so much our endless discussions about mathematics, literature and politics in Argentina.

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Oscillating Patterns in Image Processing and Nonlinear Evolution Equations

The Fifteenth Dean Jacqueline B. Lewis Memorial Lectures

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Image compression, the Navier-Stokes equations, and detection of gravitational waves are three seemingly unrelated scientific problems that, remarkably, can be studied from one perspective. The notion that unifies the three problems is that of "oscillating patterns", which are present in many natural images, help to explain nonlinear equations, and are pivotal in studying chirps and frequency-modulated signals.

The first chapter of this book considers image processing, more precisely algorithms of image compression and denoising. This research is motivated in particular by the new standard for compression of still images known as JPEG-2000. The second chapter has new results on the Navier-Stokes and other nonlinear evolution equations. Frequency-modulated signals and their use in the detection of gravitational waves are covered in the final chapter.

In the book, the author describes both what the oscillating patterns are and the mathematics necessary for their analysis. It turns out that this mathematics involves new properties of various Besov-type function spaces and leads to many deep results, including new generalizations of famous Gagliardo-Nirenberg and Poincaré inequalities.

This book is based on the "Dean Jacqueline B. Lewis Memorial Lectures" given by the author at Rutgers University. It can be used either as a textbook in studying applications of wavelets to image processing or as a supplementary resource for studying nonlinear evolution equations or frequency-modulated signals. Most of the material in the book did not appear previously in monograph literature.

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