Combinatorial Constructions in Ergodic Theory and Dynamics

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Ergodic theory studies measure-preserving transformations of measure spaces. These objects are intrinsically infinite and the notion of an individual point or an orbit makes no sense. Still there is a variety of situations when a measure-preserving transformation (and its asymptotic behavior) can be well described as a limit of certain finite objects (periodic processes).

In the first part of this book this idea is developed systematically, genericity of approximation in various categories is explored, and numerous applications are presented, including spectral multiplicity and properties of the maximal spectral type. The second part of the book contains a treatment of various constructions of cohomological nature with an emphasis on obtaining interesting asymptotic behavior from approximate pictures at different time scales.

The book presents a view of ergodic theory not found in other expository sources and is suitable for graduate students familiar with measure theory and basic functional analysis.

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