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Superdiffusions and Positive Solutions of Nonlinear Partial Differential Equations

E. B. Dynkin

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Preface

This book is devoted to the applications of the probability theory to the theory of nonlinear partial differential equations. More precisely, we investigate the class $\mathcal{U}$ of all positive solutions of the equation $Lu = \psi(u)$ in E where L is an elliptic differential operator of the second order, E is a bounded smooth domain in $\mathbb{R}^d$ and ψ is a continuously differentiable positive function.

The progress in solving this problem till the beginning of 2002 was described in the monograph [**D**]. [We use an abbreviation [**D**] for [Dy02].] Under mild conditions on ψ, a trace on the boundary ∂E was associated with every $u \in \mathcal{U}$. This is a pair (Γ, ν) where Γ is a subset of ∂E and ν is a σ-finite measure on $\partial E \setminus \Gamma$. [A point y belongs to Γ if $\psi'(u)$ tends sufficiently fast to infinity as $x \to y$.] All possible values of the trace were described and a 1-1 correspondence was established between these values and a class of solutions called σ-moderate. We say that u is σ-moderate if it is the limit of an increasing sequence of moderate solutions. [A moderate solution is a solution u such that $u \leq h$ where $Lh = 0$ in E.] In the Epilogue to [**D**], a crucial outstanding question was formulated: *Are all the solutions σ-moderate?* In the case of the equation $\Delta u = u^2$ in a domain of class C^4, a positive answer to this question was given in the thesis of Mselati [Ms02a]—a student of J.-F. Le Gall.[1] However his principal tool—the Brownian snake—is not applicable to more general equations. In a series of publications by Dynkin and Kuznetsov [Dy04b], [Dy04c], [Dy04d], [Dy04e],[DK03], [DK04], [Ku04], Mselati's result was extended, by using a superdiffusion instead of the snake, to the equation $\Delta u = u^\alpha$ with $1 < \alpha \leq 2$. This required an enhancement of the superdiffusion theory which can be of interest for anybody who works on applications of probabilistic methods to mathematical analysis.

The goal of this book is to give a self-contained presentation of these new developments. The book may be considered as a continuation of the monograph [**D**]. In the first three chapters we give an overview of the theory presented in [**D**] without duplicating the proofs which can be found in [**D**]. The book can be read independently of [**D**]. [It might be even useful to read the first three chapters before reading [**D**].]

In a series of papers (including [MV98a], [MV98b] and [MV04]) M. Marcus and L. Véron investigated positive solutions of the equation $\Delta u = u^\alpha$ by

[1]The dissertation of Mselati was published in 2004 (see [Ms04]).

purely analytic methods. Both analytic and probabilistic approaches have their advantages and an interaction between analysts and probabilists was important for the progress of the field. I take this opportunity to thank M. Marcus and L. Véron for keeping me informed about their work.

The Choquet capacities are one of the principal tools in the study of the equation $\Delta u = u^\alpha$. This class contains the Poisson capacities used in the work of Dynkin and Kuznetsov and in this book and the Bessel capacities used by Marcus and Véron and by other analysts. I am very grateful to I. E. Verbitsky who agreed to write Appendix B, where the relations between the Poisson and Bessel capacities are established, thus allowing to connect the work of both groups.

I am indebted to S. E. Kuznetsov who provided me with several preliminary drafts of his paper [Ku04] used in Chapters **8** and **9**. I am grateful to him and to J.-F. Le Gall and B. Mselati for many helpful discussions. It is my pleasant duty to thank J.-F. Le Gall for permission to include in the book as Appendix A his note which clarifies a statement used but not proved in Mselati's thesis (we use it in Chapter **8**).

I am especially indebted to Yuan-chung Sheu for reading carefully the entire manuscript and suggesting many corrections and improvements.

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References

[AH96] D. R. Adams and L. I. Hedberg, *Function Spaces and Potential Theory*, Springer-Verlag, New York, 1996.

[BHQ79] Bui Huy Qui, *Harmonic functions, Riesz potentials, and the Lipschitz spaces of Herz*, Hiroshima Math. J. **5** (1979), 245–295.

[COV04] C. Cascante, J. M. Ortega, and I. E. Verbitsky, *Nonlinear potentials and two weight trace inequalities for general dyadic and radial kernels*, Indiana Univ. Math. J. **53** (2004).

[Ch54] G. Choquet, *Theory of capacities*, Ann. Inst. Fourier (Grenoble) **5** (1953), 131–295.

[Da75] D. A. Dawson, *Stochastic evolution equations and related measure processes*, J. Multivariant Anal. **3** (1975), 1–52.

[Da93] ———, *Measure-valued Markov processes*, Springer-Verlag, 1993, Lecture Notes in Math., vol. 1541.

[DL97] J. S. Dhersin and J.-F. Le Gall, *Wiener's test for super-Brownian motion and for the Brownian snake*, Probab. Th. Rel. Fields **108** (1997), 103–129.

[DuL02] T. Duquesne and J.-F. Le Gall, *Random trees, Lévy Processes and Spatial Branching Processes*, Sociéte Mathématique de France, Astérisque **281** (2002).

[Dy88] E. B. Dynkin, *Representation for functionals of superprocesses by multiple stochastic integrals, in application to selfintersection local times*, Société Mathématique de France, Astérisque **157–158** (1988), 147–171.

[Dy91a] ———, *A probabilistic approach to one class of nonlinear differential equations*, Probab. Th. Rel. Fields **89** (1991), 89–115.

[Dy91b] ———, *Branching particle systems and superprocesses*, Ann. Probab. **19** (1991), 1157–1194.

[Dy92] ———, *Superdiffusions and parabolic nonlinear differential equations*, Ann. Probab. **20** (1992), 942–962.

[Dy93] ———, *Superprocesses and partial differential equations*, Ann. Probab. **21** (1993), 1185–1262.

[Dy94] ———, *An Introduction to Branching Measure-Valued Processes*, American Mathematical Society, Providence, R. I, 1994.

[Dy97] ———, *A new relation between diffusions and superdiffusions with applications to the equation $Lu = u^\alpha$*, C. R. Acad. Sc. Paris, Série I **325** (1997), 439–444.

[Dy98] ———, *Stochastic boundary values and boundary singularities for solutions of the equation $Lu = u^\alpha$*, J. Funct. Anal. **153** (1998), 147-186.

[Dy02] ———, *Diffusions, Superdiffusions and Partial Differential Equations*, American Mathematical Society, Providence, R.I., 2002.

[Dy04a] ———, *Harmonic functions and exit boundary of superdiffusion*, J. Funct. Anal. **206** (2004), 31–68.

[Dy04b] ———, *Superdiffusions and positive solutions of nonlinear partial differential equations*, Uspekhi Matem. Nauk **59** (2004).

[Dy04c] ———, *Absolute continuity results for superdiffusions with applications to differential equations*, C. R. Acad. Sc. Paris, Série I **338** (2004), 605–610.

[Dy04d] ———, *On upper bounds for positive solutions of semilinear equations*, J. Funct. Anal. **210** (2004), 73–100.

[Dy04e] ———, *A new inequality for superdiffusions and its applications to nonlinear differential equations*, Amer. Math. Soc., Electronic Research Announcements, to appear.

[DK96a] E. B. Dynkin and S. E. Kuznetsov, *Solutions of $Lu = u^\alpha$ dominated by L-harmonic functions*, J. Anal. Math. **68** (1996), 15–37.

[DK96b] ———, *Superdiffusions and removable singularities for quasilinear partial differential equations*, Comm. Pure Appl. Math. **49** (1996), 125–176.

[DK98a] ———, *Trace on the boundary for solutions of nonlinear differential equations*, Trans. Amer. Math. Soc. **350** (1998), 4499–4519.

[DK98b] ———, *Fine topology and fine trace on the boundary associated with a class of quasilinear differential equations*, Comm. Pure Appl. Math. **51** (1998), 897–936.

[DK03] ———, *Poisson capacities*, Math. Research Letters **10** (2003), 85–95.

[DK04] ———, *$\mathbb{N}$-measures for branching exit Markov systems and their applications to differential equations*, Prob. Th. Rel. Fields, to appear.

[EP91] S. N. Evans and E. Perkins, *Absolute continuity results for superprocesses with some aplications*, Trans. Amer. Math. Soc. **325** (1991), 661–681.

[GrW82] M. Grüter and K. O. Widam, *The Green function for uniformly elliptic equations*, Manuscripta Math. **37** (1982), 303–342.

[GT98] D. Gilbarg and N. S. Trudinger, *Elliptic Partial Differential Equations of the Second Order, Second Edition, Revised Third Printing*, Springer-Verlag, Berlin–Heidelberg–New York, 1998.

[GV91] A. Gmira and L. Véron, *Boundary singularities of solutions of some nonlinear elliptic equations*, Duke Math. J. **64** (1991), 271–324.

[HW83] L. I. Hedberg and Th. H. Wolff, *Thin sets in nonlinear potential theory*, Ann. Inst. Fourier (Grenoble) **33** (1983), 161–187.

[Ka77] O. Kallenberg, *Random Measures, 3rd. ed.*, Academic Press, 1977.

[KV99] N. J. Kalton and I. E. Verbitsky, *Nonlinear equations and weighted norm inequalities*, Trans. Amer. Math. Soc. **351** (1999), 3441–3497.

[Ke57] J. B. Keller, *On the solutions of $\Delta u = f(u)$*, Comm. Pure Appl. Math. **10** (1957), 503–510.

[Ku98] S. E. Kuznetsov, *σ-moderate solutions of $Lu = u^\alpha$ and fine trace on the boundary*, C. R. Acad. Sc. Paris, Série I **326** (1998), 1189–1194.

[Ku04] ———, *An upper bound for positive solutions of the equation $\Delta u = u^\alpha$*, Amer. Math. Soc., Electronic Research Announcements, to appear.

[Lab03] D. A. Labutin, *Wiener regularity for large solutions of nonlinear equations*, Arkiv for Mathematik **41** (2003), 203–231.

[La77] N. S. Landkov, *Foundations of Modern Potential Theory*, Springer-Verlag, Berlin, 1972.

[Le93] J.-F. Le Gall, *Solutions positives de $\Delta u = u^2$ dans le disque unité*, C. R. Acad. Sci. Paris, Série I **317** (1993), 873–878.

[Le95] ———, *The Brownian snake and solutions of $\Delta u = u^2$ in a domain*, Prob. Th. Rel. Fields **102** (1995), 393–402.

[Le97] ———, *A probabilistic Poisson representation for positive solutions of $\Delta u = u^2$ in a planar domain*, Comm. Pure Appl Math. **50** (1997), 69–103.

[Le99] ———, *Spatial Branching Processes, Random Snakes and Partial Differential Equations*, Birkhäuser, Basel, Boston, Berlin, 1999.

[Ma75] V. G. Maz'ya, *Beurling's theorem on a minimum principle for positive harmonic functions*, J. Soviet Math. **4** (1975), 367–379, First published (in Russian) in: Zap. Nauch. Sem. Leningrad. Otdel. Mat. Inst. Steklov., **30** (1972), 76–90.

[Ma85] ———, *Sobolev Spaces*, Springer-Verlag, Berlin–Heidelberg–New York, 1985.

[Me66] P. A. Meyer, *Probability and Potentials*, Blaisdell, Waltham, MA, 1966.

[Ms02a] B. Mselati, *Classification et représentation probabiliste des solutions positives de $\Delta u = u^2$ dans un domaine*, Thése de Doctorat de l'Université Paris 6, 2002.

[Ms02b] ———, *Classification et représentation probabiliste des solutions positives d'une équation elliptic semi-linéaire*, C.R. Acad. Sci. Paris,Série I **332** (2002).

[Ms04] ———, *Classification and probabilistic representation of the positive solutions of a semilinear elliptic equation*, Mem. Amer. Math. Soc. **168** (2004).

[MV98a] M. Marcus and L. Véron, *The boundary trace of positive solutions of semilinear elliptic equations, I: The subcritical case*, Arch. Rat. Mech. Anal. **144** (1998), 201–231.

[MV98b] ———, *The boundary trace of positive solutions of semilinear elliptic equations: The supercritical case*, J. Math. Pures Appl. **77** (1998), 481–524.

[MV03] ———, *Capacitary estimates of solutions of a class of nonlinear elliptic equations*, C. R. Acad. Sci. Paris, Ser. I **336** (2003), 913–918.

[MV04] ———, *Capacitary estimates of positive solutions of semilinear elliptic equations with absorbtion*, J. European Math. Soc., to appear.

[MW74] B. Muckenhoupt and R. L. Wheeden, *Weighted norm inequalities for fractional integrals*, Trans. Amer. Math. Soc. **192** (1974), 261–274.

[Os57] R. Osserman, *On the inequality $\Delta u \geq f(u)$*, Pacific J. Math. **7** (1957), 1641–1647.

[Ru73] W. Rudin, *Functional Analysis*, McGraw-Hill, 1973.

[Ru87] ———, *Real and Complex Analisis*, McGraw-Hill, 1987.

[Wa68] S. Watanabe, *A limit theorem on branching processes and continuous state branching processes*, J. Math. Kyoto Univ. **8** (1968), 141–167.

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