Three Lectures on Commutative Algebra

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Preface

The Winter School on Commutative Algebra and Applications was held from January 30 to February 3, 2006, at the Institut de Matemàtica de la Universitat de Barcelona (IMUB). The Winter School was a scientific activity of the (RTACA) Red Temática de Algebra Conmutativa y Aplicaciones. The school consisted of three series of lectures surveying recent developments in commutative algebra and applications and some additional talks and communications. The series of lectures were: Tight Closure and Vector Bundles (Holger Brenner, University of Sheffield), Combinatorics and Commutative Algebra (Jürgen Herzog, University of Essen), Constructive Desingularization (Orlando Villamayor, Universidad Autónoma de Madrid). This volume collects an enlarged version of these series of lectures.

The School was organized by G. Colomé-Nin, T. Cortadellas-Benítez, J. Elias, and S. Zarzuela. It was sponsored by Institut de Matemàtica de la Universitat de Barcelona, Societat Catalana de Matemàtiques and Ministerio de Educación y Ciencia.

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Titles in This Series

42 Holger Brenner, Jürgen Herzog, and Orlando Villamayor (Gemma Colomé-Nin, Teresa Cortadellas Benítez, Juan Elias, and Santiago Zarzuela, Editors), Three lectures on commutative algebra, 2008
41 James Haglund, The q,t-Catalan numbers and the space of diagonal harmonics (with an appendix on the combinatorics of Macdonald polynomials), 2008
40 Vladimir Pestov, Dynamics of infinite-dimensional groups. The Ramsey–Dvoretzky–Milman phenomenon, 2006
39 Oscar Zariski, The moduli problem for plane branches (with an appendix by Bernard Teissier), 2006
37 Alexander Polishchuk and Leonid Positselski, Quadratic algebras, 2005
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This book provides careful and detailed introductions to some of the latest advances in three significant areas of rapid development in commutative algebra and its applications. The book is based on courses at the Winter School on Commutative Algebra and Applications held in Barcelona: Tight closure and vector bundles, by H. Brenner; Combinatorics and commutative algebra, by J. Herzog; and Constructive desingularization, by O. Villamayor.

The exposition is aimed at graduate students who have some experience with basic commutative algebra or algebraic geometry but may also serve as an introduction to these modern approaches for mathematicians already familiar with commutative algebra.

This book is copublished by the Real Sociedad Matemática Española and the American Mathematical Society.