

Contents

Preface	xiii
Chapter 1. Patterns and Induction	1
Goals	1
1.1. Mathematics, Patterns, and Computers	1
Problems	4
1.2. Writing Mathematical Sentences and Proof by Induction	5
Problems	9
1.3. Additional Problems	11
Problems	11
Chapter 2. Divisibility	17
Goals	17
2.1. The Division Algorithm and Strange Properties of Positive Integers	17
Problems	19
2.2. Writing Mathematics in Paragraphs, Proof by Contradiction, and Irrational Numbers	22
Problems	24
2.3. Introducing the planet mod n	25
Problems	26
2.4. Additional Problems	27
Problems	27
Chapter 3. Primes	33
Goals	33
3.1. Prime Numbers	33
Problems	34
3.2. Formulas for Primes	38
Problems	41
3.3. Fermat's Theorem, Pseudo-Primes, and Carmichael Numbers	42
Problems	42
3.4. Dynamical Systems and a Proof of Fermat's Theorem	45
Problems	50
3.5. Public Key Cryptography	53
Problems	55
3.6. Open Conjectures about Primes	57
Chapter 4. Derivatives and Approximations of Functions	59
Goals	59
4.1. A Quick Review of Derivatives	59

Problems	63
4.2. Continuous Functions and Differentiability	66
Problems	70
4.3. Linearization and Approximation of Functions	73
Problems	81
4.4. Taylor Polynomials	84
Problems	87
4.5. Additional Problems	89
Problems	89
Chapter 5. Antiderivatives and Integration	91
Goals	91
5.1. Why Can Areas Be Found Using Antiderivatives? A Quick Review of Integration	91
Problems	98
5.2. Approximating Integrals: Inscribed and Circumscribed Rectangles	100
Problems	103
5.3. Functions Defined by Integrals	105
Problems	106
5.4. What if $F'(x) = G'(x)$?	108
Problems	109
Chapter 6. Distribution of Primes	111
Goals	111
6.1. Approximating $x/\pi(x)$	111
Problems	112
6.2. The Sieve of Eratosthenes	113
Problems	113
Chapter 7. Log, Exponential, and the Inverse Trigonometric Functions	115
Goals	115
7.1. The Natural Log Function and the Distribution of Primes	115
Problems	116
7.2. Properties of the Log Function	117
Problems	118
7.3. The Exponential Function	120
Problems	122
7.4. Inverse Trigonometric Functions	127
Problems	128
7.5. Additional Problems	128
Problems	128
Chapter 8. The Mean Value Theorem and Approximations	131
Goals	131
8.1. Real Numbers and Properties of Continuous Functions	131
Problems	133
8.2. The Mean Value Theorem for Integrals	134
Problems	134
8.3. The Mean Value Theorem	135
Problems	137

8.4. The Error in a Taylor Polynomial Approximation	139
Problems	142
Chapter 9. Linearization Topics	147
Goals	147
9.1. L'Hospital's Rule	147
Problems	148
9.2. Newton's Method	151
Problems	152
Chapter 10. Defining Integrals, Areas, and Arclengths	155
Goals	155
10.1. Going Beyond an Intuitive Notion of Area	155
Problems	162
10.2. Arc Length	164
Problems	165
Chapter 11. Improper Integrals and Techniques of Integration	167
Goals	167
11.1. Improper Integrals	167
Problems	168
11.2. Integration Methods	170
Problems	171
Chapter 12. The Prime Number Theorem	175
Goals	175
12.1. The Prime Number Theorem	175
Problems	176
12.2. Primes between n and $2n$	176
Problems	177
12.3. Logarithmic integral	177
Problems	178
12.4. Where approximately is the n th prime?	179
Problems	179
12.5. Primes and the Riemann Hypothesis	180
Chapter 13. Local Approximation of Functions and Integral Estimations	183
Goals	183
13.1. Taylor Polynomials and Approximations of Integrals. Are they related?	183
Problems	184
13.2. Approximating Integrals: Rectangles, Trapezoids, and Parabolas	187
Problems	188
13.3. Curvature	189
Problems	190
13.4. Padé Approximants	191
Problems	192
Chapter 14. Sequences and Series	195
Goals	195

14.1. Sequences, Convergence, and Mathematical Rigor	195
Problems	199
14.2. Series	203
Problems	206
14.3. Monotone Bounded Sequences and Limit Properties	208
Problems	210
14.4. The n th Term Test and the Comparison Test	213
Problems	214
14.5. Euler's Constant and the Alternating Harmonic Series	217
Problems	217
14.6. The Integral Test and p -series	218
Problems	219
14.7. Additional Problems	221
Problems	221
Chapter 15. Power Series and Taylor Series	223
Goals	223
15.1. Taylor Polynomials and Series	223
Problems	225
15.2. Power Series and the Ratio Test	225
Problems	226
15.3. Analytic Functions and Convergence of Taylor Series	231
Problems	232
15.4. The Interval of Convergence of a Power Series	233
Problems	235
15.5. New Power Series from Old	236
Problems	238
Chapter 16. More On Series	243
Goals	243
16.1. The Limit Comparison Test	243
Problems	243
16.2. Leibniz's Alternating Series Test	247
Problems	248
16.3. Additional Problems	249
Problems	249
Chapter 17. Limits of Functions	251
Goals	251
Introduction	251
17.1. The Precise Definition of Limits	251
Problems	255
Chapter 18. Differential Equations	259
Goals	259
18.1. Differential Equations and Modeling	259
Problems	260
18.2. Qualitative Analysis of Differential Equations	262
Problems	263
18.3. Additional Problems	266

Problems	266
Chapter 19. Logical Arguments	271
Goals	271
19.1. Logical Reasoning through Puzzles	271
Problems	271
Hints for Selected Problems	277
Bibliography	283
Index	287