Preface

A Discrete Transition to Advanced Mathematics is designed to bridge the gap between more-computational lower level courses and more-theoretical upper level courses in mathematics. While the focus is on building understanding, sharpening critical thinking skills, and developing mathematical maturity, topics from discrete mathematics provide the means.

The text contains more material than can be covered in one semester. There are several reasons for this. First, this makes the book appropriate for Discrete Mathematics courses for second- or third-year mathematics majors, as well as for Introduction to Proofs courses. Second, this will allow each instructor some flexibility in the selection of topics. Perhaps the best reason for the inclusion of so much material, however, is that the book is designed for students who should be learning to read mathematics on their own, and the extra sections should provide enjoyable reading at an appropriate level for these students. Besides more standard topics, the topics mentioned below will distinguish this text from others and, if not presented in class, would provide excellent material for independent projects.

- *Divisibility tests*, long familiar to many students, are explained and proved in Section 3.4.

- The surprising elementary *number patterns* in Section 3.5 emphasize the importance of pattern recognition.

- The *binomial coefficients* are introduced and applied geometrically in Section 4.1 before the formula for them is presented in Section 4.3.

- Modular arithmetic is introduced in Section 5.4 as a quotient construction and *quotient spaces* are used to investigate partial order relations on the blocks of a partition of a set A (i.e., quasiorders on A).

- The study of *sequences* in Chapter 8 provides a discrete version of analysis. *Finite differences* and their relation to sequences generated by polynomials are investigated. *Limits* are treated formally, providing an introduction to epsilon-N proofs for those who may have missed epsilon proofs in the calculus sequence.

- Infinite series, infinite products, and nested radicals in Section 8.5 provide an introduction to some forms of *infinite arithmetic*.

- *Fibonacci numbers* and *Pascal's triangle* in Chapter 9 provide a delightful array of surprising results that provide a unifying synthesis of topics from the previous chapters.

- *Continued fractions* and their applications are discussed in Chapter 10.

The remarkable connections between the Fibonacci numbers, Pascal's triangle, and the golden ratio in Chapter 9 were the original impetus for our writing this text. We considered a course based on these connections and patterns, many of which are very easily grasped. As we debated the appropriate level of presentation, we concluded that these ideas would serve as an excellent capstone to *A Discrete Transition to Advanced Mathematics* course.

We have taught courses based on Chapters 1–6 with Sections 3.4, 3.5, 4.5, and 6.4 optional, with additional topics and projects selected from the later chapters. Chapters 1–3 are required for all subsequent chapters and should be presented in order. The subsequent chapters need not be covered in order, but Chapter 5 is required for Chapter 6 and Sections 6.1 and 6.2 are needed for Section 7.2 and ' Chapter 8.

The material presented here should be accessible to students with the mathematical maturity provided by two or three semesters of calculus or an introductory linear algebra class. No calculus or linear algebra is used, but on a few occasions, connections to these subjects are noted.

Besides many classic results, we also include many elegant or surprising results which are not as widely known. We have tried to attain an engaging writing style that emphasizes precision through an intuitive understanding of the underlying concepts. However, simply reading the text will not be enough: Every student should work lots of exercises! There are over 650 exercises of varying difficulty designed to reinforce and extend the material presented.

We hope that the selection of topics, examples, and exercises will provide each reader with some of the marvel and amazement we still enjoy.

Ancillaries

The following ancillaries are available:

Student Solutions Manual The Student Solutions Manual provides worked out solutions to selected problems in the text.

Complete Solutions Manual The Complete Solutions Manual provides complete worked out solutions to all of the problems in the text and is available only to instructors.

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