

Contents

Preface	vii
Acknowledgments	ix
Lecture 1. Introduction	1
1A. Discrete versions of G	1
1B. Nonlinear actions	2
1C. Open questions	5
Comments	5
References	6
Lecture 2. Actions in Dimension 1 or 2	9
2A. Finite actions	9
2B. Actions on the circle	10
2C. Farb-Shalen method	11
2D. Actions on surfaces	12
Comments	13
References	14
Lecture 3. Geometric Structures	17
3A. Reductions of principal bundles	17
3B. Algebraic hull of a G -action on M	18
3C. Actions preserving an H -structure	20
3D. Groups that act on Lorentz manifolds	21
Comments	22
References	22
Lecture 4. Fundamental Groups I	25
4A. Engaging conditions	25
4B. Consequences	27
4C. Examples: Actions with engaging conditions	29
Comments	30
References	30
Lecture 5. Gromov Representation	33
5A. Gromov's Centralizer Theorem	33
5B. Higher-order frames and connections	35
5C. Ideas in the proof of Gromov's Centralizer Theorem	37
Comments	38
References	38

Lecture 6. Superrigidity and First Applications	41
6A. Cocycle Superrigidity Theorem	41
6B. Application to connection-preserving actions	43
6C. From measurability to smoothness	44
Comments	45
References	45
Lecture 7. Fundamental Groups II (Arithmetic Theory)	47
7A. The fundamental group is a lattice	47
7B. Arithmeticity of representations of the fundamental group	49
Comments	50
References	50
Lecture 8. Locally Homogeneous Spaces	53
8A. Statement of the problem	53
8B. The use of cocycle superrigidity	55
8C. The approach of Margulis and Oh	56
Comments	58
References	58
Lecture 9. Stationary Measures and Projective Quotients	61
9A. Stationary measures	61
9B. Projective quotients	63
9C. Furstenberg entropy	64
Comments	65
References	65
Lecture 10. Orbit Equivalence	67
10A. Definition and basic facts	67
10B. Orbit-equivalence rigidity for higher-rank simple Lie groups	68
10C. Orbit-equivalence rigidity for higher-rank lattice subgroups	68
10D. Orbit-equivalence rigidity for free groups	69
10E. Relationship to quasi-isometries	70
Comments	71
References	72
Appendix: Background Material	75
A1. Lie groups	75
A2. Ergodic Theory	76
A3. Moore Ergodicity Theorem	76
A4. Algebraic Groups	77
A5. Cocycle Superrigidity Theorem	78
A6. Borel Density Theorem	79
A7. Three theorems of G. A. Margulis on lattice subgroups	79
A8. Fixed-point theorems	80
A9. Amenable groups	80
References	81
Name Index	83
Index	85