Preface

This monograph is compiled from the notes of a series of ten lectures given at the NSF-CBMS Conference on Malliavin Calculus and Its Applications at Kent State University, Ohio, August 7th to 12th, 2008.

The Malliavin calculus or stochastic calculus of variations is an infinite-dimensional differential calculus on the Wiener space that has been developed from the probabilistic proof of Hörmander's hypoellipticity theorem by Paul Malliavin in 1976 (see the reference [**25**]). Contributions by Stroock, Bismut, Kusuoka, and Watanabe, among others, have expanded this theory in different directions.

The main application of Malliavin calculus is to establish the regularity of the probability distribution of functionals of an underlying Gaussian process. In this way one can prove the existence and smoothness of the density for solutions to ordinary and partial stochastic differential equations. In addition to this main application, the Malliavin calculus has proved to be a powerful tool in a variety of problems in stochastic analysis. For example, the divergence operator can be interpreted as a generalized stochastic integral, and this has been the starting point of the development of the anticipating stochastic calculus. In the last years some new applications of Malliavin calculus in areas such as central limit theorems and mathematical finance have emerged.

The purpose of these lectures is to introduce the basic results of Malliavin calculus and its applications. We have chosen the general setting of a Gaussian family of random variables associated with an arbitrary separable Hilbert space. Some of the applications are just briefly introduced, and we recommend the reader to look over additional references for more details.

In the first three chapters we introduce the fundamental operators: the derivative operator D; its adjoint δ, called the divergence operator; and the generator of the Ornstein-Uhlenbeck semigroup, denoted by L. Chapter 4 is devoted to proving the Meyer inequalities and the continuity of the divergence operator in the Sobolev spaces. The remaining chapters deal with a variety of applications of Malliavin calculus. First, in Chapter 5, we establish the general criteria for the existence and smoothness of densities for functionals of a Gaussian process. In Chapter 6 we discuss properties of the support of the law of a given Gaussian functional that can be proved using Malliavin calculus. Chapter 7 deals with the proof of Hörmander's hypoellipticity theorem. In Chapter 8 we discuss the use of the divergence as an anticipating stochastic integral with respect to the Brownian motion. This chapter also contains an introduction to the stochastic calculus with respect to the fractional Brownian motion, using techniques of Malliavin calculus. Chapter 9 presents some recent applications of Malliavin calculus to derive central limit theorems for multiple stochastic integrals, and Chapter 10 describes some applications of Malliavin calculus in mathematical finance.

Finally, I would like to express my gratitude to Oana Mocioalca, Frederi Viens, and Kazim Khan for encouraging me to prepare these lectures and for organizing a very stimulating and interesting workshop. I would also like to thank all the participants for their helpful remarks and questions.

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